Griffith Fracture in Viscoelastic Elastomers Done Right

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It has been long established that the Griffith criticality condition

$$-\frac{\partial \mathcal{W}}{\partial \Gamma_0} = T_c \tag{1}$$

describes the growth of cracks in elastomers subjected to quasi-static mechanical loads.

The left-hand side $-\partial\mathcal{W}/\partial\Gamma_0$ in expression (1) stands for the change in total (stored and dissipated) deformation energy $\mathcal{W}$ in the elastomer with respect to an added surface area to the pre-existing crack Γ_0 under conditions of fixed deformation of those parts of the boundary that are not traction-free.

The right-hand side T_c is the so-called critical tearing energy. It is a characteristic property of the elastomer. Importantly, it is *not* a constant. Much like $\mathcal{W}$, it is a function of the loading history. More specifically, experiments have established that T_c can be written in the general form

$$T_c = G_c(1 + f_c).$$

In this expression, G_c denotes the intrinsic fracture energy, or critical energy release rate, associated with the creation of new surface in the given elastomer. It is a material constant, independent of time. Its value is in the relatively narrow range $G_c \in [10, 100]$ N/m for all common hydrocarbon elastomers. On the other hand, f_c is a non-negative function of the loading history that scales with the viscosity of the elastomer. Precisely how f_c — and hence T_c — depends on the loading history for arbitrary loading conditions has remained an open problem for decades rendering the Griffith criticality condition in its ordinary form (1) essentially useless.

In a recent contribution, Shrimali and Lopez-Pamies [1] have uncovered a general formula for f_c — and hence T_c — and in so doing they have determined that (1) can in fact be reduced to a form that involves *not* the historically elusive critical tearing energy T_c, but only the intrinsic fracture energy G_c of the elastomer. The result hinges on the following two elementary observations.

i. For any viscoelastic elastomer, the total deformation energy $\mathcal{W}$ in (1) can be partitioned into three different contributions:

$$\mathcal{W} = \underbrace{\mathcal{W}^{\rm Eq} + \mathcal{W}^{\rm NEq}}_{\text{stored}} + \underbrace{\mathcal{W}^{v}}_{\text{dissipated}}.$$

Here, $\mathcal{W}^v$ represents the part of the total energy that is dissipated by the elastomer via viscous deformation. On the other hand, the combination $\mathcal{W}^{\rm Eq} + \mathcal{W}^{\rm NEq}$ represents the part of the total energy that is stored by the elastomer via elastic deformation. In this combination, $\mathcal{W}^{\rm NEq}$ stands for the part of the stored elastic energy that will be dissipated eventually via viscous dissipation as the elastomer reaches a state of thermodynamic equilibrium. On the contrary, $\mathcal{W}^{\rm Eq}$ denotes the part of the stored elastic energy that the elastomer will retain at thermodynamic equilibrium.

ii. “Pure-shear” fracture tests of common hydrocarbon elastomers, as well as of more modern types of elastomers, consistently show — rather remarkably — that fracture occurs from the pre-existing crack in the specimens at a critical stretch that is independent of the stretch rate at which the test is carried out.

Precisely, by combining the above two observations, Shrimali and Lopez-Pamies [1] have shown that the Griffith criticality condition (1) can be reduced to the fundamental form

$$-\frac{\partial \mathcal{W}^{\rm Eq}}{\partial \Gamma_0} = G_c. \tag{2}$$

In this presentation, I will talk about the derivation of the criticality condition (2) and its use to explain two of the most distinctive fractures tests for viscoelastic elastomers: the delayed fracture test and the trousers fracture test.

References

[1] B. Shrimali, O. Lopez-Pamies, The “pure-shear” fracture test for viscoelastic elastomers and its revelation on Griffith fracture, *Extreme Mechanics Letters* 58 (2023) 101944.