Vplyv geometrie na hodnotu efektívneho Poissonovho súčiniteľa auxetického metamateriálu

Semestrálna práca

Pružnosť a pevnosť

Katedra mechaniky

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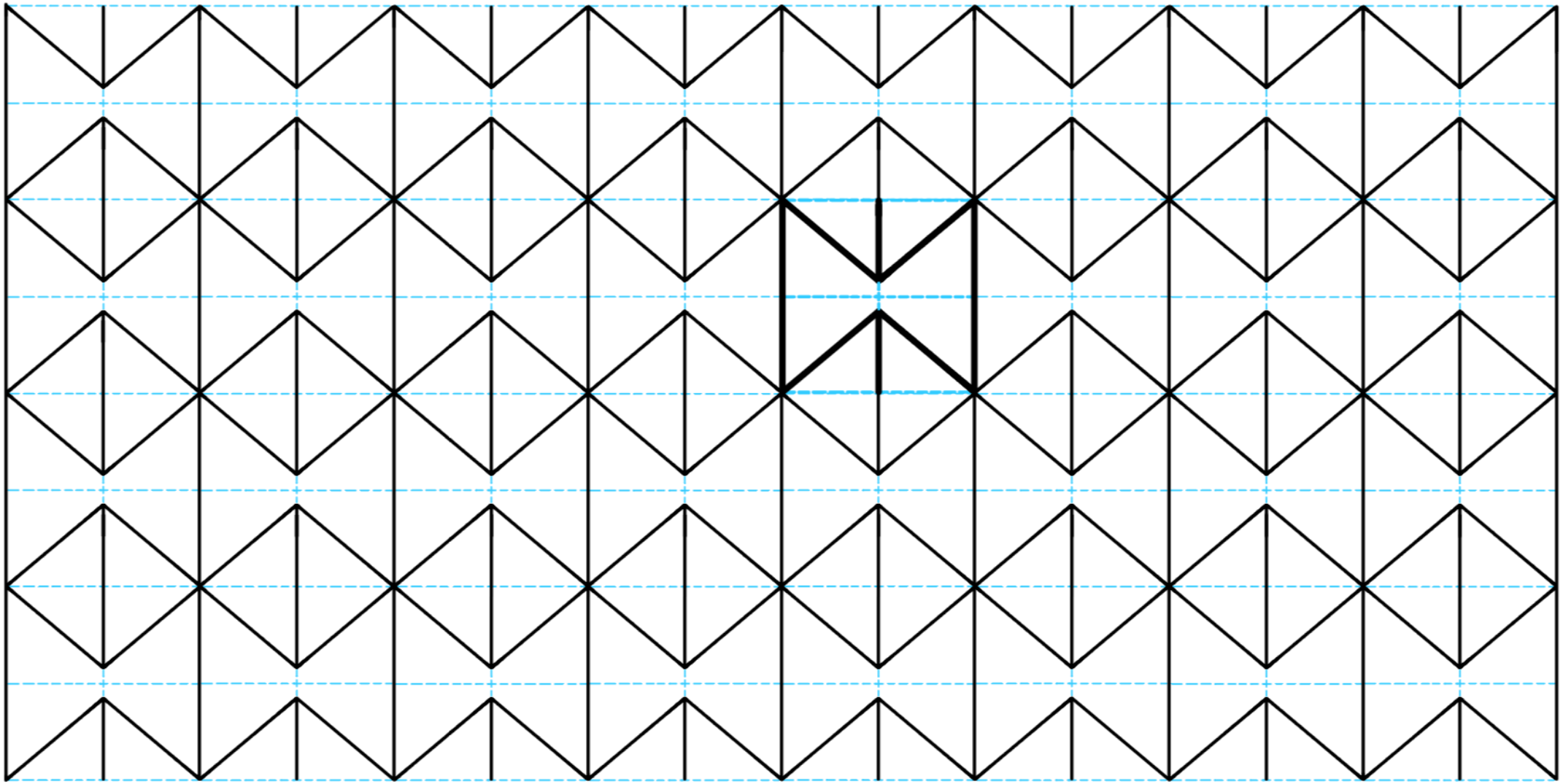
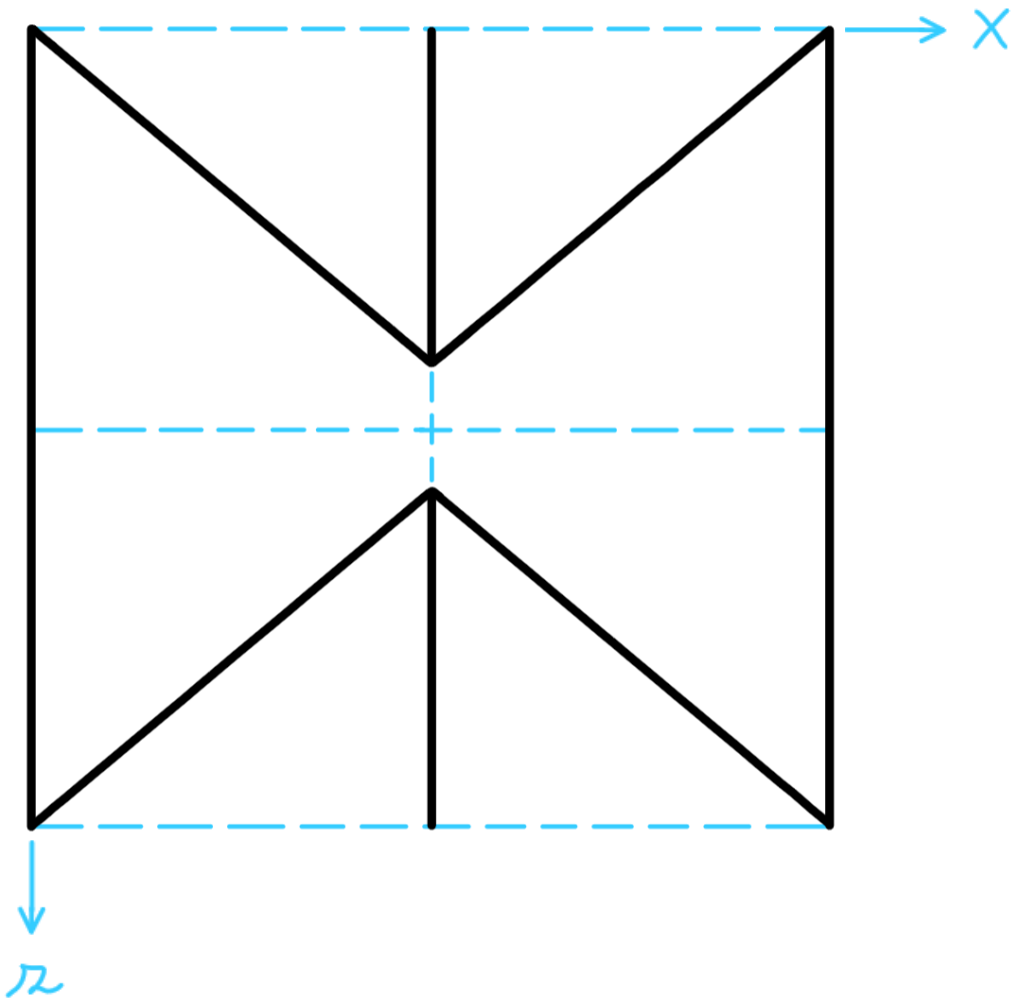
FSv ČVUT

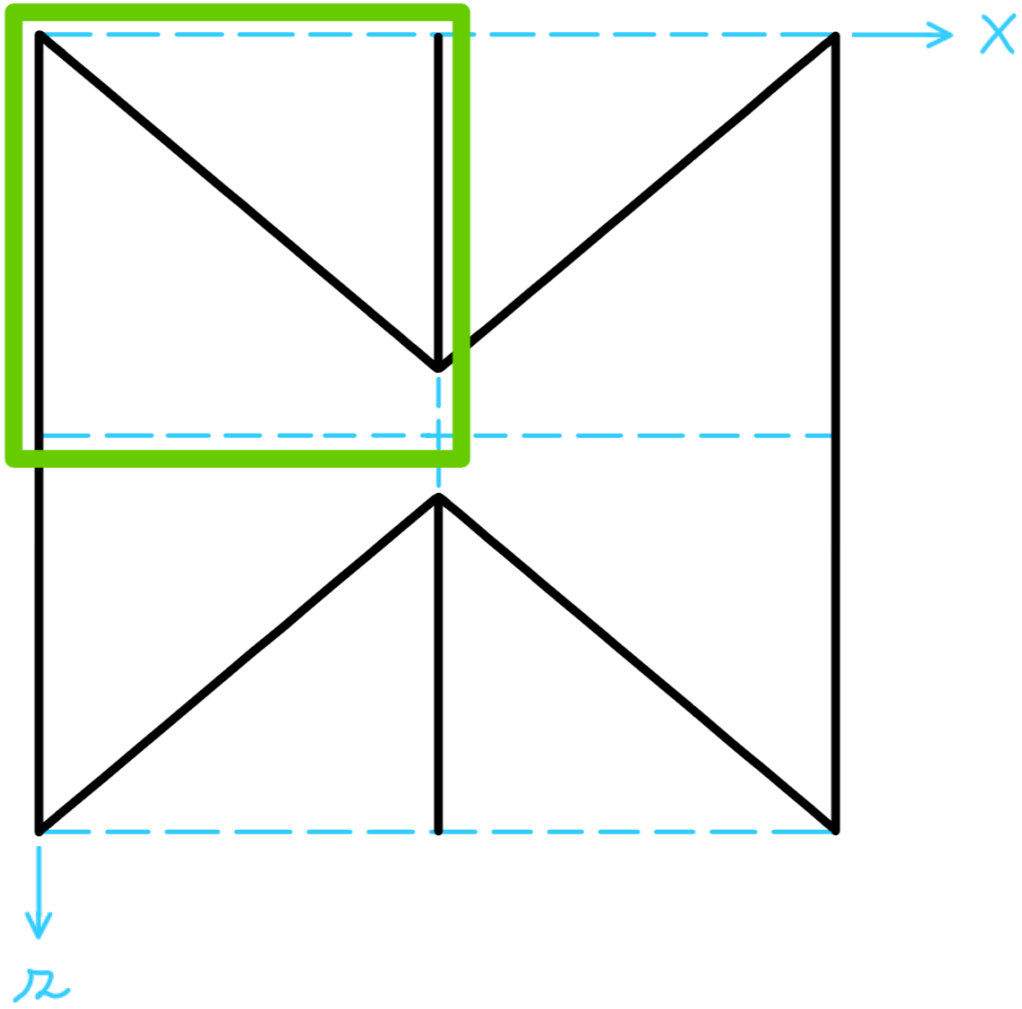
Auxetický metamateriál



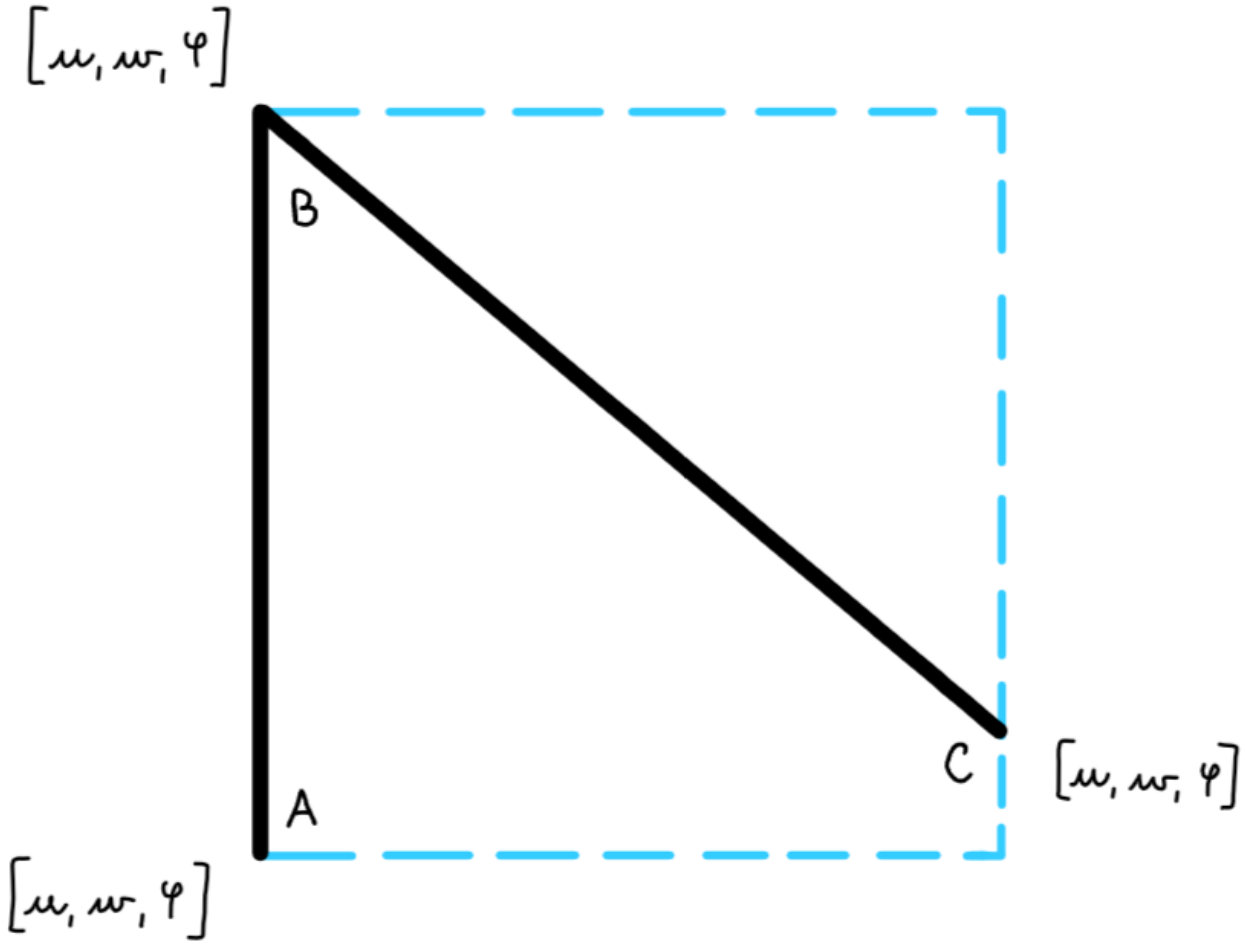
Zjednodušenie



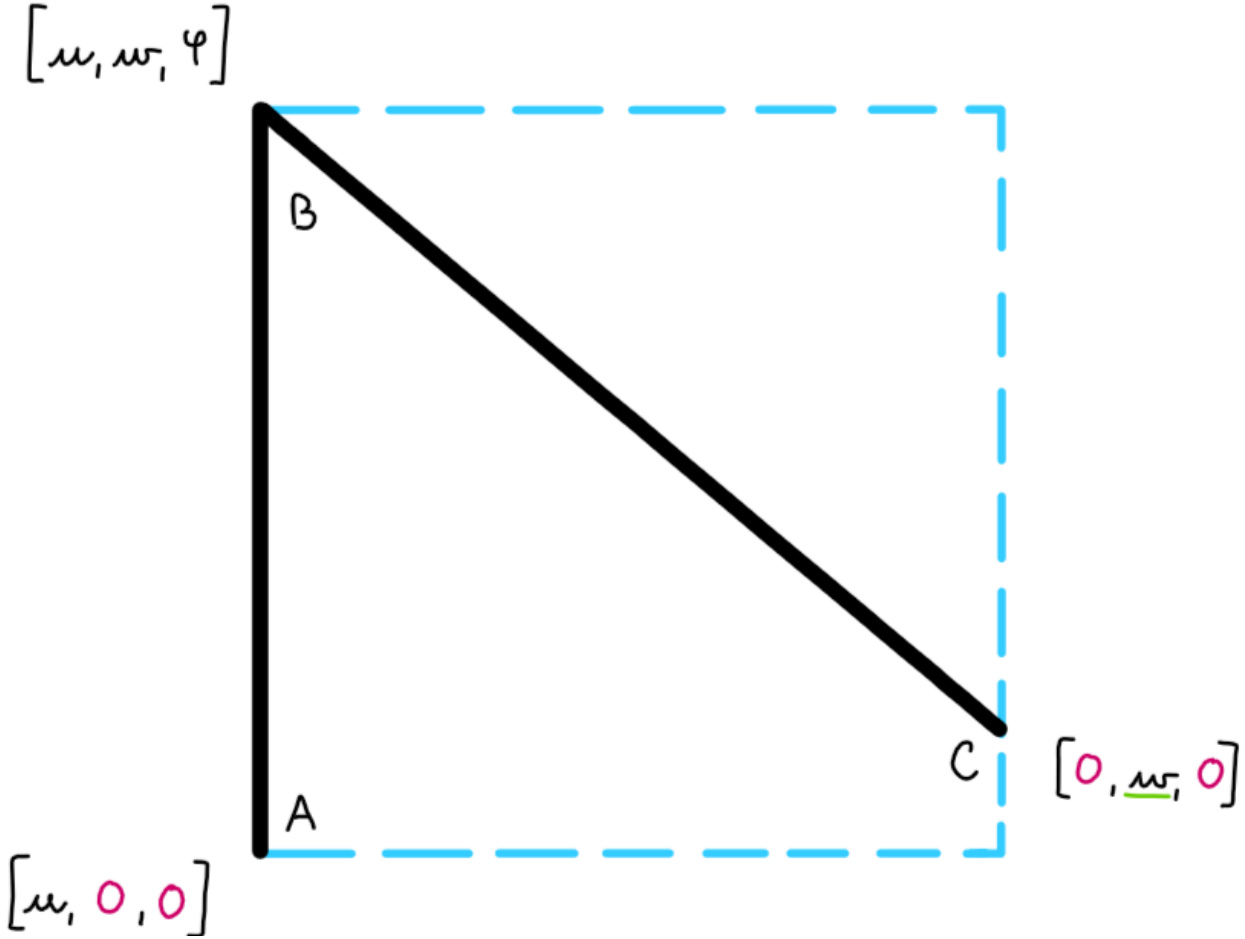
Zjednodušenie



Úvahy



Úvahy



Matica tuhosti prutu

$$\begin{Bmatrix} N_{ab} \\ V_{ab} \\ M_{ab} \\ N_{ba} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} u_a \\ w_a \\ \varphi_a \\ u_b \\ w_b \\ \varphi_b \end{Bmatrix}$$

Vyjadrenie posunu *u*

$$\frac{d}{dx}\left[EA\frac{du}{dx}(x)\right] + \bar{f}_x(x) = 0$$

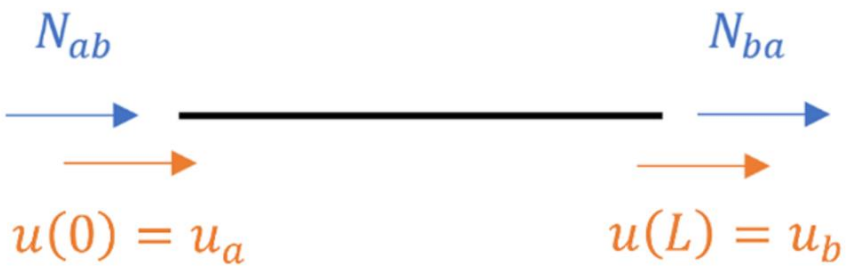
$$\varepsilon(x) = \frac{du}{dx}(x)$$

$$N = \frac{EA\Delta u}{L}$$

Normálová tuhosť prutu

$$n_{ab} = \frac{EA}{L}$$

$$\boxed{\begin{aligned} N_{ab} &= n_{ab}(u_a - u_b) \\ N_{ba} &= n_{ab}(u_b - u_a) \end{aligned}}$$



Matica tuhosti prutu

$$\begin{Bmatrix} N_{ab} \\ V_{ab} \\ M_{ab} \\ N_{ba} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = \begin{bmatrix} k_{11} & 0 & 0 & k_{14} & 0 & 0 \\ 0 & k_{22} & k_{23} & 0 & k_{25} & k_{26} \\ 0 & k_{32} & k_{33} & 0 & k_{35} & k_{36} \\ k_{41} & 0 & 0 & k_{44} & 0 & 0 \\ 0 & k_{52} & k_{53} & 0 & k_{55} & k_{56} \\ 0 & k_{62} & k_{63} & 0 & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} u_a \\ w_a \\ \varphi_a \\ u_b \\ w_b \\ \varphi_b \end{Bmatrix}$$

$$\begin{Bmatrix} N_{ab} \\ V_{ab} \\ M_{ab} \\ N_{ba} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = \begin{bmatrix} n & 0 & 0 & -n & 0 & 0 \\ 0 & k_{22} & k_{23} & 0 & k_{25} & k_{26} \\ 0 & k_{32} & k_{33} & 0 & k_{35} & k_{36} \\ -n & 0 & 0 & n & 0 & 0 \\ 0 & k_{52} & k_{53} & 0 & k_{55} & k_{56} \\ 0 & k_{62} & k_{63} & 0 & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} u_a \\ w_a \\ \varphi_a \\ u_b \\ w_b \\ \varphi_b \end{Bmatrix}$$

Matica tuhosti 4x4

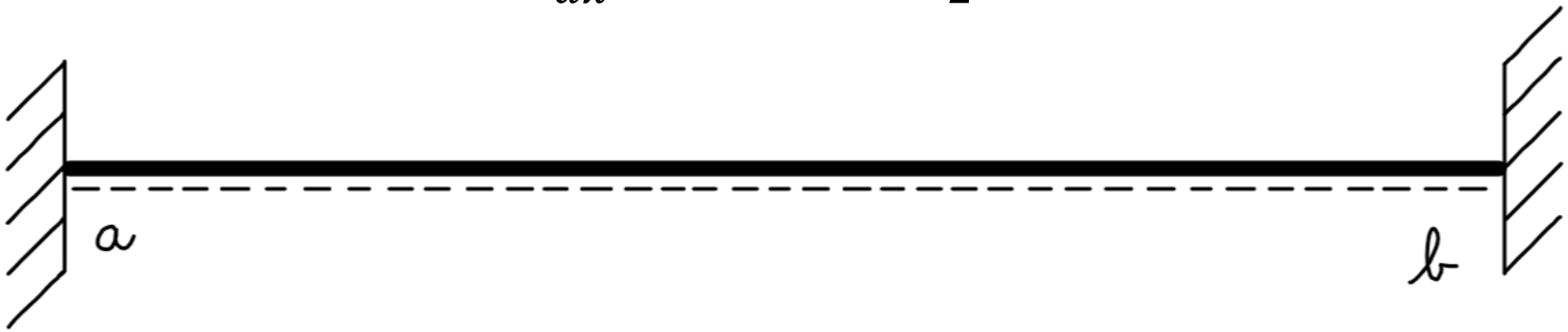
$$\begin{Bmatrix} V_{ab} \\ M_{ab} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = \begin{bmatrix} k_{22} & k_{23} & k_{25} & k_{26} \\ k_{32} & k_{33} & k_{35} & k_{36} \\ k_{52} & k_{53} & k_{55} & k_{56} \\ k_{62} & k_{63} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} w_a \\ \varphi_a \\ w_b \\ \varphi_b \end{Bmatrix}$$

Diferenciálna rovnica ohybovej čiary prutu

$$\frac{d^2}{dx^2}\left[EI_y\frac{d^2w}{dx^2}(x)\right] = \bar{f}_z(x)$$

$$V = -EI_y\frac{d^3w}{dx^3}(x) = -\bar{f}_z(x)x - c_1$$

$$M = -EI_y\frac{d^2w}{dx^2}(x) = -\bar{f}_z(x)\frac{x^2}{2} - c_1x - c_2$$



Matica tuhosti 4x4

$$\begin{Bmatrix} V_{ab} \\ M_{ab} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = \begin{bmatrix} k_{22} & k_{23} & k_{25} & k_{26} \\ k_{32} & k_{33} & k_{35} & k_{36} \\ k_{52} & k_{53} & k_{55} & k_{56} \\ k_{62} & k_{63} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} w_a \\ \varphi_a \\ w_b \\ \varphi_b \end{Bmatrix}$$

$$\begin{Bmatrix} V_{ab} \\ M_{ab} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = \begin{bmatrix} k_{22} & k_{23} & k_{25} & k_{26} \\ k_{32} & k_{33} & k_{35} & k_{36} \\ k_{52} & k_{53} & k_{55} & k_{56} \\ k_{62} & k_{63} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} V_{ab} \\ M_{ab} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = \begin{bmatrix} k_{22} & k_{23} & k_{25} & k_{26} \\ k_{32} & k_{33} & k_{35} & k_{36} \\ k_{52} & k_{53} & k_{55} & k_{56} \\ k_{62} & k_{63} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} V_{ab} \\ M_{ab} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = \begin{bmatrix} k_{22} & k_{23} & k_{25} & k_{26} \\ k_{32} & k_{33} & k_{35} & k_{36} \\ k_{52} & k_{53} & k_{55} & k_{56} \\ k_{62} & k_{63} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} V_{ab} \\ M_{ab} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = \begin{bmatrix} k_{22} & k_{23} & k_{25} & k_{26} \\ k_{32} & k_{33} & k_{35} & k_{36} \\ k_{52} & k_{53} & k_{55} & k_{56} \\ k_{62} & k_{63} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}$$

Matica tuhosti prutu

Ohybová tuhosť prutu $k_{ab} = \frac{2EI_y}{L}$

$$\begin{Bmatrix} V_{ab} \\ M_{ab} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = \begin{bmatrix} \frac{6k_{ab}}{L^2} & -\frac{3k_{ab}}{L} & -\frac{6k_{ab}}{L^2} & -\frac{3k_{ab}}{L} \\ -\frac{3k_{ab}}{L} & 2k_{ab} & \frac{3k_{ab}}{L} & k_{ab} \\ -\frac{6k_{ab}}{L^2} & \frac{3k_{ab}}{L} & \frac{6k_{ab}}{L^2} & \frac{3k_{ab}}{L} \\ -\frac{3k_{ab}}{L} & k_{ab} & \frac{3k_{ab}}{L} & 2k_{ab} \end{bmatrix} \begin{Bmatrix} w_a \\ \varphi_a \\ w_b \\ \varphi_b \end{Bmatrix}$$

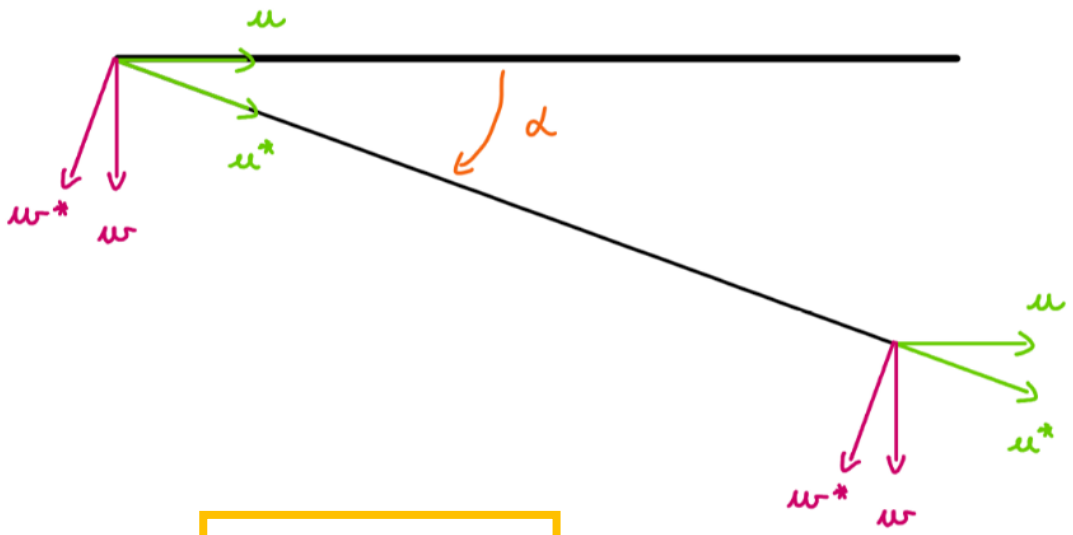
$$\begin{Bmatrix} V_{ab} \\ M_{ab} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = k_{ab} \begin{bmatrix} \frac{6}{L^2} & -\frac{3}{L} & -\frac{6}{L^2} & -\frac{3}{L} \\ -\frac{3}{L} & 2 & \frac{3}{L} & 1 \\ -\frac{6}{L^2} & \frac{3}{L} & \frac{6}{L^2} & \frac{3}{L} \\ -\frac{3}{L} & 1 & \frac{3}{L} & 2 \end{bmatrix} \begin{Bmatrix} w_a \\ \varphi_a \\ w_b \\ \varphi_b \end{Bmatrix}$$

Vyjadrená matica tuhosti prutu

$$\begin{Bmatrix} N_{ab} \\ V_{ab} \\ M_{ab} \\ N_{ba} \\ V_{ba} \\ M_{ba} \end{Bmatrix} = \begin{bmatrix} n_{ab} & 0 & 0 & -n_{ab} & 0 & 0 \\ 0 & \frac{6k_{ab}}{L^2} & -\frac{3k_{ab}}{L} & 0 & -\frac{6k_{ab}}{L^2} & -\frac{3k_{ab}}{L} \\ 0 & -\frac{3k_{ab}}{L} & 2k_{ab} & 0 & \frac{3k_{ab}}{L} & k_{ab} \\ -n_{ab} & 0 & 0 & n_{ab} & 0 & 0 \\ 0 & -\frac{6k_{ab}}{L^2} & \frac{3k_{ab}}{L} & 0 & \frac{6k_{ab}}{L^2} & \frac{3k_{ab}}{L} \\ 0 & -\frac{3k_{ab}}{L} & k_{ab} & 0 & \frac{3k_{ab}}{L} & 2k_{ab} \end{bmatrix} \begin{Bmatrix} u_a \\ w_a \\ \varphi_a \\ u_b \\ w_b \\ \varphi_b \end{Bmatrix}$$

Transformácia súradníc

$$u_a^* = u_a \cos\alpha + w_a \sin\alpha$$
$$w_a^* = -u_a \sin\alpha + w_a \cos\alpha$$
$$\varphi_a^* = \varphi_a$$
$$u_b^* = u_b \cos\alpha + w_b \sin\alpha$$
$$w_b^* = -u_b \sin\alpha + w_b \cos\alpha$$
$$\varphi_b^* = \varphi_b$$



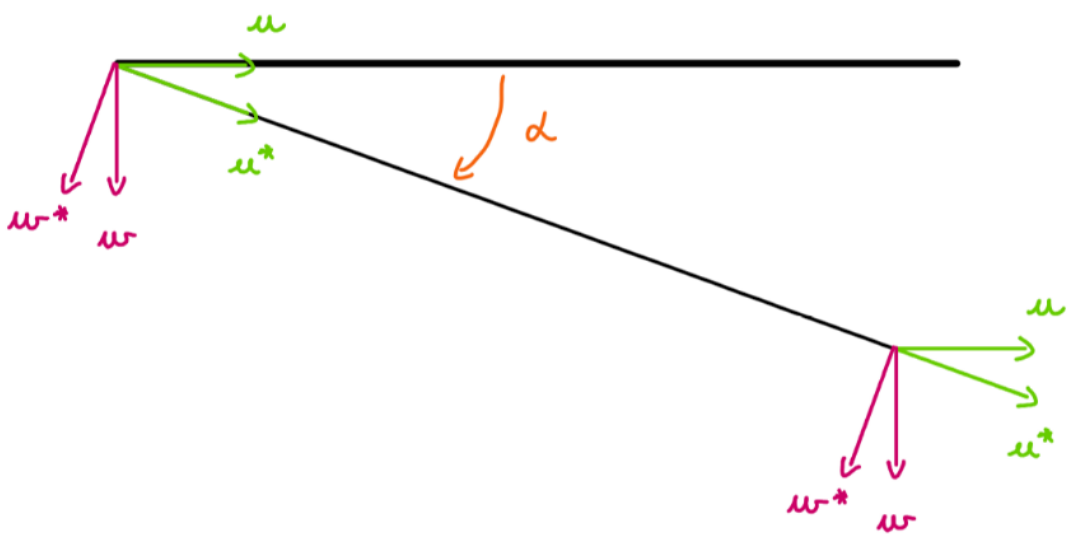
$$\begin{bmatrix} u^* \\ w^* \\ \varphi^* \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ w \\ \varphi \end{bmatrix} = \pi \begin{bmatrix} u \\ w \\ \varphi \end{bmatrix}$$

$$\boxed{T = \begin{bmatrix} \pi & 0 \\ 0 & \pi \end{bmatrix}}$$

Transformácia súradníc

$$T = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^T = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & 0 & 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

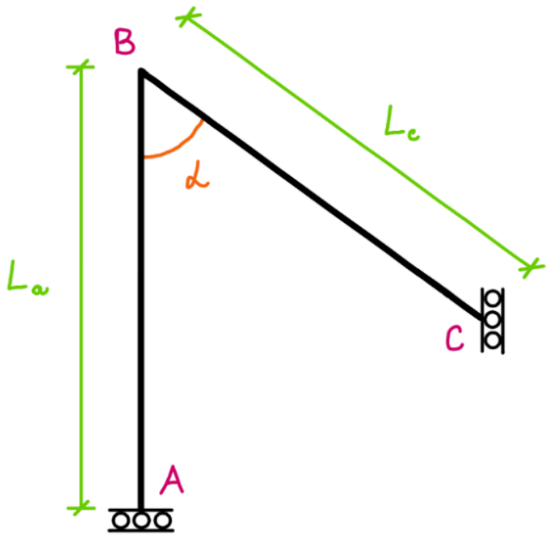


$$\boxed{K^g = T^T K^l T}$$

Matica tuhosti v globálnom SS

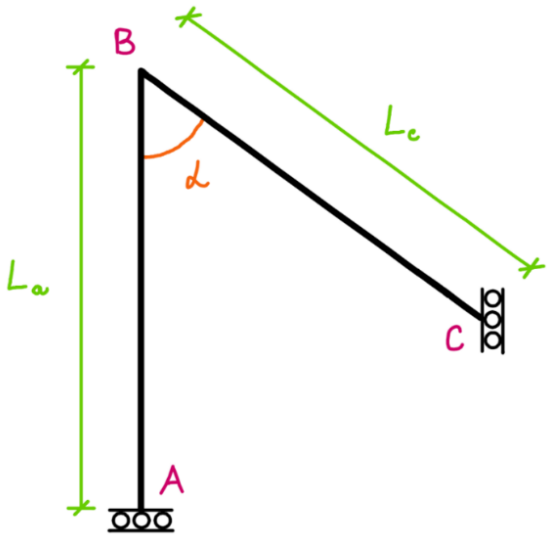
$$
\begin{Bmatrix} N_a^* \\ V_a^* \\ M_a^* \\ N_b^* \\ V_b^* \\ M_b^* \end{Bmatrix}
=
\begin{bmatrix}
n_{ab}cos^2\alpha + \frac{6k_{ab}}{L^2}sin^2\alpha & \left(n_{ab} - \frac{6k_{ab}}{L^2}\right)\sin\alpha\cos\alpha & \frac{3k_{ab}}{L}\sin\alpha & -n_{ab}cos^2\alpha - \frac{6k_{ab}}{L^2}sin^2\alpha & \left(\frac{6k_{ab}}{L^2} - n_{ab}\right)\sin\alpha\cos\alpha & \frac{3k_{ab}}{L}\sin\alpha \\
\left(n_{ab} - \frac{6k_{ab}}{L^2}\right)\sin\alpha\cos\alpha & n_{ab}sin^2\alpha + \frac{6k_{ab}}{L^2}cos^2\alpha & -\frac{3k_{ab}}{L}\cos\alpha & \left(\frac{6k_{ab}}{L^2} - n_{ab}\right)\sin\alpha\cos\alpha & -n_{ab}sin^2\alpha - \frac{6k_{ab}}{L^2}cos^2\alpha & -\frac{3k_{ab}}{L}\cos\alpha \\
\frac{3k_{ab}}{L}\sin\alpha & -\frac{3k_{ab}}{L}\cos\alpha & 2k_{ab} & -\frac{3k_{ab}}{L}\sin\alpha & \frac{3k_{ab}}{L}\cos\alpha & k_{ab} \\
-n_{ab}cos^2\alpha - \frac{6k_{ab}}{L^2}sin^2\alpha & \left(\frac{6k_{ab}}{L^2} - n_{ab}\right)\sin\alpha\cos\alpha & -\frac{3k_{ab}}{L}\sin\alpha & n_{ab}cos^2\alpha + \frac{6k_{ab}}{L^2}sin^2\alpha & \left(n_{ab} - \frac{6k_{ab}}{L^2}\right)\sin\alpha\cos\alpha & -\frac{3k_{ab}}{L}\sin\alpha \\
\left(\frac{6k_{ab}}{L^2} - n_{ab}\right)\sin\alpha\cos\alpha & -n_{ab}sin^2\alpha - \frac{6k_{ab}}{L^2}cos^2\alpha & \frac{3k_{ab}}{L}\cos\alpha & \left(n_{ab} - \frac{6k_{ab}}{L^2}\right)\sin\alpha\cos\alpha & n_{ab}sin^2\alpha + \frac{6k_{ab}}{L^2}cos^2\alpha & \frac{3k_{ab}}{L}\cos\alpha \\
\frac{3k_{ab}}{L}\sin\alpha & -\frac{3k_{ab}}{L}\cos\alpha & k_{ab} & -\frac{3k_{ab}}{L}\sin\alpha & \frac{3k_{ab}}{L}\cos\alpha & 2k_{ab}
\end{bmatrix}
\begin{Bmatrix} u_a \\ w_a \\ \varphi_a \\ u_b \\ w_b \\ \varphi_b \end{Bmatrix}
$$

Matica tuhosti pre sústavu 2 prutov



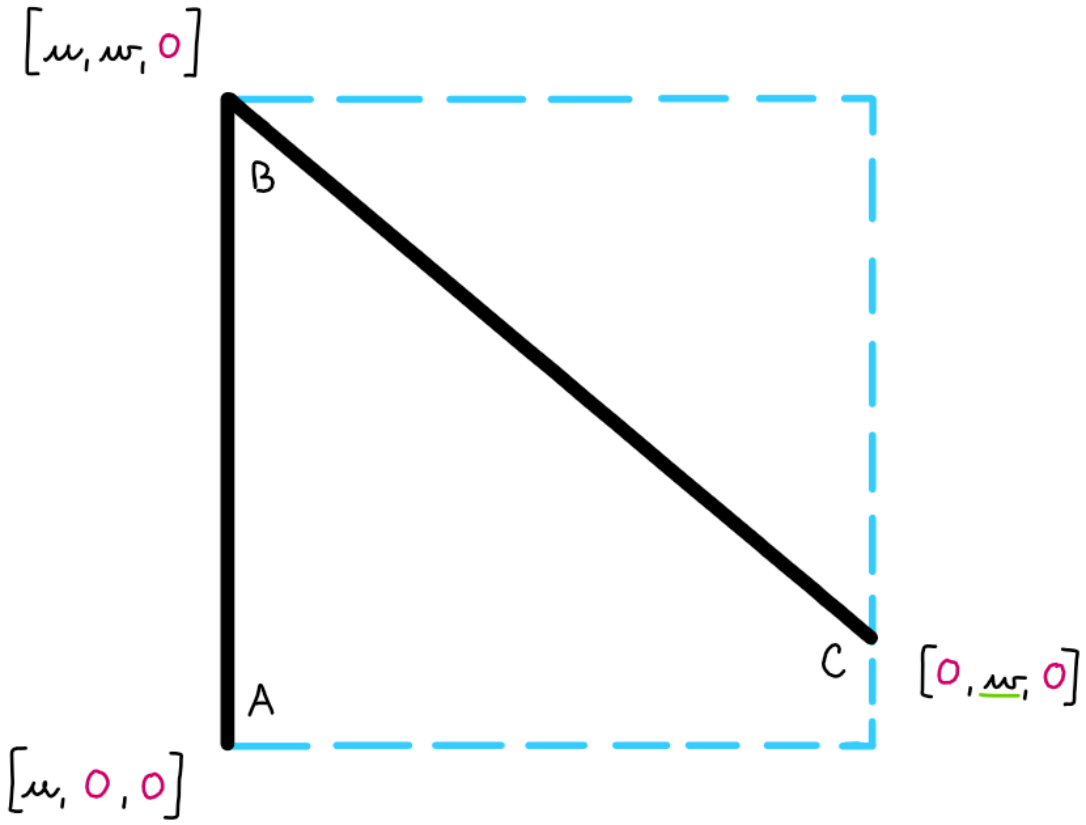
$$
\begin{Bmatrix} X_{ab} \\ Z_{ab} \\ M_{ab} \\ X_b \\ Z_b \\ M_b \\ X_{cb} \\ Z_{cb} \\ M_{cb} \end{Bmatrix}
=
\begin{bmatrix}
0 & n_{ab} & 0 & 0 & -n_{ab} & 0 & 0 & 0 & 0 \\
\frac{6k_{ab}}{L_a^2} & 0 & -\frac{3k_{ab}}{L_a} & -\frac{6k_{ab}}{L_a^2} & 0 & -\frac{3k_{ab}}{L_a} & 0 & 0 & 0 \\
-\frac{3k_{ab}}{L_a} & 0 & 2k_{ab} & \frac{3k_{ab}}{L_a} & 0 & k_{ab} & 0 & 0 & 0 \\
0 & -n_{ab} & 0 & n_{bc}cos^2\alpha + \frac{6k_{bc}}{L_c^2}sin^2\alpha & \left(n_{bc} - \frac{6k_{bc}}{L_c^2}\right)\sin\alpha\cos\alpha & \frac{3k_{bc}}{L_c}\sin\alpha & -n_{bc}cos^2\alpha - \frac{6k_{bc}}{L_c^2}sin^2\alpha & \left(\frac{6k_{bc}}{L_c^2} - n_{bc}\right)\sin\alpha\cos\alpha & \frac{3k_{bc}}{L_c}\sin\alpha \\
-\frac{6k_{ab}}{L_a^2} & 0 & \frac{3k_{ab}}{L_a} & \frac{6k_{ab}}{L_a^2} + \left(n_{bc} - \frac{6k_{bc}}{L_c^2}\right)\sin\alpha\cos\alpha & n_{bc}sin^2\alpha + \frac{6k_{bc}}{L_c^2}cos^2\alpha & \frac{3k_{ab}}{L_a} - \frac{3k_{bc}}{L_c}\cos\alpha & \left(\frac{6k_{bc}}{L_c^2} - n_{bc}\right)\sin\alpha\cos\alpha & -n_{bc}sin^2\alpha - \frac{6k_{bc}}{L_c^2}cos^2\alpha & -\frac{3k_{bc}}{L_c}\cos\alpha \\
-\frac{3k_{ab}}{L_a} & 0 & k_{ab} & \frac{3k_{ab}}{L_a} + \frac{3k_{bc}}{L_c}\sin\alpha & -\frac{3k_{bc}}{L_c}\cos\alpha & 2k_{ab} + 2k_{bc} & -\frac{3k_{bc}}{L_c}\sin\alpha & \frac{3k_{bc}}{L_c}\cos\alpha & k_{bc} \\
0 & 0 & 0 & -n_{bc}cos^2\alpha - \frac{6k_{bc}}{L_c^2}sin^2\alpha & \left(\frac{6k_{bc}}{L_c^2} - n_{bc}\right)\sin\alpha\cos\alpha & -\frac{3k_{bc}}{L_c}\sin\alpha & n_{bc}cos^2\alpha + \frac{6k_{bc}}{L_c^2}sin^2\alpha & \left(n_{bc} - \frac{6k_{bc}}{L_c^2}\right)\sin\alpha\cos\alpha & -\frac{3k_{bc}}{L_c}\sin\alpha \\
0 & 0 & 0 & \left(\frac{6k_{bc}}{L_c^2} - n_{bc}\right)\sin\alpha\cos\alpha & -n_{bc}sin^2\alpha - \frac{6k_{bc}}{L_c^2}cos^2\alpha & \frac{3k_{bc}}{L_c}\cos\alpha & \left(n_{bc} - \frac{6k_{bc}}{L_c^2}\right)\sin\alpha\cos\alpha & n_{bc}sin^2\alpha + \frac{6k_{bc}}{L_c^2}cos^2\alpha & \frac{3k_{bc}}{L_c}\cos\alpha \\
0 & 0 & 0 & \frac{3k_{bc}}{L_c}\sin\alpha & -\frac{3k_{bc}}{L_c}\cos\alpha & k_{bc} & -\frac{3k_{bc}}{L_c}\sin\alpha & \frac{3k_{bc}}{L_c}\cos\alpha & 2k_{bc}
\end{bmatrix}
\begin{Bmatrix} u_a \\ w_a \\ \varphi_a \\ u_b \\ w_b \\ \varphi_b \\ u_c \\ w_c \\ \varphi_c \end{Bmatrix}
$$

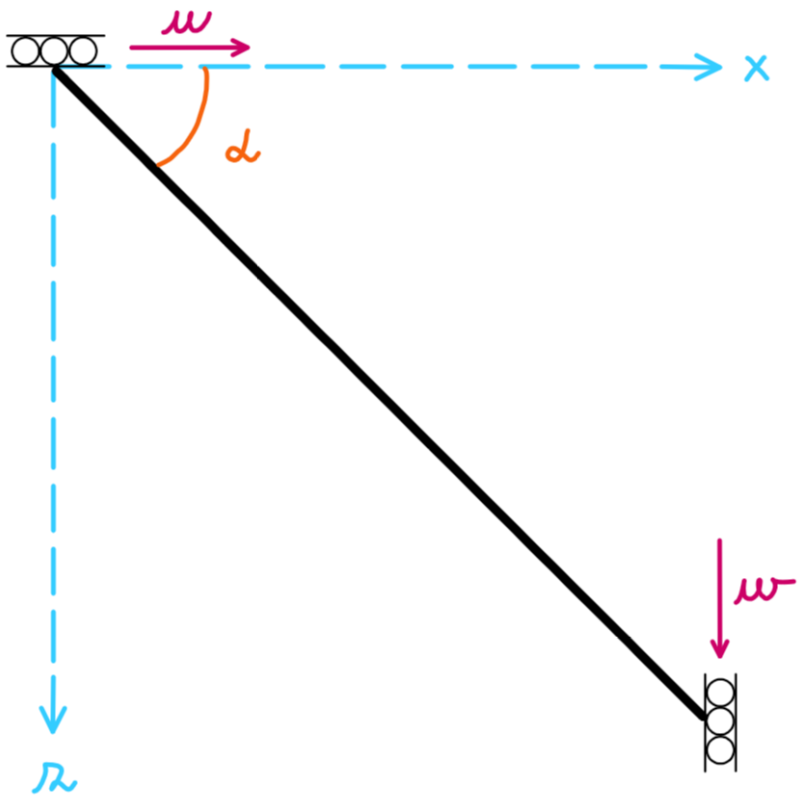
Matica tuhosti pre sústavu 2 prutov



$$
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} =
\begin{bmatrix}
0 & 0 & -\frac{EA}{L_a} & 0 & 0 \\
0 & \frac{EA}{L_c}cos^2\alpha + \frac{12EI_y}{L_c^3}sin^2\alpha & \left(\frac{EA}{L_c} - \frac{12EI_y}{L_c^3}\right)\sin\alpha\cos\alpha & \frac{6EI_y}{L_c^2}\sin\alpha & \left(\frac{12EI_y}{L_c^3} - \frac{EA}{L_c}\right)\sin\alpha\cos\alpha \\
-\frac{12EI_y}{L_a^3} & \frac{12EI_y}{L_a^3} + \left(\frac{EA}{L_c} - \frac{12EI_y}{L_c^3}\right)\sin\alpha\cos\alpha & \frac{EA}{L_c}sin^2\alpha + \frac{12EI_y}{L_c^3}cos^2\alpha & \frac{6EI_y}{L_a^2} - \frac{6EI_y}{L_c^2}\cos\alpha & -\frac{EA}{L_c}sin^2\alpha - \frac{12EI_y}{L_c^3}cos^2\alpha \\
-\frac{6EI_y}{L_a^2} & \frac{6EI_y}{L_a^2} + \frac{6EI_y}{L_c^2}\sin\alpha & -\frac{6EI_y}{L_c^2}\cos\alpha & \frac{4EI_y}{L_a} + \frac{4EI_y}{L_c} & \frac{6EI_y}{L_c^2}\cos\alpha \\
0 & \left(\frac{12EI_y}{L_c^3} - \frac{EA}{L_c}\right)\sin\alpha\cos\alpha & -\frac{EA}{L_c}sin^2\alpha - \frac{12EI_y}{L_c^3}cos^2\alpha & \frac{6EI_y}{L_c^2}\cos\alpha & \frac{EA}{L_c}sin^2\alpha + \frac{12EI_y}{L_c^3}cos^2\alpha
\end{bmatrix}
\begin{Bmatrix} u_a \\ u_b \\ w_b \\ \varphi_b \\ w_c \end{Bmatrix}
$$

Model auxetického metamateriálu s jedným prutom





Matica tuhosti modelu s jedným prutom

$$\begin{Bmatrix} X_b^* \\ Z_b^* \\ M_b^* \\ X_c^* \\ Z_c^* \\ M_c^* \end{Bmatrix} = \begin{bmatrix} n_{ab}cos^2\alpha + \frac{6k_{ab}}{L^2}sin^2\alpha & \left(n_{ab} - \frac{6k_{ab}}{L^2}\right)\sin\alpha\cos\alpha & \frac{3k_{ab}}{L}\sin\alpha & -n_{ab}cos^2\alpha - \frac{6k_{ab}}{L^2}sin^2\alpha & \left(\frac{6k_{ab}}{L^2} - n_{ab}\right)\sin\alpha\cos\alpha & \frac{3k_{ab}}{L}\sin\alpha \\ \left(n_{ab} - \frac{6k_{ab}}{L^2}\right)\sin\alpha\cos\alpha & n_{ab}sin^2\alpha + \frac{6k_{ab}}{L^2}cos^2\alpha & -\frac{3k_{ab}}{L}\cos\alpha & \left(\frac{6k_{ab}}{L^2} - n_{ab}\right)\sin\alpha\cos\alpha & -n_{ab}sin^2\alpha - \frac{6k_{ab}}{L^2}cos^2\alpha & -\frac{3k_{ab}}{L}\cos\alpha \\ \frac{3k_{ab}}{L}\sin\alpha & -\frac{3k_{ab}}{L}\cos\alpha & 2k_{ab} & -\frac{3k_{ab}}{L}\sin\alpha & \frac{3k_{ab}}{L}\cos\alpha & k_{ab} \\ -n_{ab}cos^2\alpha - \frac{6k_{ab}}{L^2}sin^2\alpha & \left(\frac{6k_{ab}}{L^2} - n_{ab}\right)\sin\alpha\cos\alpha & -\frac{3k_{ab}}{L}\sin\alpha & n_{ab}cos^2\alpha + \frac{6k_{ab}}{L^2}sin^2\alpha & \left(n_{ab} - \frac{6k_{ab}}{L^2}\right)\sin\alpha\cos\alpha & -\frac{3k_{ab}}{L}\sin\alpha \\ \left(\frac{6k_{ab}}{L^2} - n_{ab}\right)\sin\alpha\cos\alpha & -n_{ab}sin^2\alpha - \frac{6k_{ab}}{L^2}cos^2\alpha & \frac{3k_{ab}}{L}\cos\alpha & \left(n_{ab} - \frac{6k_{ab}}{L^2}\right)\sin\alpha\cos\alpha & n_{ab}sin^2\alpha + \frac{6k_{ab}}{L^2}cos^2\alpha & \frac{3k_{ab}}{L}\cos\alpha \\ \frac{3k_{ab}}{L}\sin\alpha & -\frac{3k_{ab}}{L}\cos\alpha & k_{ab} & -\frac{3k_{ab}}{L}\sin\alpha & \frac{3k_{ab}}{L}\cos\alpha & 2k_{ab} \end{bmatrix} \begin{Bmatrix} u_b \\ w_b \\ \varphi_b \\ u_c \\ w_c \\ \varphi_c \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ R_c \end{Bmatrix} = \begin{bmatrix} n_{ab}cos^2\alpha + \frac{6k_{ab}}{L^2}sin^2\alpha & \left(\frac{6k_{ab}}{L^2} - n_{ab}\right)sin\,\alpha\,cos\,\alpha \\ \left(\frac{6k_{ab}}{L^2} - n_{ab}\right)sin\,\alpha\,cos\,\alpha & n_{ab}sin^2\alpha + \frac{6k_{ab}}{L^2}cos^2\alpha \end{bmatrix} \begin{Bmatrix} u_b \\ w_c \end{Bmatrix}$$

Vzťah medzi posunmi u_a a w_b

$$0 = \frac{L^2 EAcos^2\alpha + 12EI_y sin^2\alpha}{L^3} u_b + \left(\frac{12EI_y - L^2 EA}{L^3}\right) sin\,\alpha\, cos\,\alpha\, w_c$$

$$\boxed{u_b = -\frac{\left(12I_y - L^2 A\right) sin\,\alpha\, cos\,\alpha}{L^2 Acos^2\alpha + 12I_y sin^2\alpha} w_c}$$

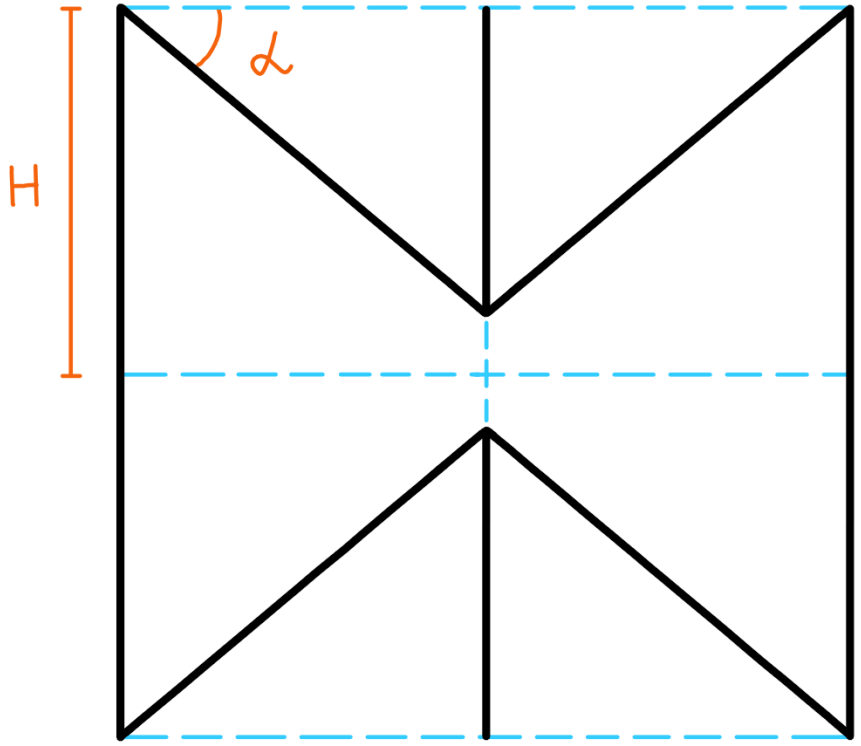
Vzťah medzi posunmi u_a a w_b

polomer zotrvačnosti

$$i^2 = \frac{I_y}{A}$$

$$L = \frac{H}{\cos\alpha}$$

$$u_b = -\frac{\left(12i^2 - \frac{H^2}{\cos^2\alpha}\right)\sin\alpha\cos\alpha}{H^2 + 12i^2\sin^2\alpha} w_c$$

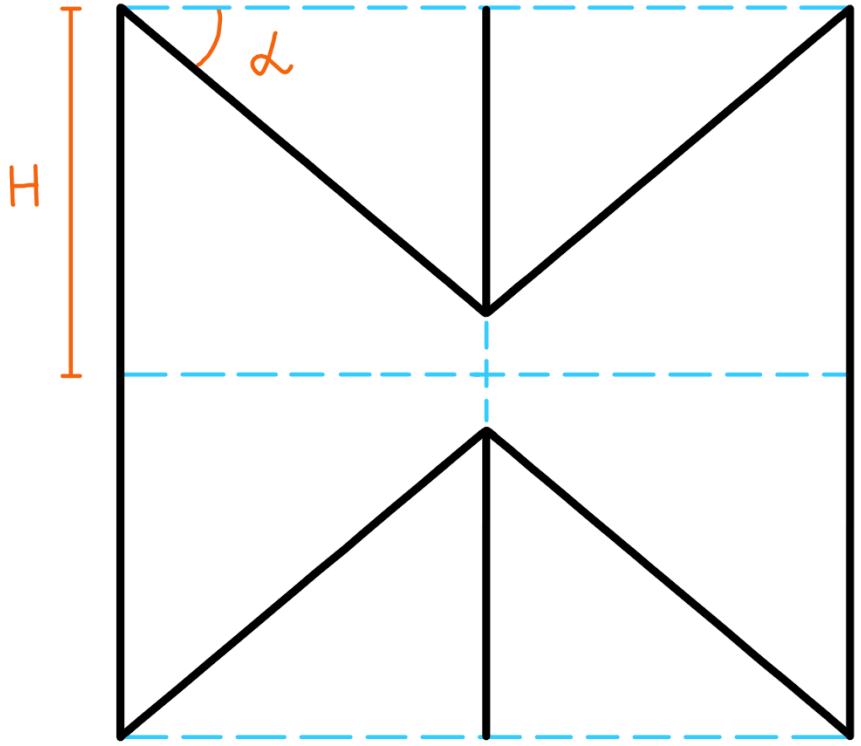


Vzťah medzi posunmi u_a a w_b

parameter λ

$$\lambda = \frac{i}{H}$$

$$u_b = -\frac{\left(12\lambda^2 - \frac{1}{cos^2\alpha}\right) sin\,\alpha\, cos\,\alpha}{1 + 12\lambda^2 sin^2\alpha} w_c$$



Poissonovo číslo ν

$$\nu = -\frac{\frac{u_b}{L}}{\frac{w_c}{L}}$$

$$\nu = -\frac{u_b}{w_c}$$

$$\boxed{\nu = -\frac{\varepsilon_x}{\varepsilon_z}}$$

Pomerná deformácia

$$\varepsilon_x \approx \frac{u_b}{L}$$

$$\varepsilon_z \approx \frac{w_c}{L}$$

$$\boxed{\nu = \frac{\left(12\lambda^2 - \frac{1}{cos^2\alpha}\right) sin\,\alpha\, cos\,\alpha}{1 + 12\lambda^2 sin^2\alpha}}$$

Poissonovo číslo ν

$$\boxed{\lambda << 1}$$

$$\nu \cong -\frac{1}{cos^2\alpha} sin\,\alpha\, cos\,\alpha$$

$$\nu \cong -\frac{sin\,\alpha}{cos\,\alpha}$$

$$\boxed{\nu \cong -\tan\alpha}$$

Vplyv geometrie auxetického metamateriálu

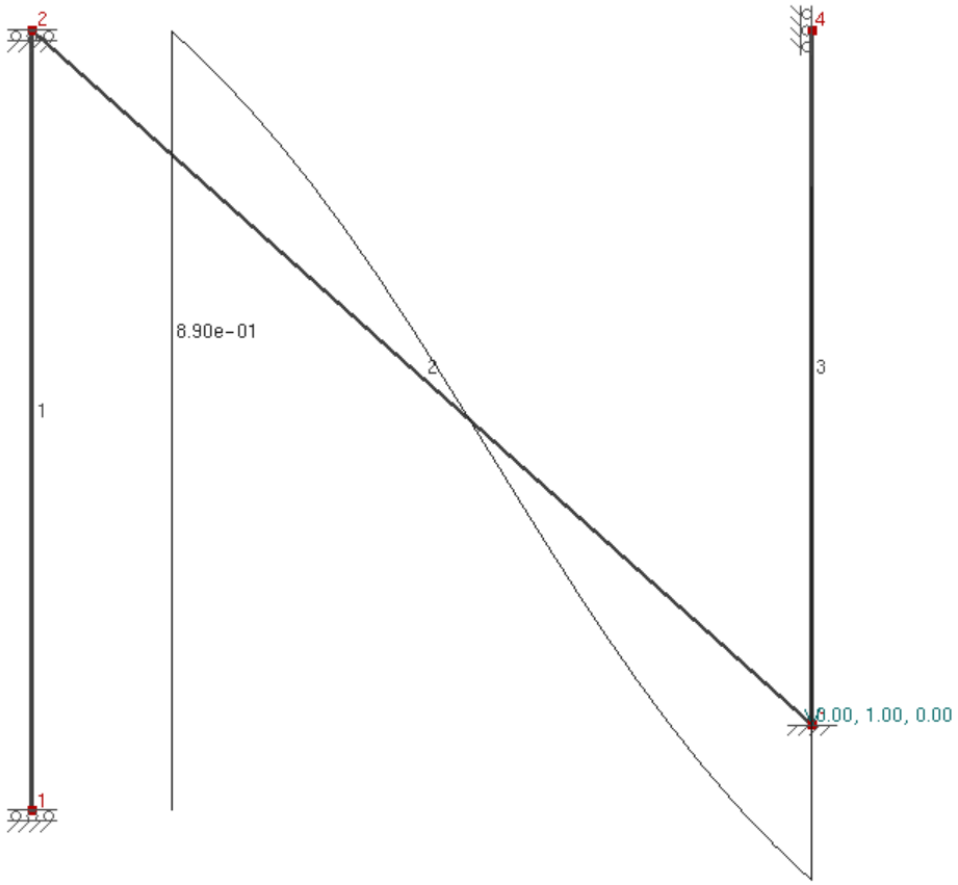
$G = \infty$

$w = 1\ \mathrm{m}$

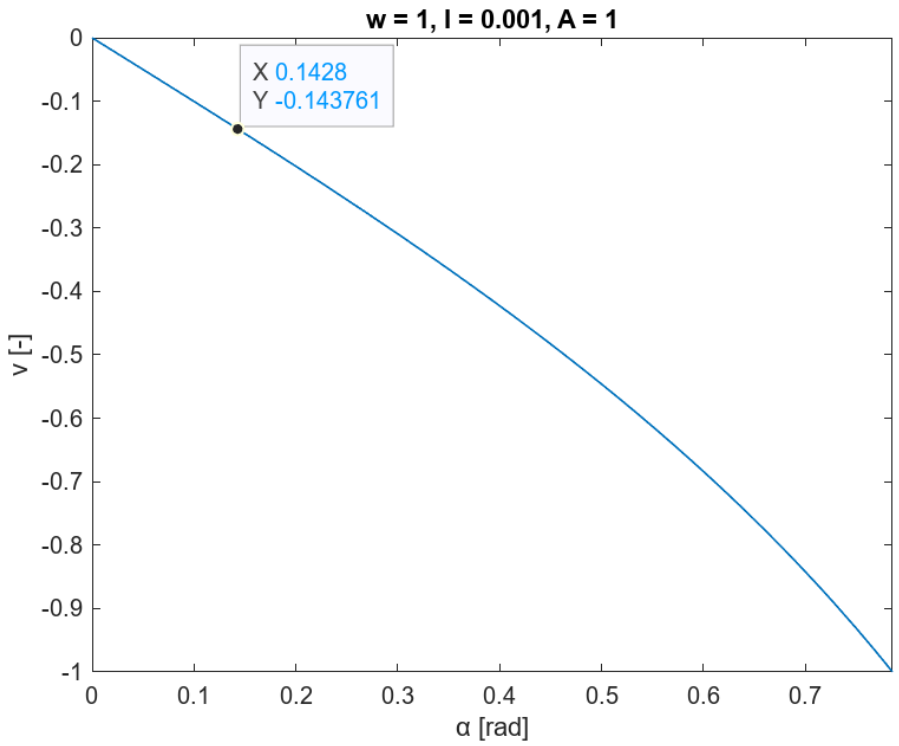
$H = 10\ \mathrm{m}$

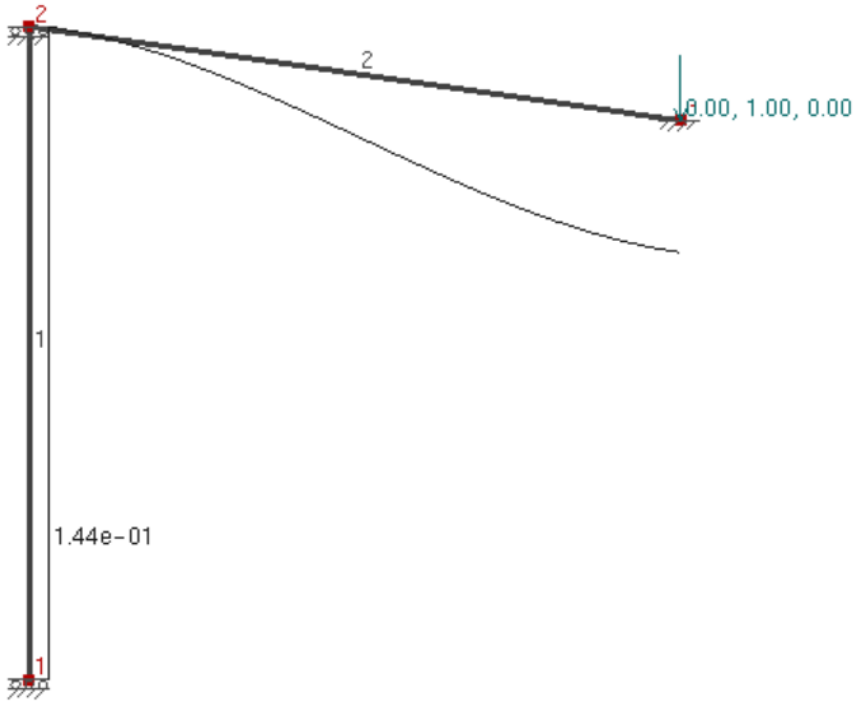
$I_y = 0.001\ \mathrm{m}^4$

$A = 1\ \mathrm{m}^2$



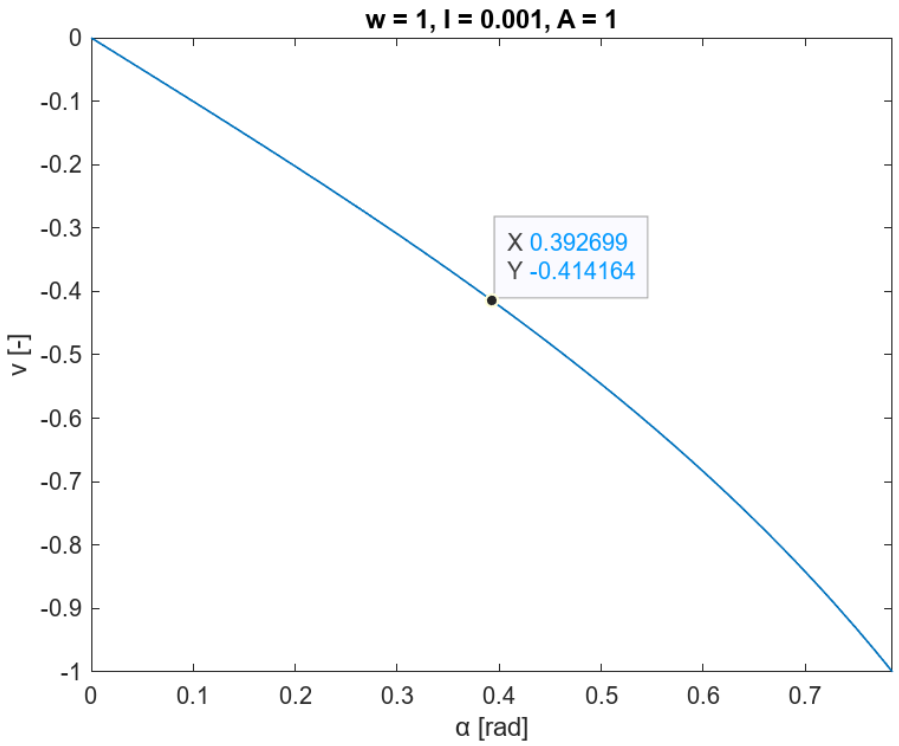
α = 8.1818°

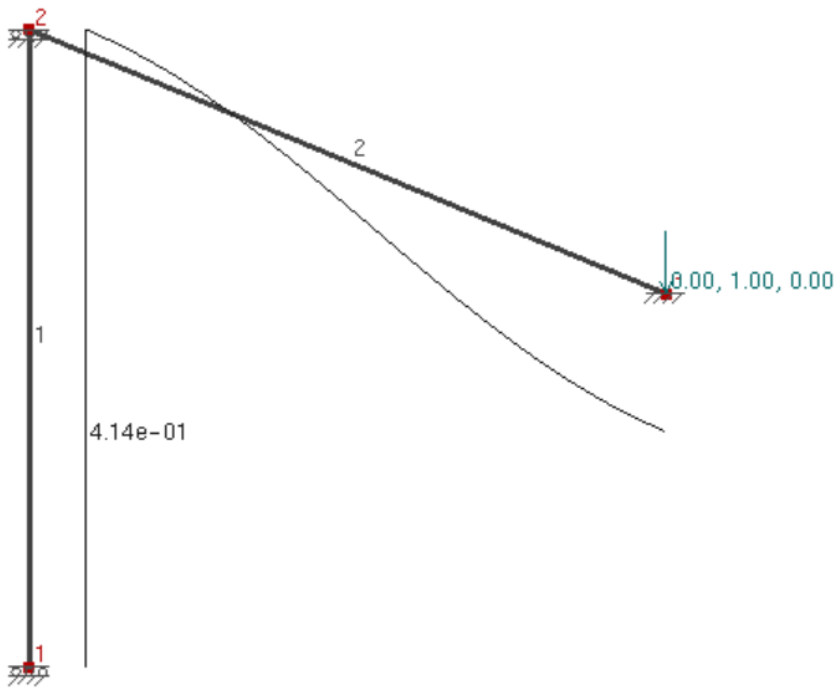




Uzel	x [~m]	y [~m]	z [~m]	u [~m]	w [~m]	phi [~rad]
1	0	0	10	0.14376275	0	0
2	0	0	0	0.14376275	0	0
3	10	0	1.4378	0	1	0

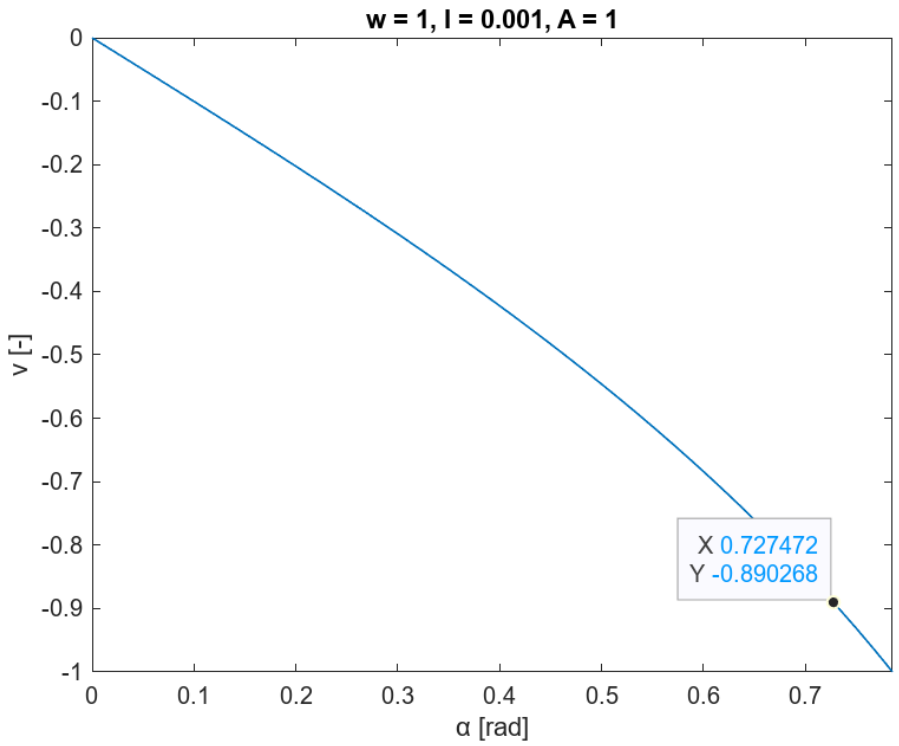
α = 22.5°

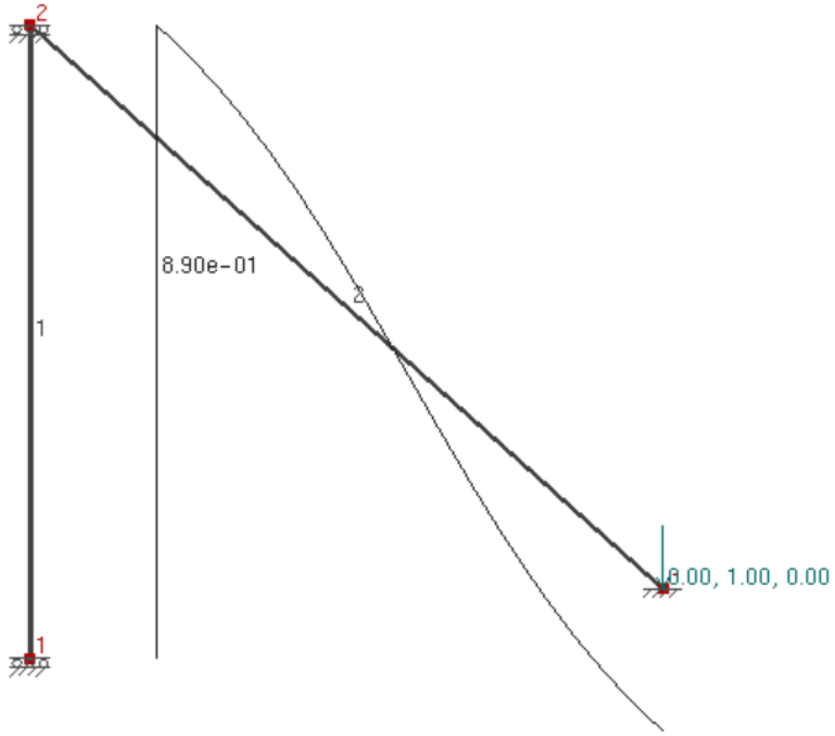




Uzel	x [~m]	y [~m]	z [~m]	u [~m]	w [~m]	phi [~rad]
1	0	0	10	0.4141603	0	0
2	0	0	0	0.4141603	0	0
3	10	0	4.1421	0	1	0

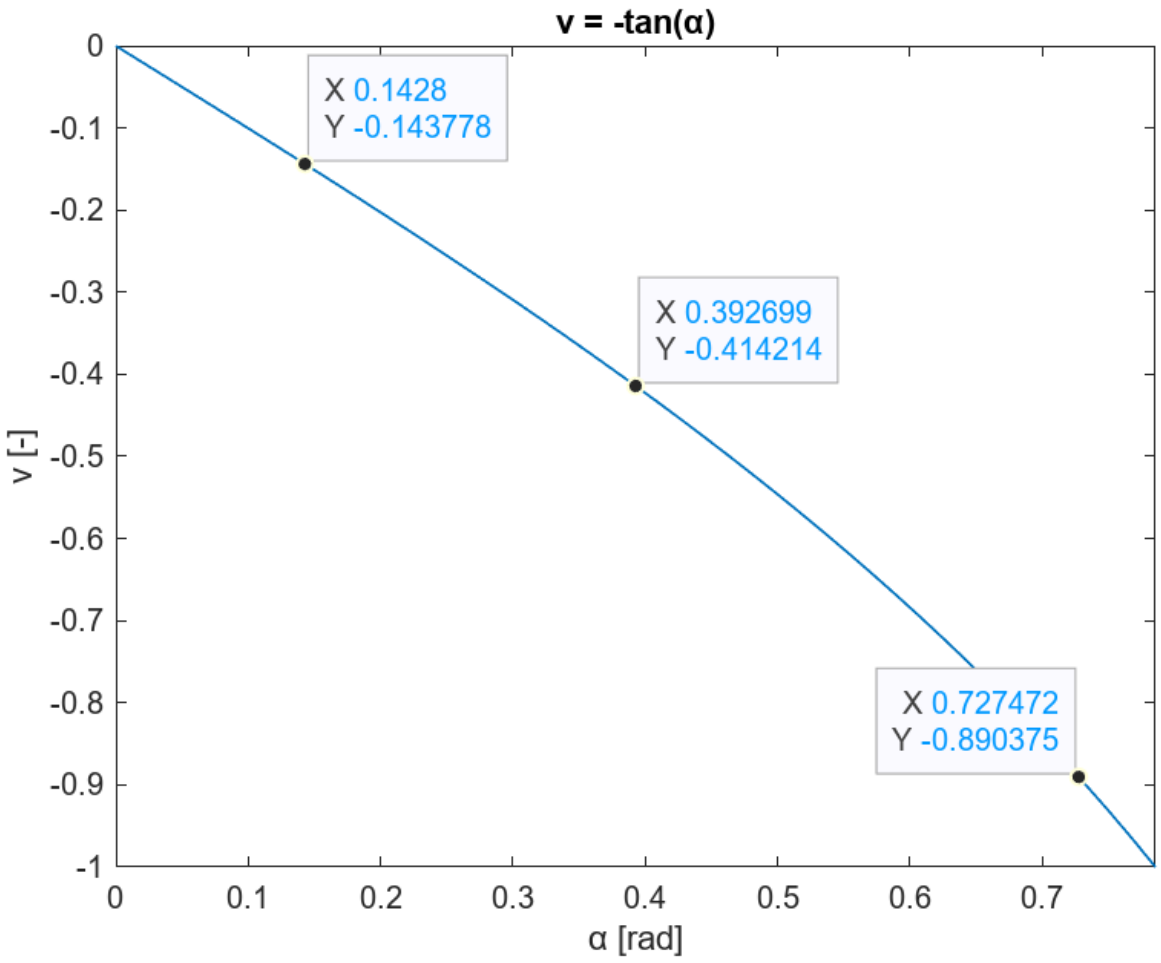
α = 41.6811°

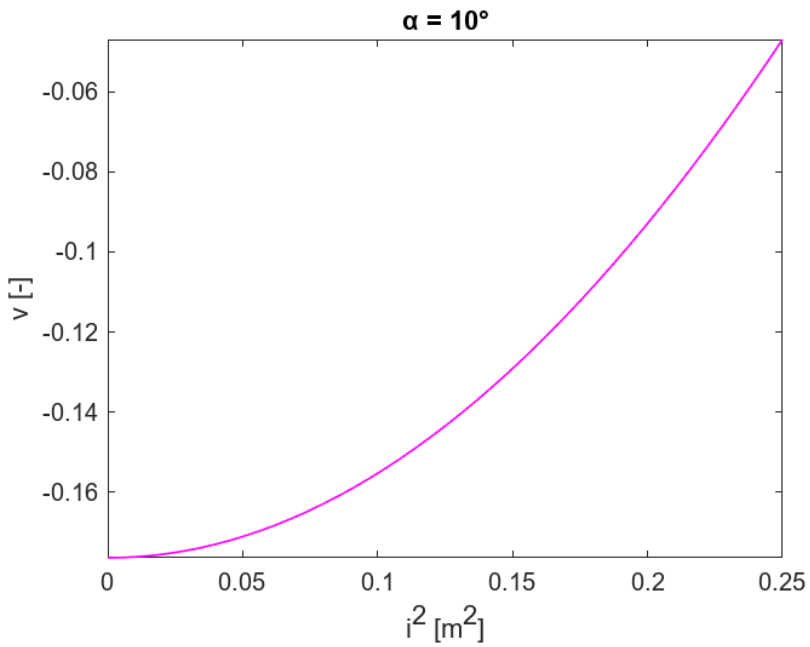


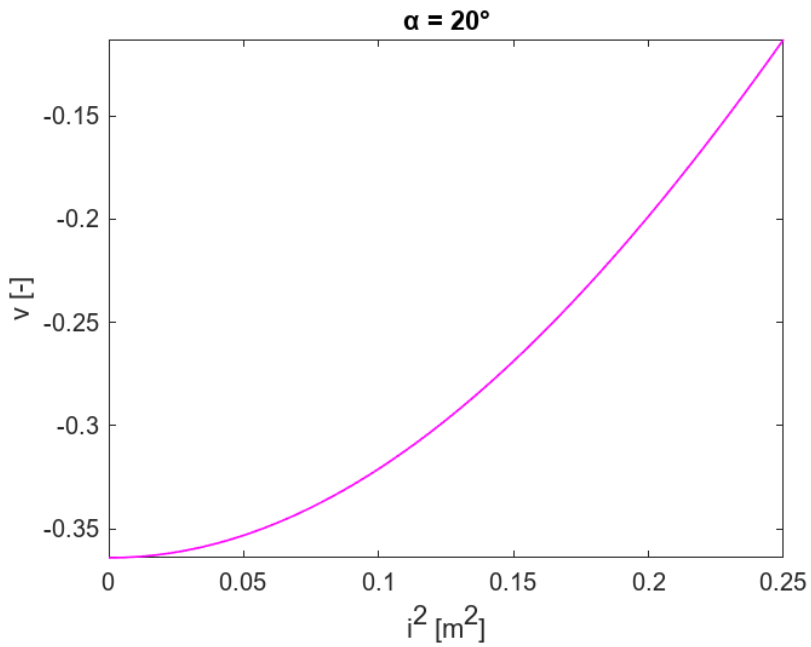


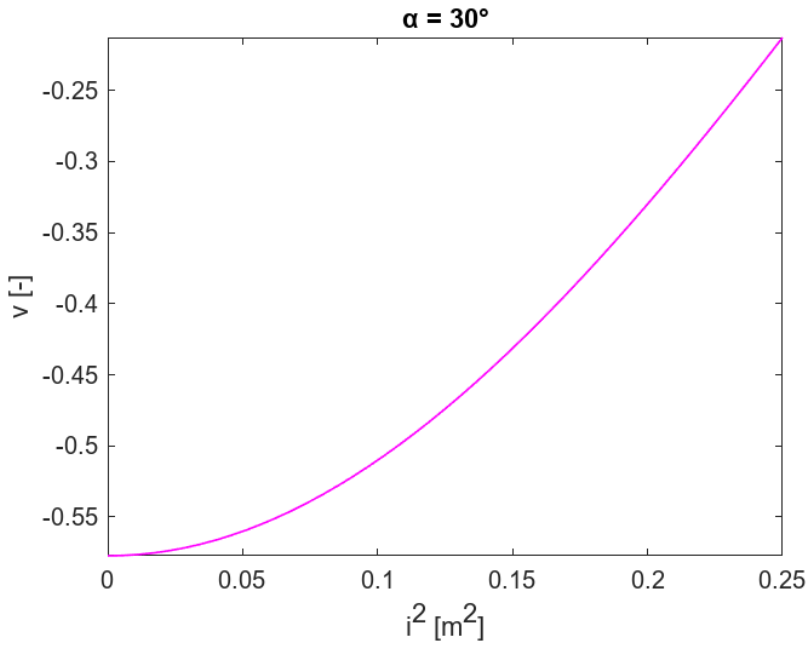
Uzel	x [~m]	y [~m]	z [~m]	u [~m]	w [~m]	phi [~rad]
1	0	0	10	0.89027316	0	0
2	0	0	0	0.89027316	0	0
3	10	0	8.9038	0	1	0

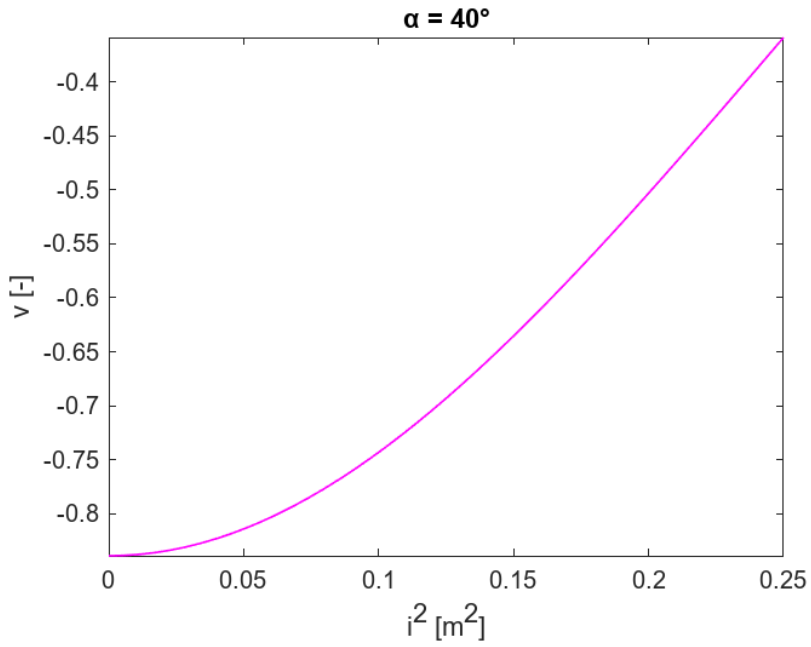
Zanedbanie λ

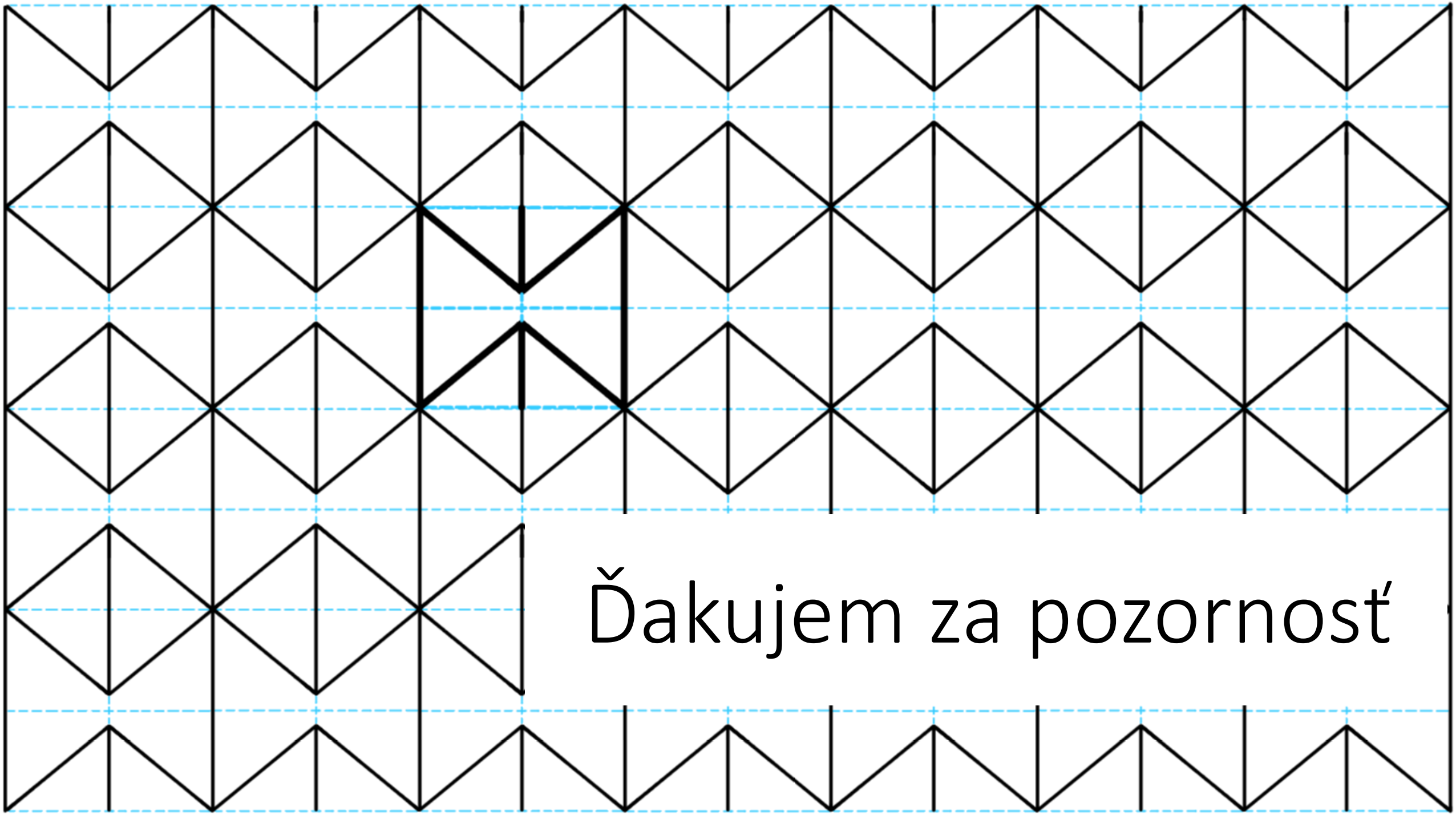



α = 10°
-0.06
-0.08
-0.1
-0.12
-0.14
-0.16
v [-]
0
0.05
0.1
0.15
0.2
0.25
i2 [m2]


α = 20°
-0.15
-0.2
-0.25
-0.3
-0.35
v [-]
0
0.05
0.1
0.15
0.2
0.25
i2 [m2]


α = 30°
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
-0.55
v [-]
0
0.05
0.1
0.15
0.2
0.25
i2 [m2]


α = 40°
-0.4
-0.45
-0.5
-0.55
-0.6
-0.65
-0.7
-0.75
-0.8
v [-]
0
0.05
0.1
0.15
0.2
0.25
i2 [m2]



Ďakujem za pozornosť