An efficient phase field implementation of fracture analysis of functionally graded materials

P. C. Sidharth[1*], **B. N. Rao**[1],

[1] Structural Engineering Laboratory, Indian Institute of Technology Madras, Chennai, Tamil Nadu, India, 600036 sidharthpccalicut@gmail.com

Innovative materials known as functionally gradient materials (FGM) exhibit gradual spatial changes in their properties. However, compared to homogeneous materials, FGMs' fracture behaviors are more complex due to graded distributions of their material properties, which makes modeling their failure process extremely difficult. Phase field approaches [1] completely avoid the necessity for monitoring discontinuities explicitly in the domain and the remeshing process, in contrast to other numerical methods like the extended finite element method (XFEM). Low computing efficiency, however, continues to be a problem for the phase-field model's numerical simulation [2], especially in FGMs where material properties are to be evaluated at the integration point level.

The current work makes use of special shape functions and informed mesh refinement schemes to render the fracture computations faster at the same time retaining accuracy in the prediction. To this end, exponential shape finite element shape functions [3] are introduced instead of standard bilinear shape functions conventionally used in finite element calculations. Bilinear shape functions offer a linear interpolation of solution variables inside an element and require closely spaced refined meshes to accurately resolve sharp gradients. Sharp gradients of solution variables are expected along the crack propagation path itself. On the other hand, exponential shape finite element shape functions allow sharp changes of solution variables inside elements. This exponential characteristic can be made use of in reducing the mesh refinement level at crack propagation paths. Although exponential finite element shape functions can be very useful, they offer good approximations only when the shape functions are oriented concerning the crack propagation path [4]. This study suggests a learned orientation scheme for these shape functions, informed by an approximate analysis using bilinear shape functions carried out during the analysis itself. Functionally graded materials, in the interest of fracture predictions, have variations in stiffness, fracture resistance, and Poisson's ratio dependent on special coordinates. A homogenization strategy can be used to infer this from the spatial variation of the volume fractions of the constituent materials. Fracture simulations are carried out in functionally graded plates with different gradation schemes, and their effect on fracture resistance is investigated. Computational efforts incurred in the present implementation are compared with existing schemes using bilinear shape functions.

Several pragmatic examples are considered that show the effect of a material gradient, crack location, and the resulting mode mixity. The results obtained provide fundamental and quantitative insight into the role of the material property gradation on the crack propagation response. The ability of the phase field model in conjunction with exponential finite element shape functions to predict complex crack patterns is proven.

References

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