

## Dual-mesh approach to discretisation for phase-field fracture method

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The Phase-field (PF) method for fracture problems was the subject of extensive research during the last decade. Its popularity can be attributed to two advantageous properties that arise from the diffusive crack description and variational approach to fracture, namely simplicity and versatility.

However, some specific disadvantages are inherited from the method's regularized formulation that contains a length-scale parameter. Besides governing the PF diffusivity, often the length-scale is considered as a material parameter, coupled with the material's critical stress, leading to the fact that the length-scale cannot be chosen arbitrarily. As a smaller length-scale leads to sharper gradients of PF and material stiffness, mesh density requirements are also determined by the length-scale. Hence, the disadvantage comes from the fact that simulations with a small length-scale require very dense meshes, and therefore lead to large computational costs.

Several works addressed the problem of computational costs due to mesh requirements. The most obvious approach is managing a number of degrees of freedom by means of adaptive remeshing. So far, the most promising adaptive approaches are anisotropic ones [1]. Further, computational costs can be reduced by utilizing phase-field methods that exhibit better convergence, like the fourth-order methods [2] that have a smooth PF profile at the peak, but unfortunately require discretization with the  $C^1$  continuity of PF. Lastly, the utilization of special discretization techniques can be utilized. Herein, the work of Olesch [3] can be noted, where exponential shape functions were utilized for a better approximation of PF profile. On the other hand, Sargado [4] hypothesized that mesh sensitivity within the Finite Element Method (FEM) framework comes from the fact that FEM is not optimal since the PF profile contains a cusp. To resolve such a problem, a combined discretization with FEM for displacements and the Finite Volume (FV) method for PF was proposed, where PF data were stored in the center of the element/volume. The resulting method reduced spurious fracture

energy and showed better convergence in comparison to the standard FEM approach.

This contribution follows the ideas of Sargado [4], but instead of using FV, the new approach is based on the construction of a new (secondary) mesh from nodes positioned at the element centers of mesh used for displacements (primary mesh). The new mesh contains linear polygonal finite elements or linear triangular elements obtained by triangulation of polygons. The benefit over the FEM-FV approach is that the local part of fracture energy is not constant over the primary element, i.e., does not contain PF plateaus, and hence the local part of critical energy is not overestimated as in the FEM-FV approach. Results of benchmark examples show that the proposed method clearly leads to better convergence of critical force for problems with a sudden crack propagation, while for a stable crack propagation, results can be considered only marginally.

### References

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