Variational formulation with dissipation-gradient regularization for softening plasticity models

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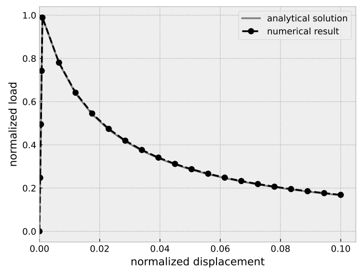
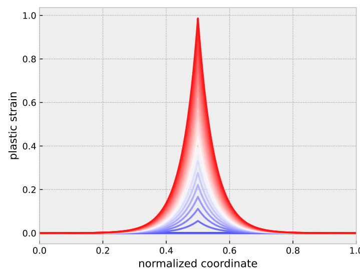
Constitutive plasticity models with softening effects require a regularization framework to describe strain localization in an objective way. One broad class of methods uses gradient theory, which includes both strain-gradient models and models with gradients of internal variables. In most cases, gradient terms are included as an additional contribution to the free-energy potential. However, these models have been shown to exhibit undesirable properties such as unlimited spreading of localization bands in later stages of the softening process [1, 2]. Alternatively, one may modify the expression of the dissipation potential by introducing additional gradient terms, as has been proposed to model size-dependent strengthening behavior of metals in agreement with micron-scale tests [3, 4]. To our knowledge, this strategy has not been explored for dealing with softening, and it is the focus of our work. We propose regularization of softening plasticity models obeying the normality rule using the gradient of the plastic strain increment in the dissipation potential, followed by an incremental variational formulation. We solve the corresponding optimization problem using dedicated algorithms for convex variational problems represented as discrete conic programs. We carry out numerical finite element simulations in one and two-dimensional settings, using nonlinear softening laws for von Mises plasticity. These examples illustrate the emergence and development of localization bands. Our contribution shows that the band width is bounded even in the vicinity of the ultimate state. Finally, we compare the numerical results with analytical solutions to support the well-posedness of both our dissipation-gradient regularization and its computational treatment.



(a)



(b)

Figure 1: Bar under traction after [1]. (a): Load-displacement responses. (b): Plastic strain profiles.

References

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