# Backward mode sensitivity analysis based multi-scale phase-field modeling

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In this contribution, we address the issue of generalization of formulation and efficient numerical treatment of multi-scale phase-field modeling of brittle fracture. The high numerical efficiency of the proposed formulation relies on backward mode sensitivity analysis. Sensitivity analysis has become an indispensable part of modern computational algorithms. Nowadays, the automation of sensitivity analysis enables efficient evaluation of sensitivities that are exact except for the round of errors. We propose a hybrid symbolic-automatic differentiation approach with code-to-code transformation and simultaneous stochastic expression optimization implemented in AceGen (www.fgg.uni-lj.si/symech/) as one of the most efficient approaches.

The automatic differentiation-based form (ADB form) of a classical phase-field formulation of brittle fracture [1] will be presented first. Next, the paper presents a unified approach to the development of an arbitrary mesh-in-element (MIEL) or $FE^2$ [2] computational scheme for two scale phase-field formulations. Implementation is based on efficient first-order (for $FE^2$ formulations) and second-order (for MIEL formulations) analytical sensitivity analysis, for which automatic-differentiation-based formulation [2] of essential boundary condition sensitivity analysis is derived. A generalized essential boundary condition sensitivity analysis-based implementation of $FE^2$ and MIEL multi-scale methods is derived as an alternative to standard implementations of multi-scale analysis, where the calculation of Schur complement of the microscopic tangent matrix is needed for bridging the scales. A fully consistently linearized two-level path-following algorithm is introduced as a solution algorithm for the strongly nonlinear multi-scale problems. Sensitivity analysis allows each macro step to be followed by an arbitrary number of micro sub-steps while retaining quadratic convergence of the overall solution algorithm [2]. The approach reduces the need for recalculation of global problems, consequently reducing the overall computational time. The approach also increases the concurrency of micro problems which can significantly improve the overall speed of the execution in multi-processor and multi-core systems. The paper also compares the benefits and drawbacks of the second-order forward and backward automatic differentiation approaches when applied to multi-scale phase-field modeling.

## References

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