Přehled vzorců z teorie pružnosti a plasticity – povolená pomůcka ke zkoušce (2010/2011)

Základní rovnice teorie pružnosti

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \qquad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \qquad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \overline{X} = 0 \qquad \sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu\varepsilon_y + \nu\varepsilon_z\right] \qquad \tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \frac{\partial \tau_{xy}}{\partial x} + \overline{Y} = 0 \qquad \sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y + \nu\varepsilon_z + \nu\varepsilon_x\right] \qquad \tau_{xz} = \frac{E}{2(1+\nu)}\gamma_{xz}$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \overline{Z} = 0 \qquad \sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_z + \nu\varepsilon_x + \nu\varepsilon_y\right] \qquad \tau_{yz} = \frac{E}{2(1+\nu)}\gamma_{yz}$$

za rovinné napjatosti: $$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$\sigma_x' = \frac{\sigma_x + \sigma_z}{2} + \frac{\sigma_x - \sigma_z}{2}\cos 2\alpha + \tau_{xz}\sin 2\alpha, \quad \tau_{xz}' = -\frac{\sigma_x - \sigma_z}{2}\sin 2\alpha + \tau_{xz}\cos 2\alpha$$

Tah-tlak, ohyb, smyk za ohybu

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(EA\frac{\mathrm{d}u}{\mathrm{d}x}\right) + f_x = 0 \qquad \frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI_y\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right) - f_z = 0 \qquad E_{\mathrm{int}} = \frac{1}{2}\int_0^L EA\,u'^2(x)\,\mathrm{d}x + \frac{1}{2}\int_0^L EI_y\,w''^2(x)\,\mathrm{d}x$$

$$\sigma_x = \frac{N_x}{A} - \frac{M_z I_y + M_y D_{yz}}{I_y I_z - D_{yz}^2}\,y + \frac{M_y I_z + M_z D_{yz}}{I_y I_z - D_{yz}^2}\,z \qquad 1 + \frac{y_c y}{i_z^2} + \frac{z_c z}{i_y^2} = 0 \qquad t_{sx} = \frac{Q_y \overline{S}_z}{I_z} + \frac{Q_z \overline{S}_y}{I_y}$$

Kroucení

$$M_x = GI_k\Theta$$

kruh $\quad I_k = I_p = \pi r^4/2, \quad \tau_{xy} = -G\Theta z, \quad \tau_{xz} = G\Theta y$

masivní průřez $\quad I_k \approx \dfrac{A^4}{40 I_p}$ (obecně), elipsa $\quad I_k = \dfrac{\pi a^3 b^3}{a^2 + b^2}$, čtverec $\quad I_k = 0{,}1406\,b^4$, $\quad \tau_{\max} = 0{,}6755\,G\Theta b$

úzký obdélník $\quad I_k \approx \dfrac{1}{3}b^3 h\left(1 - 0{,}63\dfrac{b}{h}\right), \quad \tau_{\max} \approx G\Theta b$

tenkostěnný otevřený průřez $\quad I_k \approx \dfrac{1}{3}\sum_n \delta_n^3 h_n, \quad \tau_{\max,n} \approx G\Theta\delta_n$

tenkostěnný uzavřený průřez $\quad I_k = \dfrac{\Omega^2}{\oint \dfrac{\mathrm{d}s}{\delta(s)}}, \quad \Omega = \oint \mathrm{d}\omega(s), \quad \tau_{xs}(s) = \dfrac{t}{\delta(s)}, \quad t = \dfrac{M_x}{\Omega}$

Plasticita

$M_{el} = W_{el}\sigma_0,\ M_{pl} = W_{pl}\sigma_0,\ N_{pl} = A\sigma_0$ $\qquad$ obdélník: $\quad W_{el} = \dfrac{1}{6}bh^2, \quad W_{pl} = \dfrac{1}{4}bh^2, \quad \dfrac{|M|}{M_{pl}} + \left(\dfrac{N}{N_{pl}}\right)^2 = 1$

Stabilita

$$F_k = \frac{EI\pi^2}{L_{vz}^2}, \quad \delta = \frac{\delta_0 F_k}{F_k - F}, \quad \lambda = \frac{L_{vz}}{i}, \quad \sigma_k = \frac{E\pi^2}{\lambda^2}$$