Some remarks on a new space of generalised functions of bounded variation

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The variational approach to rate-independent evolution problems relies on a time discretization scheme, where the approximate solution at a given time is obtained by solving an incremental minimum problem which involves the solution at the previous time. In this framework, the study of crack growth in linearly elastic-perfectly plastic materials in the small strain regime leads to incremental minimization problems that involve the crack as well as the elastic and the plastic parts of the strain, which in general lack of regularity. In the generalized antiplane case the crack is represented by a rectifiable set, the elastic part of the strain is a square integrable function, while the plastic part is a measure. It is convenient to express these problems in terms of a functional which depends on the displacement. However, since the functional to be minimized does not provide a control on the jump of the displacement, there are boundary conditions for which the minimum problem does not have a solution in the space of functions with bounded variation.

In order to obtain the existence of a solution to such problems we introduced in [1] a new space of generalized functions of bounded variation. We studied the fine properties of the functions belonging to this space and proved a compactness result. We also proved the lower semicontinuity of the functionals involved in the incremental minimum problems. Moreover, by adapting a nontrivial argument introduced by Friedrich in [3], we showed that every minimizing sequence can be modified to obtain a new minimizing sequence that satisfies the hypotheses of our compactness result. Therefore the Direct Method of the Calculus of Variations can be applied providing the existence of a solution to the incremental minimum problems.

In this talk I would also like to mention some results on the Gamma convergence of related functionals recently obtained together with Gianni Dal Maso [2]. In particular, we proved that a class of integral functionals defined on this space is invariant under Gamma convergence. This result can be applied to the study of some homogenization problems, both in the periodic and in the stochastic case.

References

[1] G. Dal Maso, R. Toader, A new space of generalised functions with bounded variation motivated by fracture mechanics, Nonlinear Differ. Equ. Appl. 29 (2022) Article number: 63.

[2] G. Dal Maso, R. Toader, Γ-convergence and integral representation for a class of free discontinuity functionals, Preprint SISSA (2023) .

[3] M. Friedrich, A compactness result in $GSBV^p$ and applications to Γ-convergence for free discontinuity problems, Calc. Var. Partial Differential Equations 58 (2019) Paper No. 86.