Nucleation of cracks in variational phase-field models of fracture

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Phase-field models of fracture were originally proposed as regularized versions of the Griffith brittle fracture model [1]. Their success lies in their ability to accurately simulate the nucleation and propagation of cracks without prior assumptions regarding their shape or location, while also being relatively easy to implement numerically. The variational approach to fracture has established a precise link between the toughness of the sharp interphase Griffith model and that of the smeared phase-field model, in terms of the convergence of the global minima of the energy (gamma-convergence). However, brittle fracture is characterized by two key material parameters: fracture toughness and strength. Fracture toughness is the energy dissipated during crack propagation, while strength is the maximum allowable stress that the material can sustain before failure through crack nucleation. From the energetic viewpoint, the strength can be related to the energy barrier between the purely elastic solution and the solution with cracks, an information that gamma-convergence cannot provide. There is still ongoing debate as to whether and how phase-field approaches can correctly account for strength and crack nucleation criteria.

Phase-field models introduce an additional material parameter that can be assimilated to an internal length, which is absent in the Griffith model. In a series of studies [2-5], we have related this length to the stability margin of local energy minima, and thus to the nucleation thresholds. This viewpoint has enabled us to understand crack nucleation phenomena and the morphogenesis of complex crack patterns as a structural stability problem [2,3]. In the first part of the talk, I will review the main results obtained in this framework and present the tools we have developed to perform the stability analysis numerically. I will then discuss the difficulties that arise under multi-axial loading, where the strength is a surface that delimits the domain of admissible stress tensors before material failure. In variational approaches, the shape of this surface is closely related to the strain energy density and its dependence on the phase-field variable. I will show how the theoretical stability analysis will disclose the subtle influence of softening, energy decompositions, and loading modes on the strength observed in numerical simulations. Finally, I will present new ideas that combine variational phase-field models and nonlinear elasticity to address the limitations of existing approaches, as highlighted in a recent paper [6]. I will discuss specifically the case of crack nucleation in almost incompressible materials.

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