

## Multi-axial loadings in phase field model of fracture: Part 1

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Phase field modeling of fracture is gaining popularity in the fracture mechanics community, particularly for its ability to generate cracks with arbitrarily complex geometries and topologies in two and three dimensions without the need for ad hoc criteria. The model first introduced in [1] has a clear connection with Griffith's propagation criterion via Gamma convergence tools and further results [2] have shown that, in addition to propagation, it can quantitatively predict crack nucleation for mode-I loading. However, the initial model cannot reproduce with flexibility the experimentally measured strengths under multiaxial loads. Moreover, a modification is necessary to avoid the interpenetration of crack surfaces in compression and reflect the physical asymmetry of fracture behavior between tension and compression [3]. New models are often validated through different case studies that show specific potentials, making their comparison not immediate and analysis of limitations not straightforward. Among these contributions, some preserve the variational nature of the phase field model [3, 4, 5, 6] while others seek flexibility by stepping outside the variational framework [7]. The most popular variational solutions [3, 4] are based on elastic energy decompositions. This idea is adopted in [5, 6], justified through structured deformation theory.

In this first part, we present a study that sorts the wealth of literature based on specific criteria. We define these criteria as the ability to flexibly reproduce multiaxial strength and avoid interpenetration of crack faces, and so we perform a systematic review of some available models. Based on these concepts, we propose numerical benchmarks to evaluate the behavior of solutions in literature. In particular, the proposed tests provide an assessment of a phase field model for both nucleation and propagation.

### References

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