Strain localization in elasto-plastic beams based on a phase-field approach

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Structures composed of strain softening materials are prone to failure because of strain localization in relative small critical zones. As loads increase, structures develop localization bands with thin thicknesses, which can be physically separated from the rest of the volume by interfaces. The strain rate becomes discontinuous in correspondence with the band.
It is also possible that strain localization occur in some structures that exhibit perfect plastic behavior as a result of a non-uniform distribution of forces inside them. This can be the case of a beam subjected to a non uniform distribution of the bending moment such that the inelastic curvature develops along the beam axis in a portion that contains a critical section, i.e. the section that experiences maximum bending moment. As the loading process continues, the critical section becomes fully plasticized, the curvature in the same section becomes discontinuous, and the so-called plastic hinge is formed.

As a result of the introduction of the ultimate limit state concept into several building codes for framed structures, engineers began to apply nonlinear structural analysis to these kinds of structures under static and dynamic conditions. Specifically, the nonlinear static analysis method, called *pushover* became very popular due to its ability to provide relevant information, including limit loads, displacement capacities, and overall ductility at an acceptable computational cost. In terms of the assessment of existing structures, this tool is particularly useful since it can be used to identify areas of potential weakness prior to the development of a rehabilitation plan.
The nonlinear finite element analysis can be performed assuming a distributed or lumped plasticity model for the beam element. In the distributed plasticity model, inelastic responses are diffused along a part of the beam whose lenght is comparable to the element length. The response of the element is determined by weighting the behavior at fixed sections, whose number and location are defined by the quadrature rule for numerical integration. The constitutive behaviour of the cross section is either formulated in accordance with classical plasticity theory in terms of stresses and strain resultants [1] or is explicitly derived by discretization of the cross section into fibers [2].
The present work is devoted to the implementation, in a finite element environment, of an elastoplastic Euler–Bernoulli beam element showing possible slope discontinuities at any position along the beam span, in the framework of a modified lumped plasticity. The theoretical treatment of the problem has been developed in a classical way making use of the principles of thermodynamics.
The strain jump at the fully plasticized section is regularized, i.e. smeared along the beam span, taking into account the length of the plastic hinge. This regularization is performed according to the phase-field approach presented in [3]. With respect to other phase-field models, the novelty consists in the homogenization of the inelastic strains through a weak Dirac delta function which, for a multi-axial problem, takes the shape of the Mumford-Shah functional.

References

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