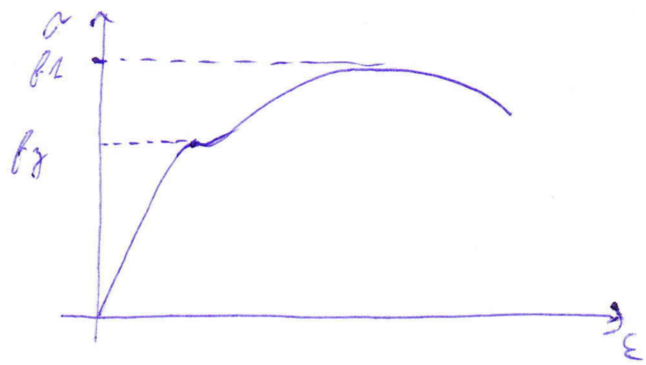
# VIII PLASTICITY

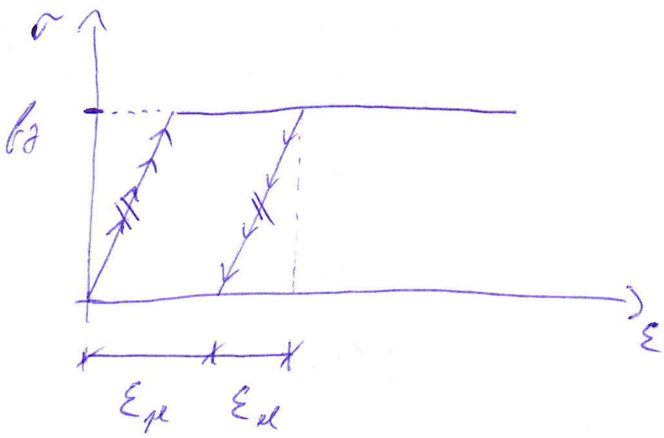


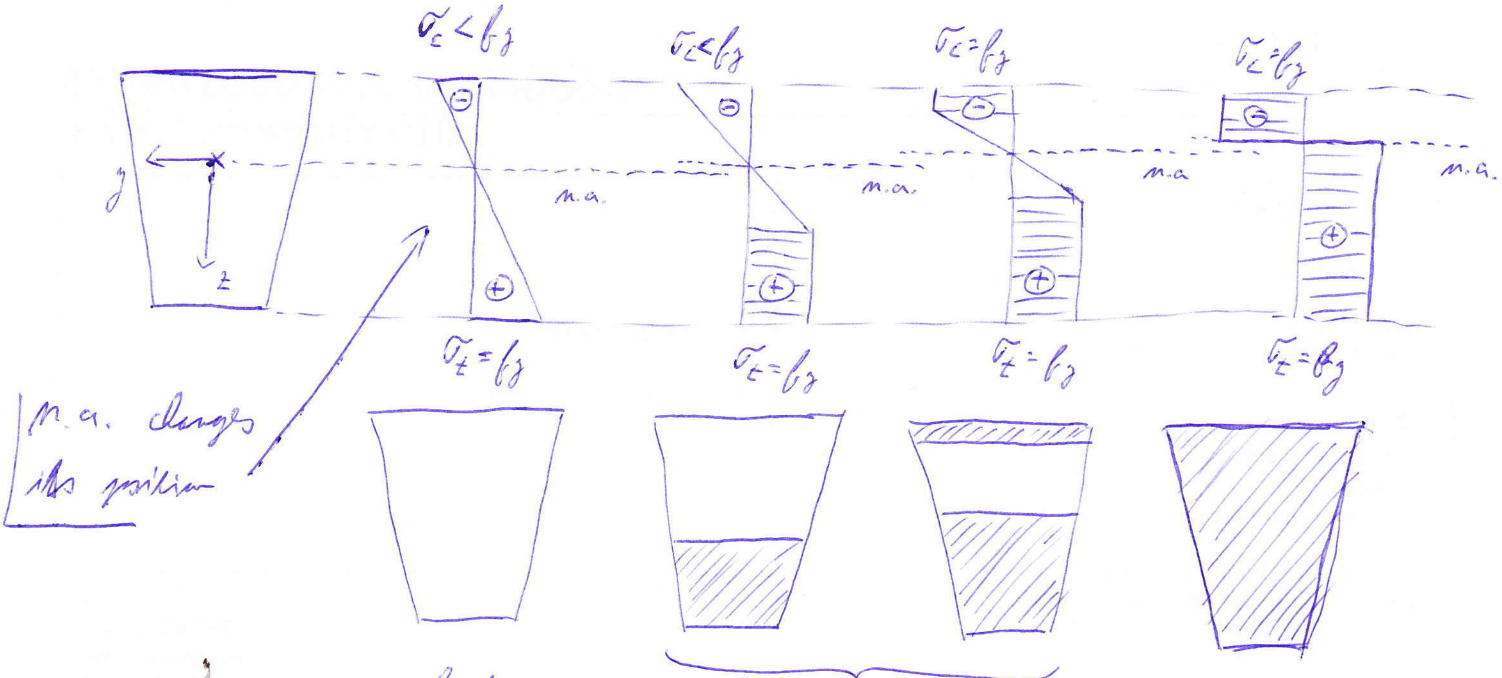
- by additional loading of the elastically loaded beam, the normal stress $\sigma_x$ reaches yield strength $f_y$

• real stress - strain diagram




• ideal elastic - plastic diagram ( for more simple calculations)






elastic state

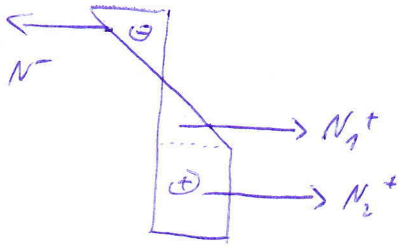
elastic - plastic state

plastic state

load-bearing capacity of the C/S is then defined by: $M_{el}$ $M_{elpl}$ $M_{pl}$

- simple and important condition for all the states: !

$$N^+ = N^-$$




$$\sigma_t \cdot A_t = \sigma_c \cdot A_c$$

areas of C/S in tension / compression

## $M_{el}$

$$M_{el} = W_{min} \cdot f_y$$

where $W_{min} = \min \left\{ \frac{I_y}{e_u} ; \frac{I_y}{e_l} \right\}$

$e_u$ - distance from c.g. to upper fibres

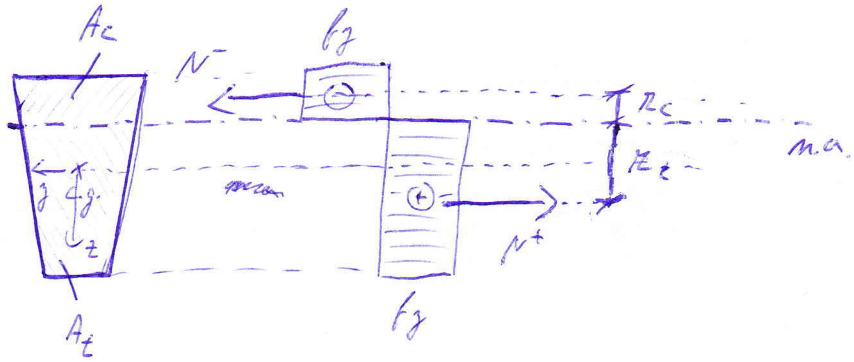
$e_l$ - —— " —— lower fibres

## $M_{el,pl}$

$$M_{el,pl} = \iint_A \sigma_x \cdot z \, dA$$

→ you'll understand in example, simple...

## $M_{pl}$




$$M_{pl} = f_y \cdot \underbrace{A_t \cdot r_t}_{S_{y,t}} + f_y \cdot \underbrace{A_c \cdot r_c}_{S_{y,c}}$$

$$M_{pl} = f_y \left( S_{y,t} - S_{y,c} \right)$$

because $r_c$ is negative

$$\underline{M_{pl} = f_y \cdot W_{pl}}$$

where $W_{pl} = |S_{y,t}| + |S_{y,c}|$

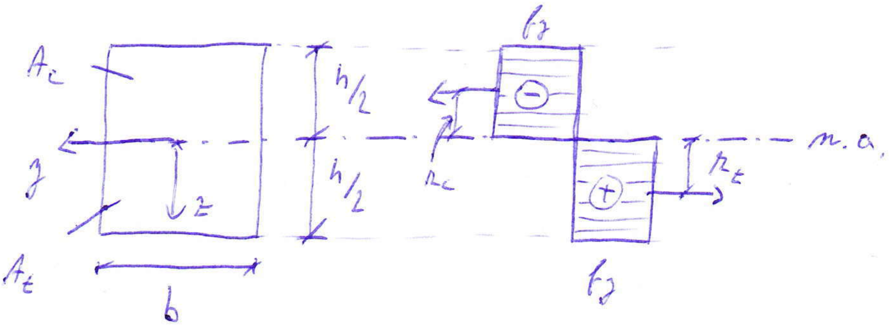
↑ plastic section modulus

$N^+ = N^-$

$\cancel{f_y} \cdot A_t = \cancel{f_y} \cdot A_c$

$\underline{\underline{A_t = A_c}}$

**ex 1** determine $W_{pl}$ of the c/s




NOTE: in case of double-symmetric c/ss, the n.a. doesn't change its position (X general c/s)

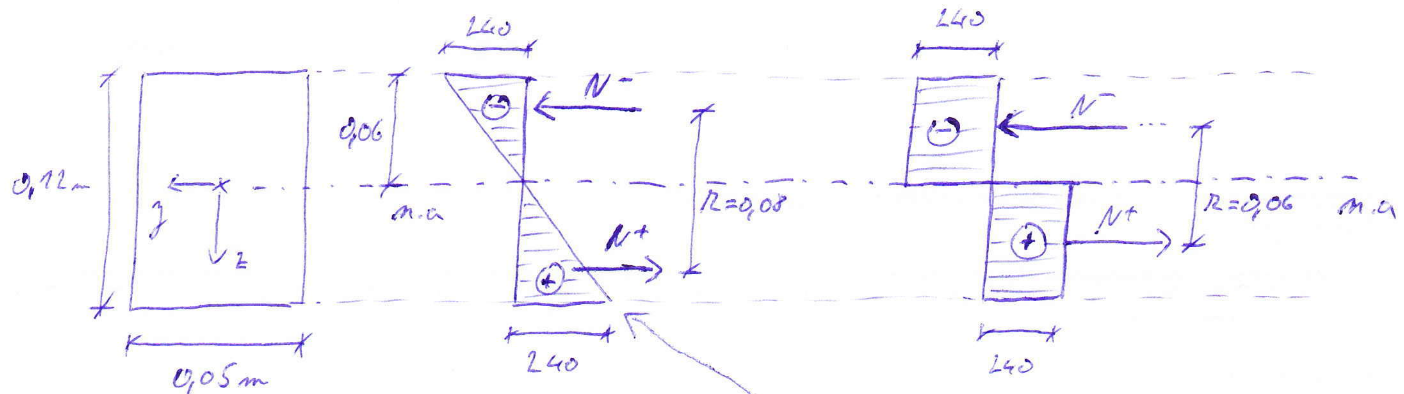
$$W_{pl} = |S_{y,t}| + |S_{y,c}| = \underbrace{b \cdot \frac{h}{2}}_{A_t} \cdot \underbrace{\frac{h}{4}}_{r_t} + \underbrace{b \cdot \frac{h}{2}}_{A_c} \cdot \underbrace{\frac{h}{4}}_{r_c} = \underline{\underline{\frac{bh^2}{4}}}$$

ex 2

a) C/S is loaded by bending moment $M_y = 36{,}8\ kNm$.

determine if the C/S is in elastic or plastic state

$f_y = 240\ MPa$




① we determine $M_{el}$

$$N^+ = \underbrace{\frac{1}{2}\cdot 240 \cdot 0{,}06}_{\text{area of ⊕ part of diagram}} \cdot \underbrace{0{,}05}_{\text{width of C/S}} = 0{,}36\ MN = |N^-|$$

$$M_{el} = N^+ \cdot R/2 + N^- \cdot R/2 = N^+ \cdot R/2 + N^+ \cdot R/2 = N^+(R/2 + R/2) = N^+ \cdot R =$$

$$= 0{,}36 \cdot 0{,}08 = 0{,}0288\ MNm = 28{,}8\ kNm < M_y = 36{,}8\ kNm$$

$$\left( \text{or alternatively: } M_{el} = f_y \cdot W_{min} = f_y \cdot \frac{I_y}{e_n} = f_y \cdot \frac{\frac{bh^3}{12}}{h/2} = f_y \cdot \frac{bh^2}{6} = = 240 \cdot \frac{0{,}05 \cdot 0{,}12^2}{6} = 0{,}0288\ MNm \right)$$

$\Longrightarrow$ C/S is not in elastic state

2 we determine $M_{pl}$

$$N^+ = \underbrace{240 \cdot 0{,}06}_{\text{area of (+) part of diagram}} \cdot \underbrace{0{,}05}_{\text{width of C/S}} = 0{,}72\ MN = |N^-|$$

$$M_{pl} = N_i^+ \cdot r/_2 + N^- \cdot r/_2 = N^+ \cdot r = 0{,}72 \cdot 0{,}06 = 0{,}0432\ MNm = 43{,}2\ kNm$$

$$M_{pl} = 43{,}2\ kNm > M_y = 36{,}8\ kNm$$

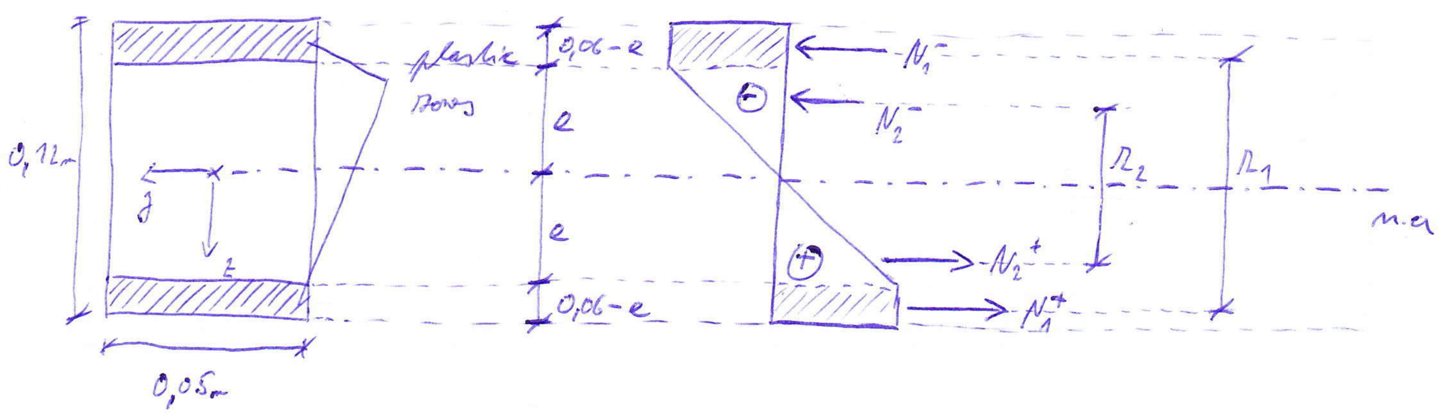
$$\left( \text{or alternatively:} \quad M_{pl} = f_y \cdot W_{pl} = f_y \cdot \underbrace{\frac{bh^2}{4}}_{\text{from ex 1}} = 240 \cdot \frac{0{,}05 \cdot 0{,}12^2}{4} = 0{,}0432\ MNm \right)$$

$\Rightarrow$ C/S is not in plastic state

$$M_{el} < M_y < M_{pl}$$

$\Rightarrow$ C/S is in elastic - plastic state

b) $\rightarrow$ it means that the C/S looks like this




$\hookrightarrow$ determine the height of the elastic zone „e"

$$M_{el,pl} = M_y = 0{,}0368 = N_1^+ \cdot r_1/2 + N_1^- \cdot r_1/2 + N_2^+ \cdot r_2/2 + N_2^- \cdot r_2/2 =$$

$\Rightarrow$ it is obvious that $N_1^+ = |N_1^-|$ and $N_2^+ = |N_2^-|$ $\Rightarrow$

$$= N_1^+ \cdot r_1 + N_2^+ \cdot r_2$$

$$N_1^+ = 240 \cdot (0{,}06 - e) \cdot 0{,}05 = 12(0{,}06 - e)$$

$$N_2^+ = \frac{1}{2} \cdot 240 \cdot e \cdot 0{,}05 = 6e$$

$$r_1 = 0{,}12 - \frac{(0{,}06 - e)}{2} \cdot 2 = 0{,}06 + e$$

$$r_2 = \frac{2}{3} e \cdot 2 = \frac{4}{3} e$$

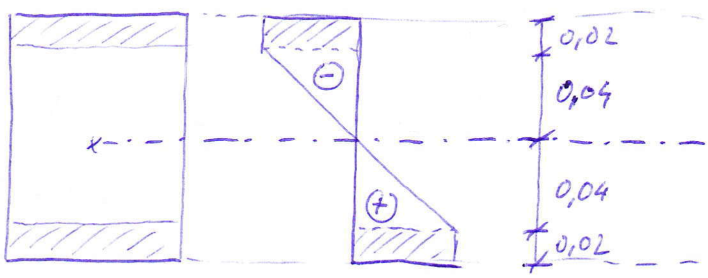
$$0{,}0368 = M_{el,pl} = 12(0{,}06 - e) \cdot (0{,}06 + e) + 6e \cdot \frac{4}{3} e = 12(0{,}06^2 - e^2) + 8e^2 =$$

$$= 0{,}0432 - 12e^2 + 8e^2 = 0{,}0432 - 4e^2$$

$$0{,}0368 = 0{,}0432 - 4e^2$$

$$4e^2 = 0{,}0064$$

$$\underline{\underline{e = 0{,}04\ m}}$$

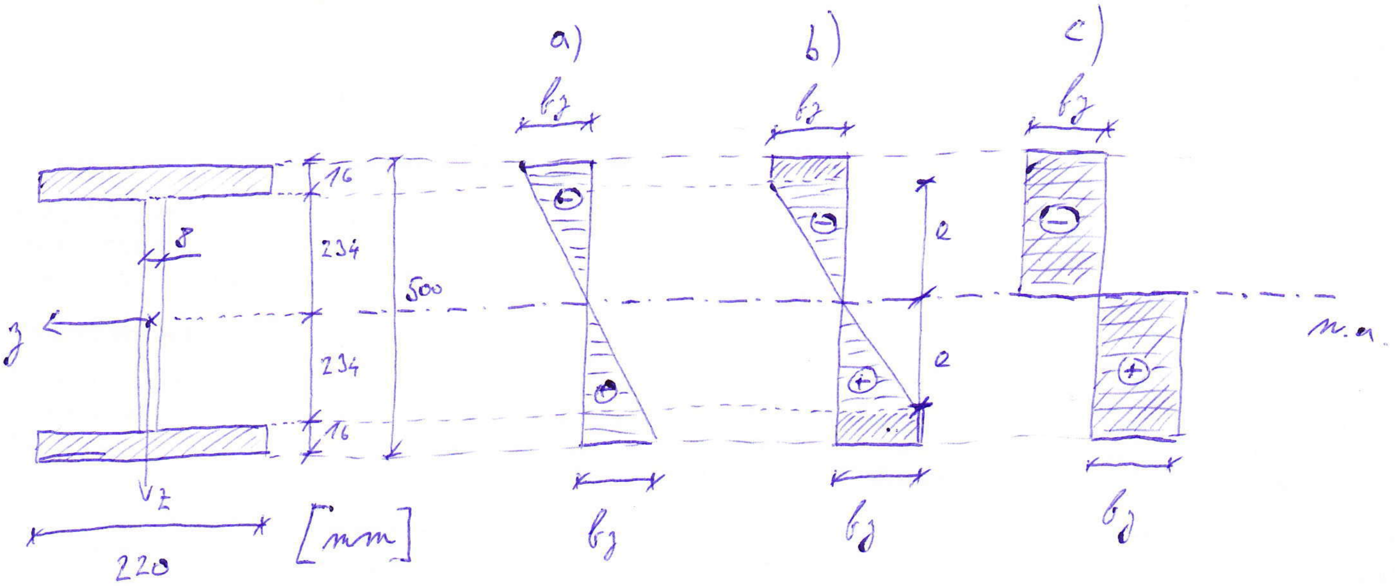


1) determine:

a) $M_{el}$ (hint: use the formula $M_{el} = f_y \cdot W_{min}$)

b) $M_{el,pl}$ if the flanges are plastified

c) $M_{pl}$




$f_y = 240\ MPa$

2) a beam is loaded in such a way that just flanges are plastified. Determine the magnitude of force F necessary to reach this state. $f_y = 300\ MPa$