Nonlocal damage models: Practical aspects and open issues

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Abstract:

The purpose of this paper is to discuss certain difficulties encountered when nonlocal constitutive models are applied in practical engineering analyses. Some of the problems remain unnoticed in theoretical studies but they may invalidate the results of large-scale simulations.

1. Introduction

Quasibrittle materials, such as concrete, rock, tough ceramics, or ice, are characterized by the development of large nonlinear fracture process zones. The progressive growth of microcracks and their gradual coalescence can be described by constitutive laws with softening, i.e., with a descending branch of the stress-strain diagram. In the context of standard continuum mechanics, softening leads to serious mathematical and numerical difficulties. The boundary value problem (BVP) becomes ill-posed, and numerical solutions exhibit a pathological sensitivity to the computational grid (finite-element mesh). Well-posedness of the BVP and objectivity of the numerical results can be restored by enriching the continuum theory. Regularization techniques that supply additional information on the internal structure of the material can enforce a realistic and mesh-independent size of the region of localized strain; therefore they are called localization limiters.

A wide class of localization limiters is based on the concept of a nonlocal continuum. Generally speaking, the nonlocal approach consists in replacing a certain state variable by its nonlocal counterpart obtained by weighted averaging over a spatial neighborhood of each point under consideration. This idea originally appeared in elasticity (Eringen, 1966). Its early extensions to strain-softening materials, leading to the so-called imbricate continuum (Bažant, 1984), were later improved by the nonlocal damage theory (Pijaudier-Cabot and Bažant, 1987) and adapted for concrete (Saouridis and Mazars, 1992). Nonlocal formulations have been elaborated for a wide spectrum of models. Their detailed overview is out of scope of the present paper; the reader can find a number of references in Bažant and Planas (1998).

2. Nonlocal Rotating Crack Model

In the present study, we shall use the nonlocal version of the rotating crack model as formulated by Jirásek and Zimmermann (1998). This model can capture the cracking-induced anisotropic degradation of the material stiffness while remaining relatively simple compared to general anisotropic damage formulations. The starting point is the standard smeared-crack concept based on a decomposition of the total strain into an elastic part and an inelastic part due to cracking. The cracking strain appears only after the stress has reached a certain critical level, usually specified by a Rankine-type criterion in terms of the maximum principal stress. Crack opening and sliding contribute only to the strain normal to the crack and to the shear strain but not to the strain parallel to the crack. The normal and shear cracking strains are linked to the normal and tangential components of the traction transmitted by the crack. It is assumed either that the crack direction remains constant (fixed-crack concept) or that it can vary (rotating-crack concept). Here we consider the latter case, in which the principal axes of stress and strain are kept aligned. The tangential components of cracking strain and crack traction remain zero, and the softening process is described by a simple relation between the normal traction across the crack and the inelastic strain due to crack opening. The crack opening strain is an internal variable that can be eliminated, either analytically (for a linear softening law), or numerically (for more general laws). The total stress-strain law can be written in the secant format resembling damage mechanics,

$$\boldsymbol{\sigma} = \boldsymbol{D}(\boldsymbol{\varepsilon})\,\boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{\sigma}$ = stress, $\boldsymbol{\varepsilon}$ = strain, and $\boldsymbol{D}(\boldsymbol{\varepsilon})$ = secant (damaged) material stiffness. The nonlocal version of the model replaces (1) by

$$\boldsymbol{\sigma} = \boldsymbol{D}(\bar{\boldsymbol{\varepsilon}})\,\boldsymbol{\varepsilon} \tag{2}$$

where

$$\bar{\boldsymbol{\varepsilon}}(\boldsymbol{x}) = \int_V \alpha(\boldsymbol{x},\boldsymbol{\xi})\boldsymbol{\varepsilon}(\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi} \tag{3}$$

is the nonlocal strain, and $\alpha(\boldsymbol{x},\boldsymbol{\xi})$ = nonlocal weight function, decaying with the growing distance between points $\boldsymbol{x}$ and $\boldsymbol{\xi}$.

The subsequent sections shall discuss certain difficulties encountered when the nonlocal approach is applied to structural models with a general geometry and loading. The examples have been calculated with the nonlocal rotating crack model but many conclusions are valid for other damage-type models as well.

3. Spurious Shifting of Process Zone due to Gravity Forces

The growth and coalescence of microcracks in the fracture process zone eventually lead to the formation of a stress-free macroscopic crack. Traditionally, this has been taken into account by softening laws for which the stress across the crack tends to zero, either at a finite value of cracking strain, or asymptotically. The latter case is more convenient from the numerical point of view, since the material keeps a nonzero stiffness and the secant stiffness matrix of the structure remains regular. A typical example is the exponential softening law

$$\sigma = f_t \exp\left(-\frac{\varepsilon_c}{\varepsilon_f}\right) \tag{4}$$

where σ = normal stress transmitted by the crack, f_t = uniaxial tensile strength, ε_c = cracking strain, and ε_f = material parameter related to the fracture energy.

The structure and evolution of the fracture process zone simulated with various nonlocal damage models were studied by Jirásek (1998a) on a simple one-dimensional test problem. It was shown that the zone of increasing local strain gets thinner as the loading process continues while the zone of increasing nonlocal strain keeps an approximately constant width. The secant material stiffness and the residual strength tend to zero in a finite zone whose thickness corresponds to the support diameter of the nonlocal weight function. This zone must always contain several finite elements, otherwise the nonlocal interaction would not be properly captured by the numerical model. The degradation of the residual strength in a band containing several elements across its thickness causes serious numerical problems if the structure is subjected to distributed body forces. This is the case, e.g., in the analysis of gravity dams, where the effect of gravity forces is essential and cannot be neglected. The presence of gravity forces inside the fracture process zone leads to spurious shifting of the center of the zone and finally to the divergence of the equilibrium iteration process. This is documented in Fig. 1.

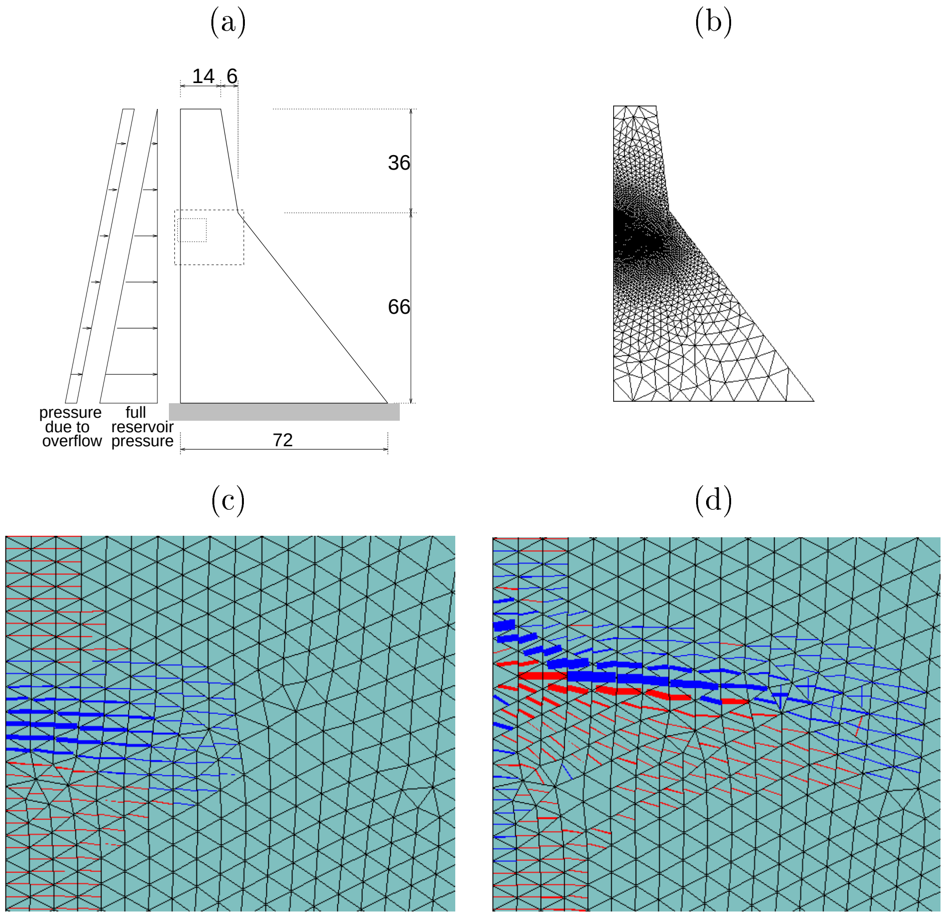

Figure 1: Analysis of a gravity dam: (a) Geometry and loading, (b) finite element mesh, (c-d) evolution of the crack pattern (close-up area is marked in (a) by a dotted rectangle)

A gravity dam with dimensions corresponding to the well-known Koyna Dam is loaded by its own weight, full reservoir pressure, and an increasing hydrostatic pressure due to reservoir overflow (Fig. 1a). The computational mesh, consisting of 4295 linear triangular elements, is shown in Fig. 1b. In a simulation using the nonlocal rotating crack model with exponential softening, the initially diffuse cracking zone at the upstream face eventually localizes and propagates into the interior of the dam (Fig. 1c). Blue and red rectangles represent opening and closing smeared cracks, resp. When a critical degradation level is reached at the mouth of the macroscopic crack, the (constant) body forces start pulling the material down, which results into a shifting of the numerical crack (Fig. 1d) and later into the loss of convergence.

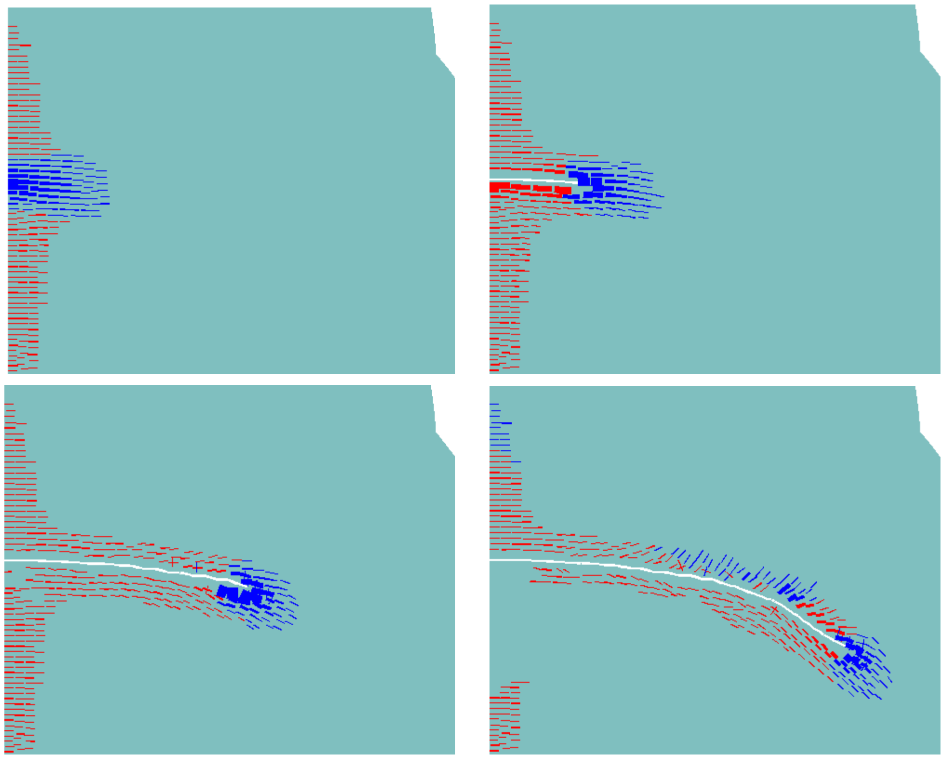

Figure 2: Fracture process zone development and propagation of a discrete crack modeled as a displacement discontinuity embedded in finite elements (close-up area is marked in Fig. 1a by a dashed rectangle)

An elegant remedy is a gradual transition from a smeared to a discrete description of material failure. Here we exploit the recently proposed combination of the nonlocal model with strong discontinuities embedded into the interior of finite elements (Jirásek, 1998b). This approach is appealing from the physical point of view. It is intuitively clear that diffuse damage at early stages of material degradation is adequately described by a model dealing with inelastic strain while highly localized fracture is better modeled by a displacement discontinuity. It can also be argued that, as fracture progresses, long-range interaction between material points becomes more difficult and finally impossible. This would be best reflected by a nonlocal model with an evolving (decreasing) characteristic length. However, such a model would be computationally very expensive, since the interaction weights for all interacting pairs of integration points would have to be continuously recomputed. The approach proposed in Jirásek (1998b) can be considered as a reasonable approximation, with a constant interaction length in the early stage of degradation and a zero interaction length in the final stage.

The simulation of a gravity dam has been repeated using the model with transition from nonlocal rotating cracks to embedded displacement discontinuities (discrete cracks). The transition takes place when the cracking strain reaches a certain critical value, and the softening laws describing the smeared and discrete parts of the model are matched such that a smooth transition and correct energy dissipation are ensured. The initial cracking pattern is the same as before, but soon after localization the first discrete crack appears and propagates along the center of the fracture process zone (see the white lines in Fig. 2). The partially damaged material in the wake of the discrete crack unloads, and its residual strength does not diminish anymore. Therefore, the correct crack trajectory can be captured without any numerical instabilities.

4. Pathological Interaction Between Tensile and Compressive Regions

Another interesting issue related to nonlocal damage models is the spurious interaction between tensile and compressive regions. This phenomenon is illustrated by the example in Fig. 3. A rectangular sheet with a pre-existing slit-like crack is loaded by pressure applied on the crack faces. Only one fourth of the specimen is simulated; see Fig. 3a. Due to the specific type of loading, the region of high tensile stresses around the crack tip is very close to the compressed region around the crack faces. If the usual nonlocal formulation is used, two spurious interaction effects appear. Firstly, the nonlocal strains just at the crack tip can be smaller than at a certain distance from the crack tip, and so the first numerical crack does not necessarily ap-

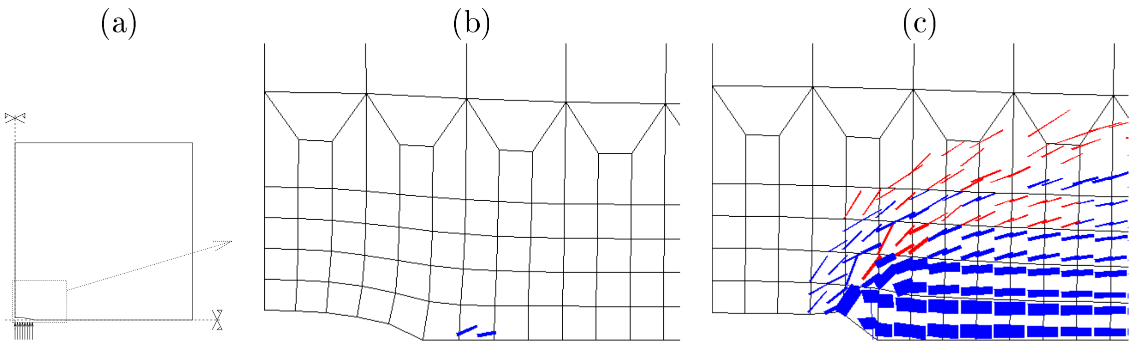

Figure 3: Cracked plate loaded on the crack faces: (a) geometry and loading (1/4 model), (b) initial stage of simulation (displacements exaggerated 500×), (c) final stage of simulation (displacements exaggerated 100×)

pear in the element closest to the tip (Fig. 3b). Secondly, as the macroscopic crack opens wide, the nonlocal strains in the compressed region can become positive (even though the local strains are of course negative), and cracking appears below the compressive load (Fig. 3c). It should be emphasized that this cracking is not primarily caused by the lateral expansion of the compressed material but by the proximity of a region that experiences large tensile strains. This spurious effect finally leads to a completely wrong failure mechanism—instead of splitting of the specimen we obtain compressive failure at the faces of the initial crack.

5. Concluding Remarks

We have briefly described and illustrated two aspects of nonlocal damage models that are not apparent in simple academic examples, namely spurious shifting due to gravity forces and pathological interaction between tensile and compressive regions. Due to the limited size of this paper, other related issues are left for the oral presentation.

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References

Bažant, Z.P. (1984). "Imbricate continuum and its variational derivation." *J. Eng. Mech. ASCE* **110**, 1693–1712.

Bažant, Z.P., and Planas, J. (1998). *Fracture and Size Effect in Concrete and Other Quasibrittle Materials.* CRC Press, Boca Raton, FL.

Eringen, A.C. (1966). "A unified theory of thermomechanical materials." *Int. J. Eng. Sci.* **4**, 179–202.

Jirásek, M. (1998a). "Comparison of nonlocal models for damage and fracture." LSC Internal Report 98/02, Swiss Federal Institute of Technology at Lausanne, Switzerland.

Jirásek, M. (1998b). "Embedded crack models for concrete fracture." In *Computational Modelling of Concrete Structures*, ed. by R. de Borst et al., Balkema, Rotterdam, pp. 291–300.

Jirásek, M., and Zimmermann, T. (1998). "Rotating crack model with transition to scalar damage." *J. Eng. Mech. ASCE* **124**, 277–284.

Pijaudier-Cabot, G., and Bažant, Z.P. (1987). "Nonlocal damage theory." *J. Eng. Mech. ASCE* **113**, 1512–1533.

Saouridis, C., and Mazars, J. (1992). "Prediction of the failure and size effect in concrete via a bi-scale damage approach." *Eng. Computations* **9**, 329–344.