

Finite Element implementation of the Coupled Criterion based on the Principle of Minimum Total Energy subjected to a Stress Condition to predict crack onset and growth

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In the framework of Finite Fracture Mechanics (FFM), Leguillon introduced the coupled criterion of the FFM (CCFFM) [1], which is the base of this work. According to CCFFM, the stress and energy criteria are two necessary conditions to allow an abrupt onset of a crack in a finite extension. A new formulation of the CCFFM based on the principle of minimal total energy subjected to a stress condition (PMTE-SC), suitable for solving complex fracture problems, was introduced by Mantič [2]. This is primarily due to the fact that it is better suited for a generic computational implementation of a load-stepping technique that addresses issues with the initiation and propagation of multiple cracks. In addition, the total energy can be expressed as a separately convex function in the displacements and the damage variable fields. This makes it possible to apply optimisation techniques that are both efficient and stable to achieve the objective of minimising the total energy with constraints.

For implementing PMTE-SC in FEM code ABAQUS, we use UINTER, a subroutine used to define the interaction between two surfaces. It is called at points on the slave surface of a contact pair with a user-defined constitutive model describing the interaction between the surfaces. Here, the interaction between the cracked surfaces is modelled by a continuous distribution of springs with a linear elastic behaviour, Linear Elastic Spring-Surface Interaction (LES-SI). Therefore, the springs interact with the two surfaces of the crack, which act linearly during tension and shear. The mixed mode constitutive law of the active surface springs was described, e.g., by Mantič et al. [3]. The change of the potential energy of the system is calculated by the code using the incremental virtual crack closure technique (VCCT), as it provides accurate results. It is based on the idea that the change in potential elastic energy due to specific crack growth is identical to the work necessary to close the crack with an equivalent extension. In the context of finite

fracture mechanics, this idea is applied from an incremental perspective. Several fundamental problems for crack onsite and propagation under quasistatic load in mode I, such as a circular hole in an infinite plate under remote tensile load and biaxial load, rhombus hole specimens under compression, and others in mixed mode, are solved. Therefore, this method opens new possibilities for studying the onset and propagation of cracks.

In this work, a new method to characterise the crack onset and propagation has been developed based on the Coupled Criterion of Finite Fracture Mechanics (CCFFM). This method predicts an instantaneous crack onset or propagation without requiring an infinitesimal crack growth. This allows the appearance of several fractures simultaneously in the same problem. A computational algorithm based on the new formulation, i.e., PMTE-SC, has been implemented using the Finite Element Method in ABAQUS and Python scripts.

References

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