An Extended Phase-Field Method (XPFM) for 3D Fracture Simulations

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The simulation of cracks and their initiation, propagation, branching and coalescence can be an important assistance to evaluate real structures and assess their robustness and resilience. However, for this the application to 3D may be necessary. Especially for more challenging cracking processes like branching and coalescence, this poses a difficulty, since complex crack geometries can occur. This makes the discrete representation of cracks in three dimensions highly complicated.

To remedy this, a smeared approach to cracking can be helpful, like within the phase-field method (PFM) [1]. Here, the crack is reproduced with the help of a scalar field, the phase-field, indicating the crack. Its propagation is calculated using an energy approach, preventing the need for additional crack propagation criteria. One drawback, however, is the high computational effort necessary to be able to approximate the phase-field, the displacement field and their respective gradients sufficiently accurate, which is amplified vastly in 3D.

With the presented extended phase-field method (XPFM) [2] this difficulty is overcome due to an ansatz transformation of the phase-field. Embedding a standard polynomial ansatz into an exponential function enables an improved approximation of the phase-field even if rather coarse meshes are employed. Still, the crack is allowed to develop freely with no restrictions with regard to the mesh geometry. The degrees of freedom themselves attain a level set type quality. This is useful when it comes to the displacement approximation, which requires special attention due to its high gradient across the crack. Here, similar to the extended finite element method (XFEM) [3], the displacement ansatz is enriched by a function, containing properties of the expected solution. Since the enrichment function is directly coupled to the phase-field ansatz, the steep slope across the crack can be reproduced, even without a discrete approximation of the crack.

Special attention is given to the integration method. The standard Gaussian approach is not sufficient here, due to the exponential ansatz space of the phase-field on one hand and due to the non-polynomial enrichment function of the displacement field on the other hand. Additionally, the location of the crack within the finite element is generally not known a priori. In 2D, an adaptive approach based on the even subdivision of the triangular elements is chosen. However, the adaptation to 3D of this scheme is not straightforward, since the subdivision of tetrahedral elements does not yield even-sized subtetrahedra. With iterative refinement, the subelements grow evermore distorted and the integration scheme becomes less efficient as a consequence. Therefore, an adjusted approach to the subdivision of the tetrahedra has to be taken [4].

The XPFM combines the advantages of the classical PFM and the XFEM and is therefore able to reduce the necessary number of degrees of freedom significantly in comparison to other methods while still producing accurate results. This is shown by its application to several examples for simulating fracture in 3D.

References

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