A Discontinuity-Enriched Finite Element Method for Crack Growth in Brittle Materials

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The standard finite element method (FEM) can be used to model brittle fracture, but cracks have to be aligned with edges of finite elements. As an alternative, the eXtended/Generalized Finite Element Method (XFEM/GFEM) elegantly decouples the crack geometry from the discretization [1]. However, some important properties of standard FEM are lost, e.g., the condition number of the stiffness matrix can be arbitrarily high when cracks are very close to standard FEM nodes, and standard degrees of freedom (DOFs) loose their physical meaning. The computer implementation is also far from trivial, since non-standard procedures are required for prescribing nonzero essential boundary conditions (BCs) when enrichments are nonzero along Dirichlet boundaries (e.g., shifting or penalty formulations). Finally, the formulation is very intricate when dealing with complex discontinuity cases such as branching and merging.

As an alternative to XFEM/GFEM, the Discontinuity-Enriched Finite Element Method (DE-FEM) [2, 3] can solve problems with both material interfaces and cracks and with a unified formulation. DE-FEM places enriched DOFs to nodes created directly along discontinuities, thereby solving many issues of XFEM/GFEM. Because enrichment functions in DE-FEM vanish at standard mesh nodes, standard DOFs retain their physical meaning and there are no issues in blending elements. The method is also stable with regards to the condition number, and nonzero essential BCs can be enforced strongly. Finally, DE-FEM's computer implementation in displacement-based FEM codes is straightforward. DE-FEM thus keeps the most salient feature of XFEM/GFEM—decoupling between mesh and cracks—while retaining some desirable properties of standard FEM.

DE-FEM has only been studied so far for stationary cracks under stationary loading cases. In this presentation, we demonstrate DE-FEM for both quasi-static and dynamic brittle fracture propagation, including branching and merging. For dynamic fractures, implicit time integration methods are used and their parameters are tuned to mitigate numerical instabilities. Dynamic stress intensity factors are obtained by a path-independent dynamic interaction integral [4] and propagation directions are determined according to maximum circumferential stress criterion. The methodology is demonstrated on a set of complex crack growth problems.

References

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