A phase-field model describing Paris' fatigue law

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Fatigue is one of the most critical and less predictable phenomena in nature, affecting the mechanical behavior of materials. Engineers currently use the Griffith's fracture law to assess the structural limits under monotonically increasing loads and Paris' law to estimate the structural life under cyclic loads. These two fundamental laws governing the failure of structures in the short and long run have often been seen as disconnected in the scientific literature. The first formal link between the two fields was established in some seminal, but somewhat unknown, papers by Marigo and coworkers; see [1, 2, 3] and citations therein. In [1], the authors present a detailed derivation of this link and its fundamental ingredients: in the case of a peeling test, it is shown how a sequence of cohesive fracturing processes due to cyclic loading at the micro time scale can accumulate to form a steady-state propagation at the macroscopic time scale. The prefixes micro and macro for the time scales are related to the small parameter d/L measuring the size of the cohesive zone with respect to the size of the structure. An implicit law is derived connecting the macroscopic time rate of the crack length to the ratio G/G_c between the actual energy release rate G and the material toughness G_c. The very same law reduces to Paris' fatigue law when G is much smaller than G_c, while it tends to the Griffith's law when G approaches G_c from below.

From these results [1, 2, 3], we have deduced a phase-field model having this dual capability of describing both sudden and accumulated fracture phenomena. The approach is based on the least energy principle where the total energy of the structure, to be minimized, is the sum of the elastic energy and a dissipation potential, taken as a power function of the newly created crack surface. Then, we implemented the model via FEniCSx [4], observing promising results: in a pre-notched rectangular sample with mode I cyclic loading conditions was possible to retrieve standard pre-assigned path cracks. The code was also tested in a less trivial case considering, instead, a mixed I-II mode solicitation, where we observed the initiation of the crack and its subsequent complex path propagation.

References

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