# Free crack propagation through nonlocal modeling

**R Lipton**[1]*,

[1] Department of Mathematics, Louisiana State University, Baton Rouge, LA, 70803 USA lipton@lsu.edu

The fracture of brittle solids is a particularly interesting collective interaction connecting both large and small length scales. Apply enough stress or strain to a sample of brittle material and one eventually snaps bonds at the atomistic scale leading to fracture of the macroscopic specimen. In this talk a nonlocal mesoscopic fracture model is presented in which fractures emerge from the initial boundary value problem as part of the solution. The mesoscopic model eliminates the need for separate mathematical treatment of crack and intact material seen in classic (macroscopic) fracture models.

The nonlocal model carries details of the process zone seen at the mesoscopic length scale. In the limit of vanishing nonlocality, solutions of the model converge to solutions of the wave equation with evolving boundary formulated in Dal Maso and Toader [1], see [6].

The nonlocal dynamic initial value problem implicitly encodes the features of the classic model and delivers them in the limit of vanishing nonlocality [7].

In this talk the kinetic relation of LEFM is recovered from the nonlocal model noting that the same equation of motion applies everywhere in the body for the nonlocal model [5]. This is used to show that local power balance is given by the stationarity in time of the internal energy of a small domain containing the crack tip. The change in internal energy is shown to be the difference between the elastic energy flowing into the crack and the kinetic energy and stress work flux flowing into the domain. To leading order the stress work flux is precisely the rate of energy needed to create new surface. These results are obtained directly and exclusively from the dynamics governed by the nonlocal Cauchy equations of motion for a continuum body. This is the explicit connection between the nonlocal Cauchy equations of motion derived from double well potentials and the energy rate required to make new surface. For remote boundary loading we apply energy balance and pass to the local limit to recover the celebrated kinetic relation for the modern theory of dynamic fracture mechanics articulated in Freund [2], Ravi-Chandar [3].

Numerical simulations are given that illustrate the kinetic relation for roughly constant velocity traveling in a long strip. The crack velocity approaches steady state value of 0.6 which is consistent with the experimental result in Goldman et al. [4].

## References

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