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Updating of meta-models
Uniform space-filling designs in hyperspheres

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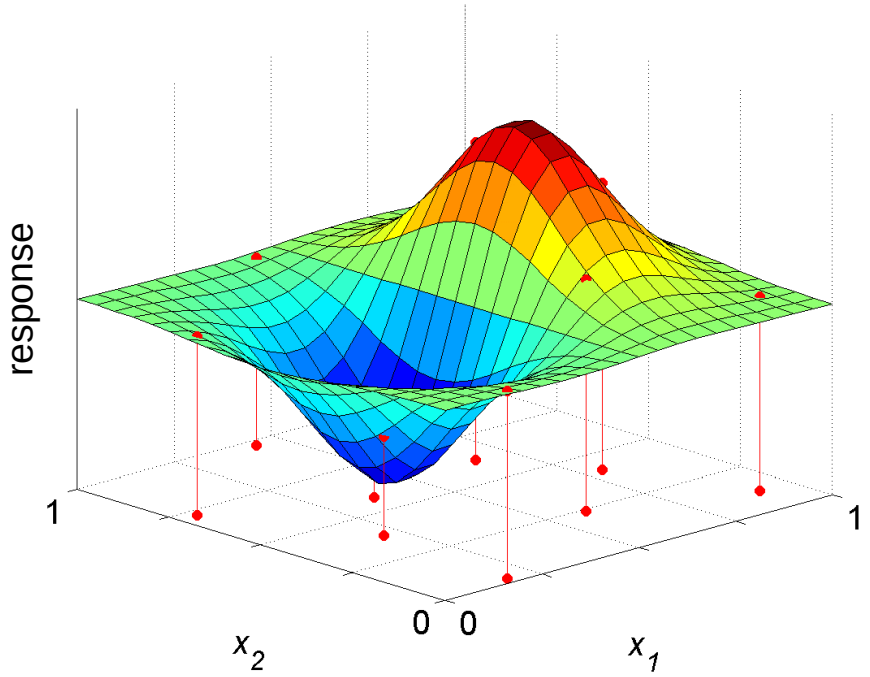
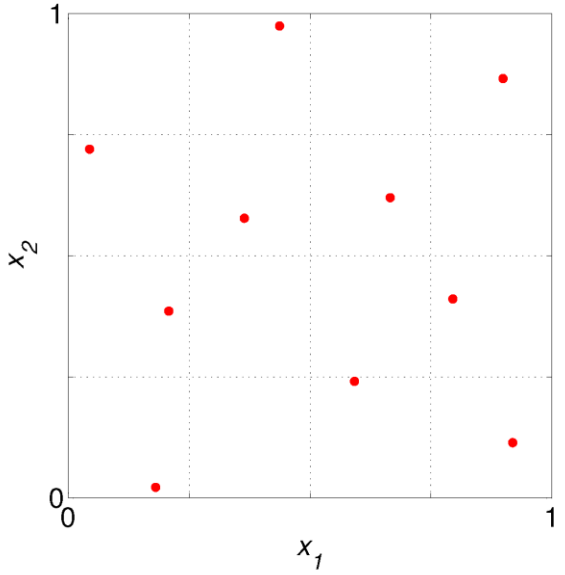
Content

1. Design of experiments (DoE)
2. Adaptive updating of meta-models (addition of points into DoE)
3. Uniform space-filling designs in hyperspheres

Design of experiments (DoE)

- coordinates of the design points correspond to the combinations of values of input variables

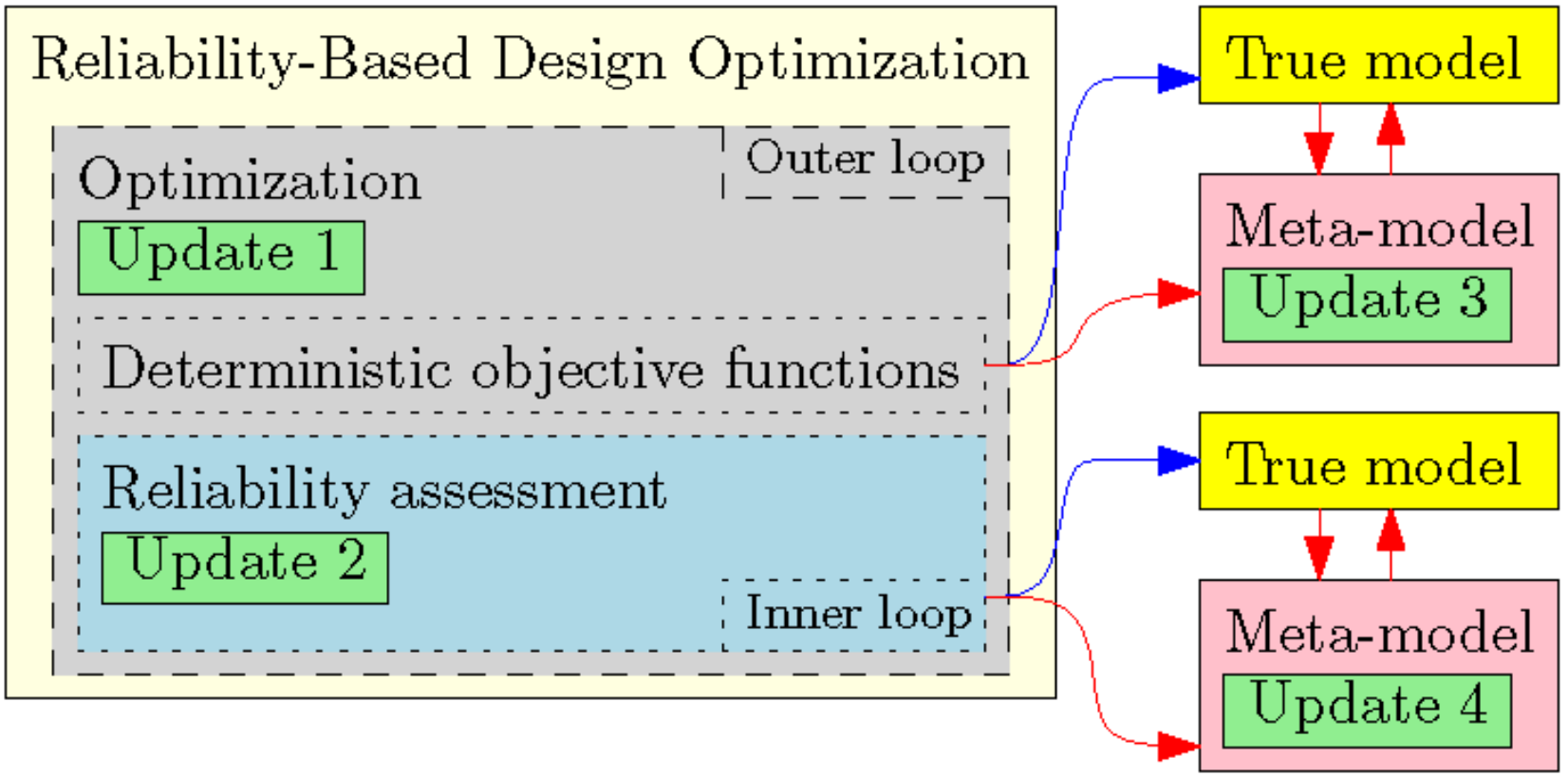




Design of experiments (DoE)

- an essential part of experimentation, meta-modeling, sensitivity analysis or probability computations
- basic requirements
  - orthogonality
  - space-filling properties
- quality of the designs is evaluated by criteria

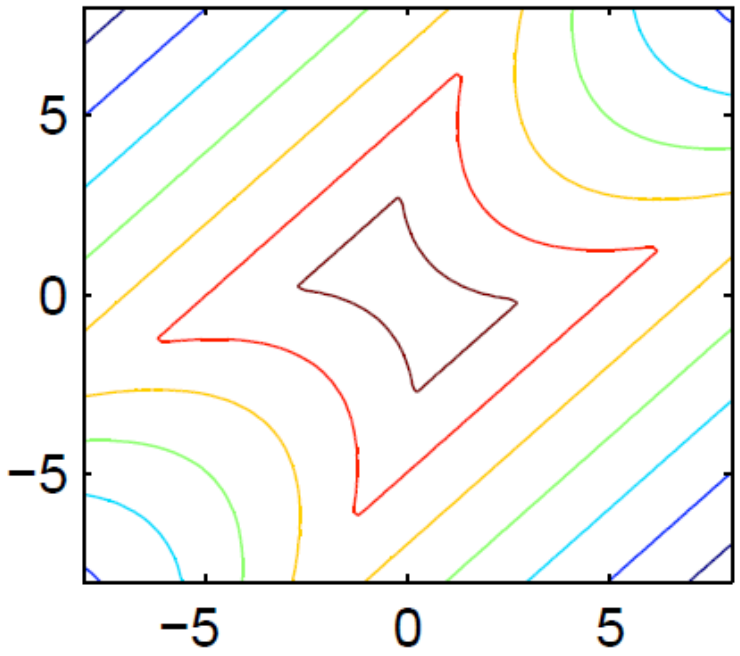
Overview

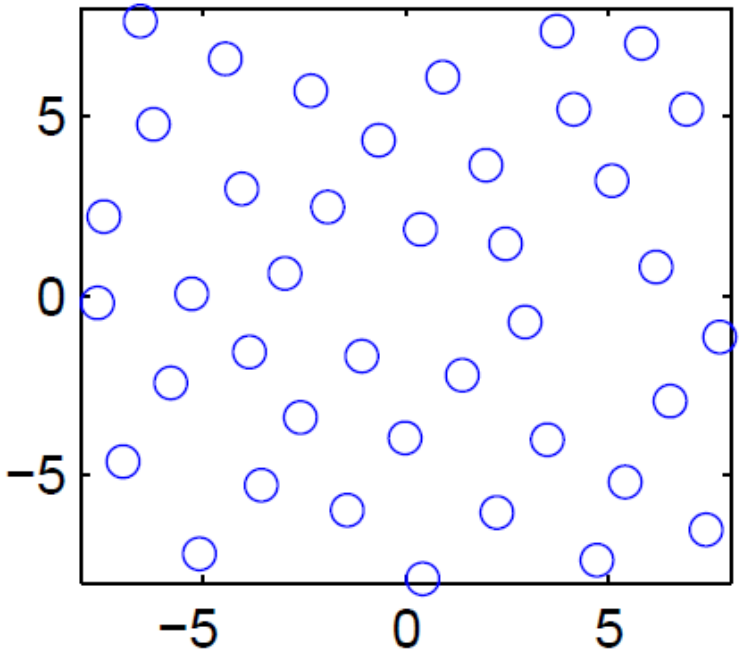

Reliability-Based Design Optimization
Optimization
Outer loop
Update 1
Deterministic objective functions
Reliability assessment
Update 2
Inner loop
True model
Meta-model
Update 3
True model
Meta-model
Update 4

Surrogate model (meta-model)

1. initial DoE
2. true responses of the full model in design points
3. construction of meta-model
4. Adaptive updating procedure:
   - multi-objective optimization problem

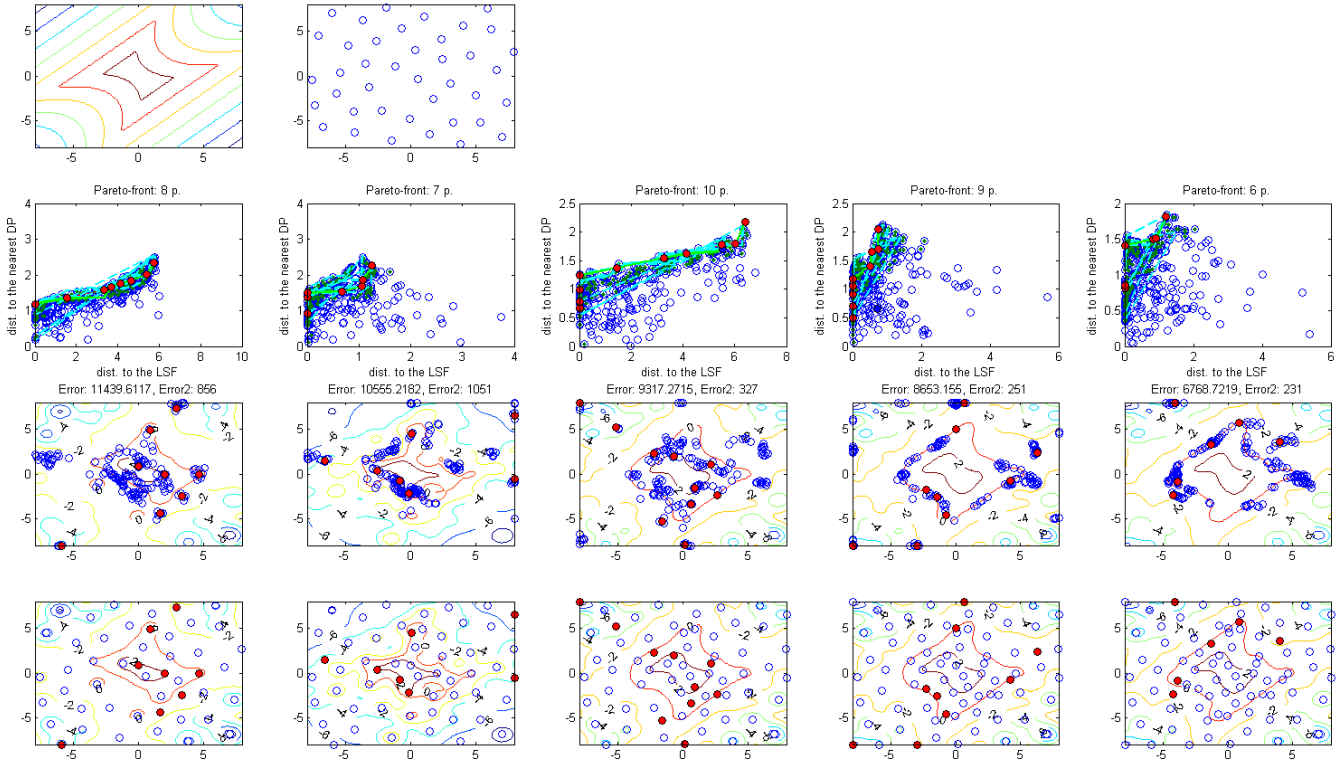
Adaptive procedure for meta-model updating





Adaptive procedure for meta-model updating

- new points added into initial DoE in:
  1. unsampled region („holes“ in the design)
  2. interresting area from model-response point of view



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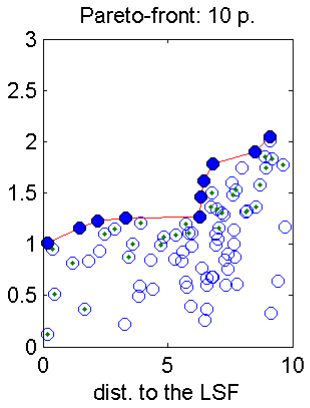
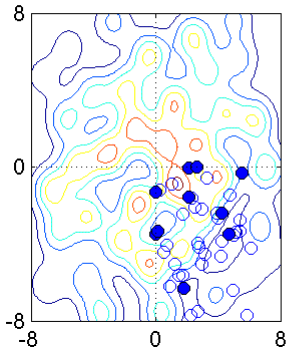
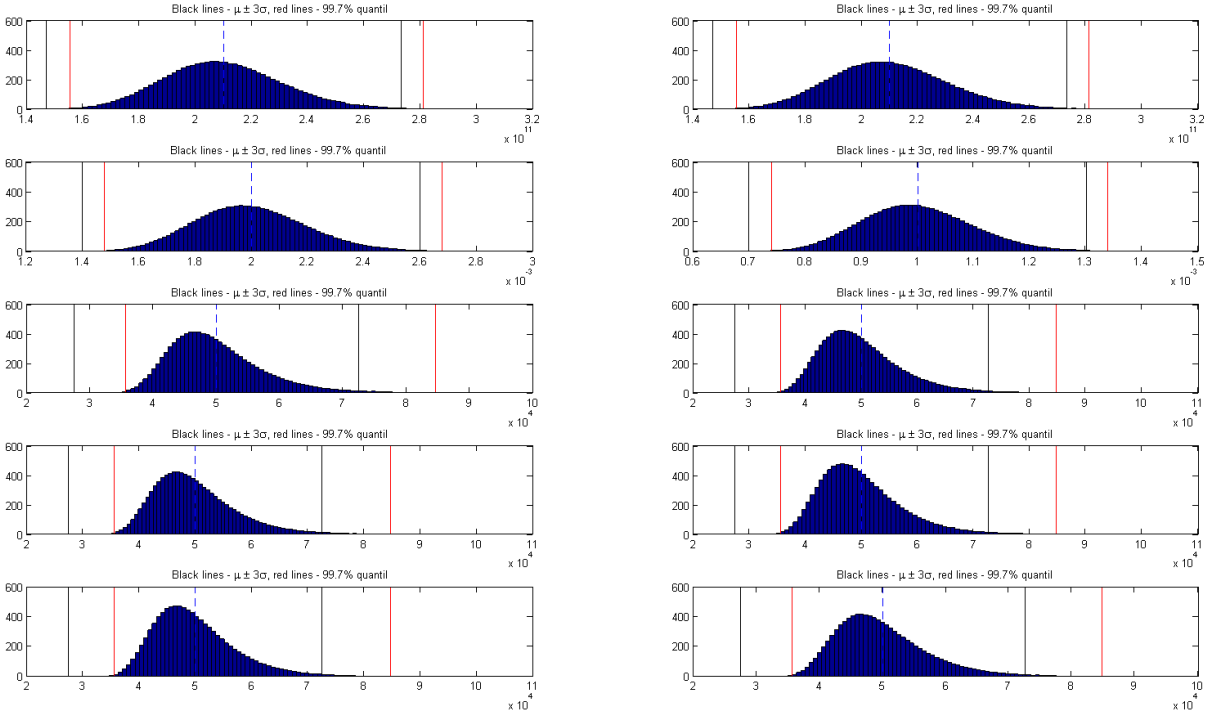




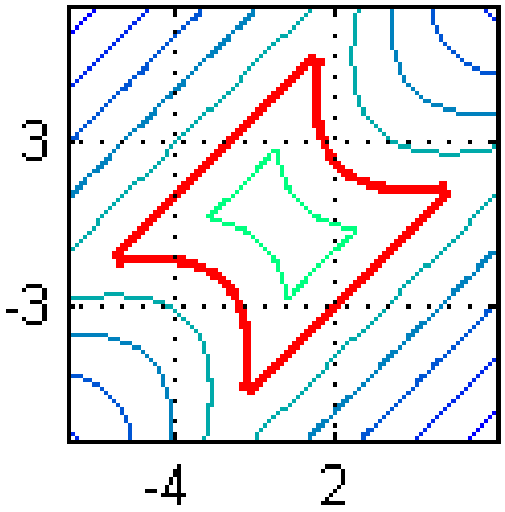
Restricted domain for meta-model construction

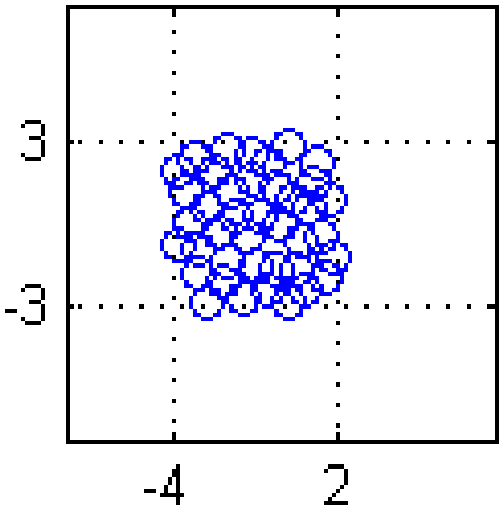
- selection of the bounds of the domain
  - $\mu \pm 3\sigma$
  - <0,0015; 0,9985> - quantile (covers approx. 99,7 % of realizations)



Advanced adaptive procedure for meta-model construction

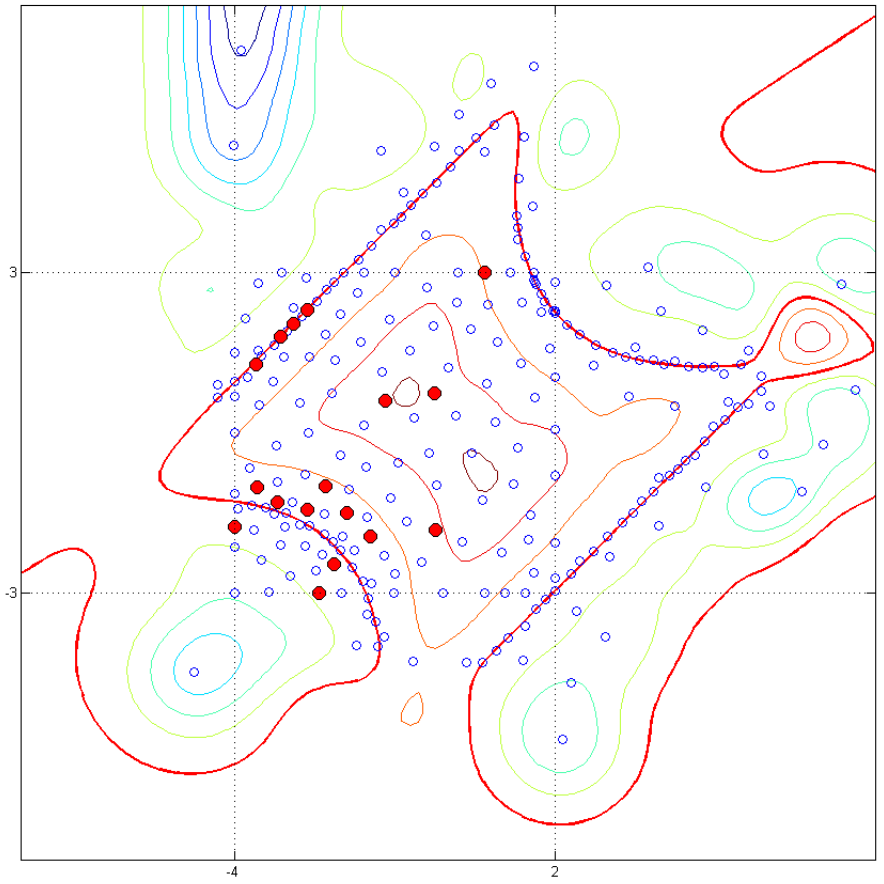
- risk of omitting important regions
- sampling outside the bounds is needed





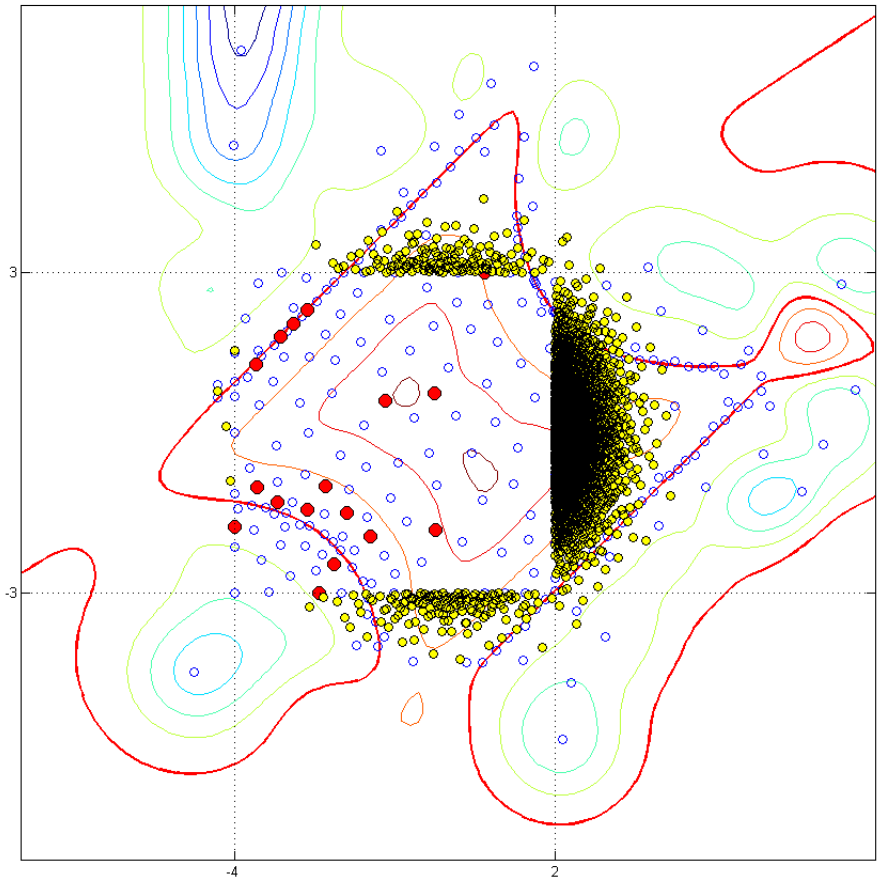
Advanced adaptive procedure for meta-model construction

1. new points from the inside of the domain are added (found by the two-criterial procedure)



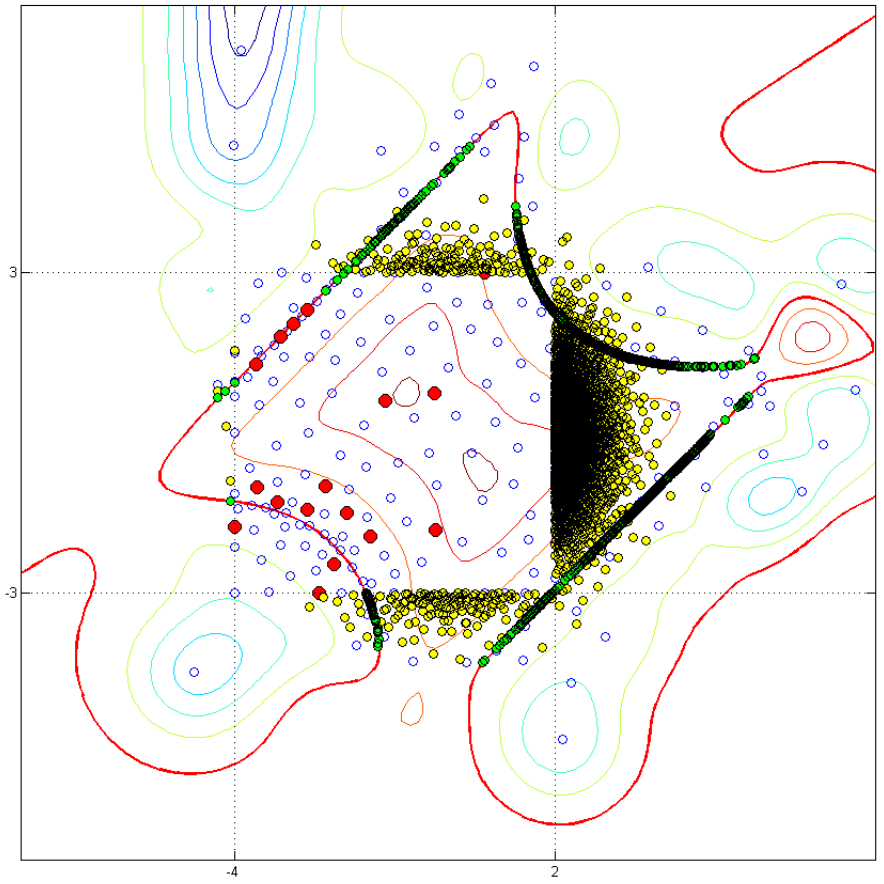
Advanced adaptive procedure for meta-model construction

2. the sampling from the prescribed distribution is performed



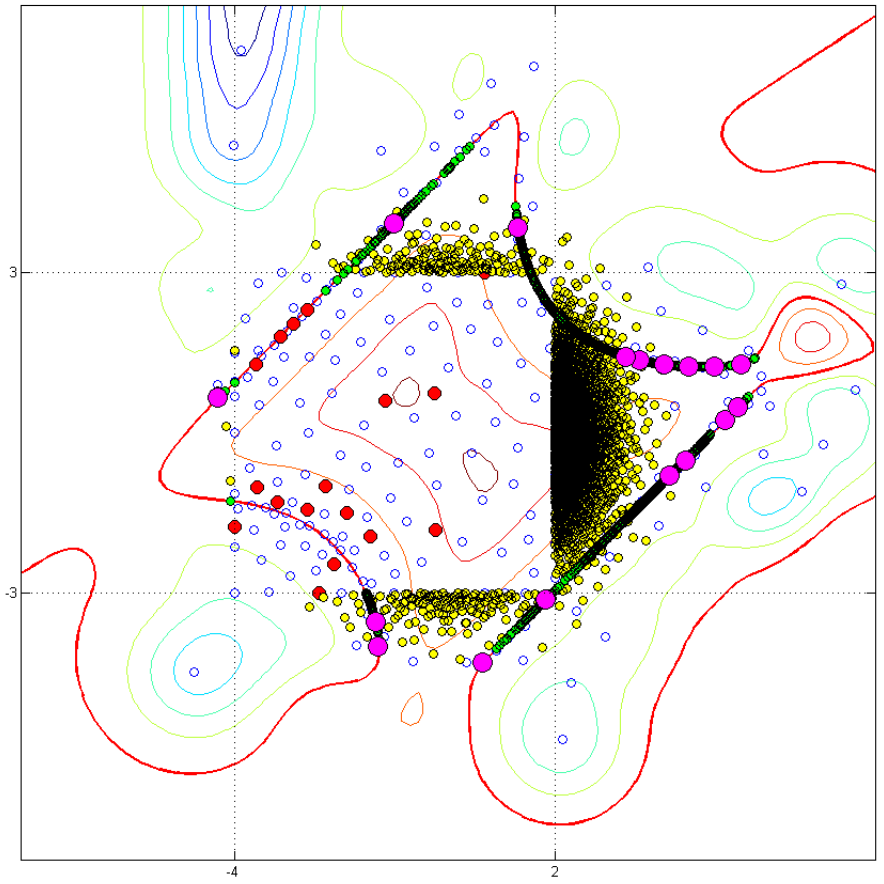
Advanced adaptive procedure for meta-model construction

3. for the outlying points the closest point on the predicted limit state is found



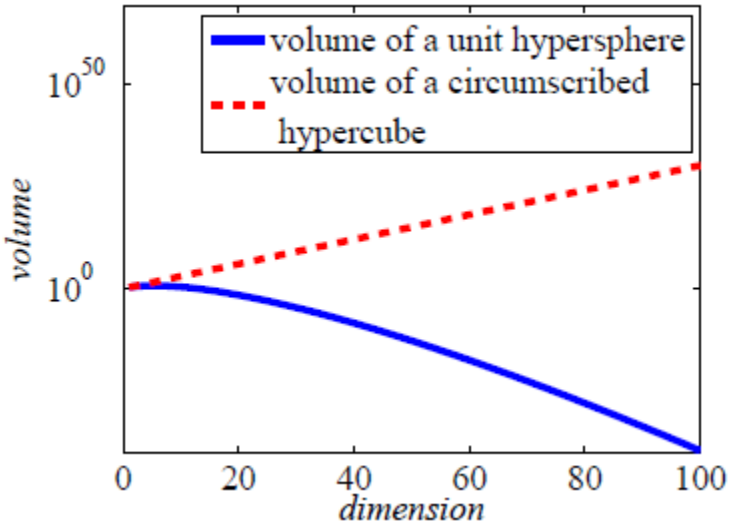
Advanced adaptive procedure for meta-model construction

4. some of these points are also added into the design



DoE in hyperspheres

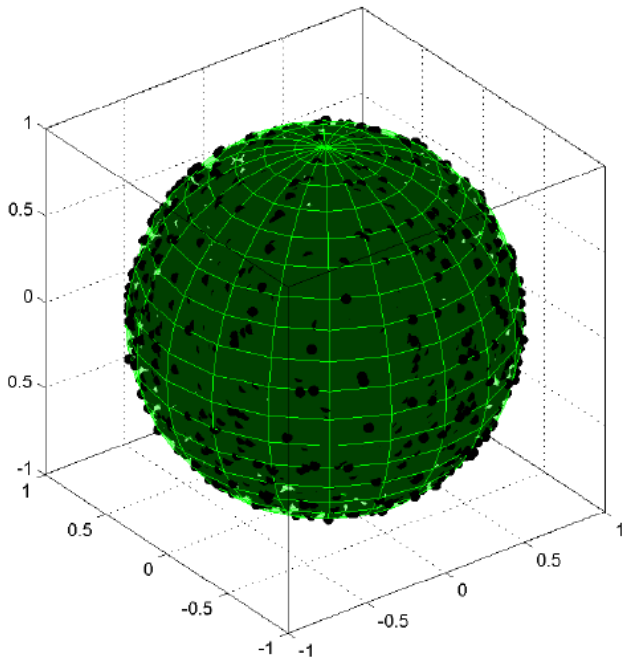
- an alternative to the most commonly used design domain – hypercube
- the simplest way:
  1. repeatedly generate points in the circumscribed hypercube
  2. exclude those lying outside the hypersphere

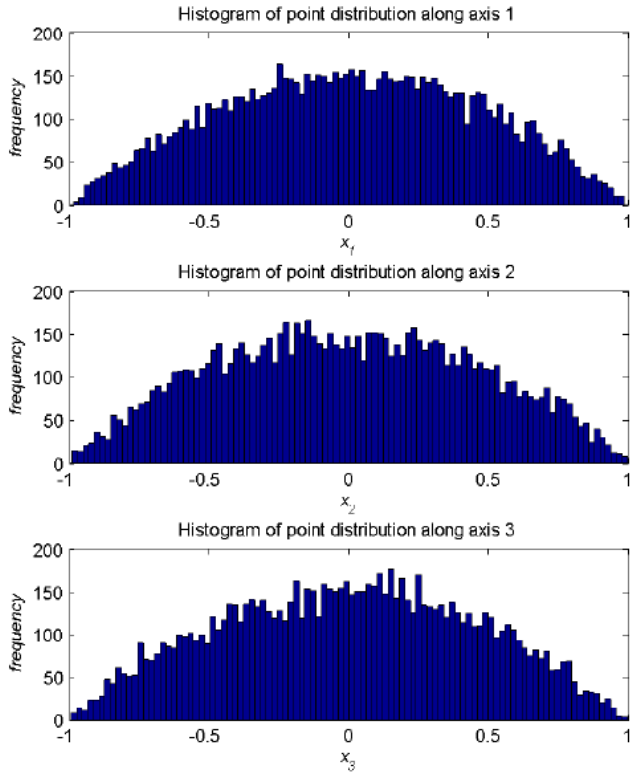


DoE in hyperspheres

- uniform point set:

$$X = U^{\frac{1}{n}} \cdot \frac{Y}{\|Y\|_2}$$





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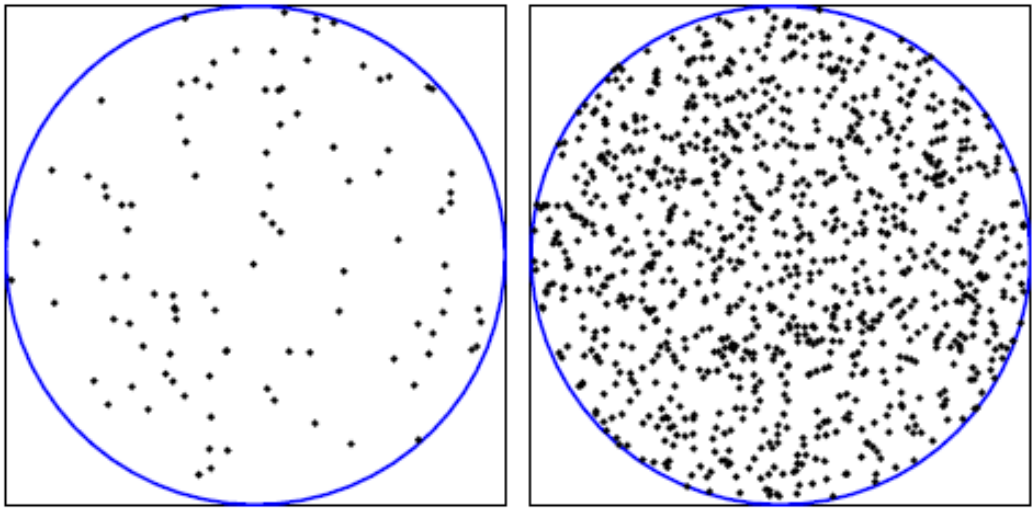


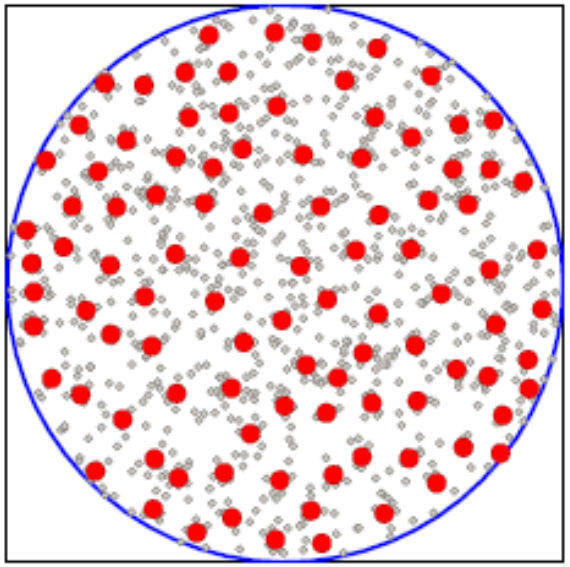
Space-filling DoE in hyperspheres

- reducing of the number of points → DoE:
  - clustering [Dubourg, 2011]
  - removal of superfluous points from intentionally overcrowded initial design
    - in each step one of points from the actual closest pair is removed
      - removal
      - removal_NEW

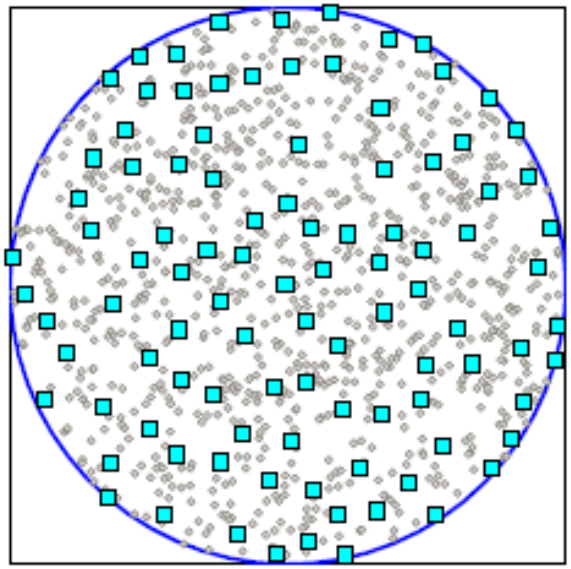
Space-filling DoE in hyperspheres

- reducing of the number of points → DoE:

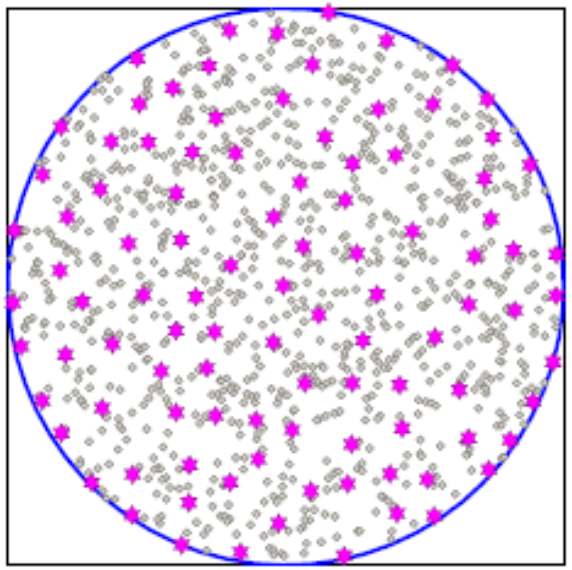
a) clustering



b) removal



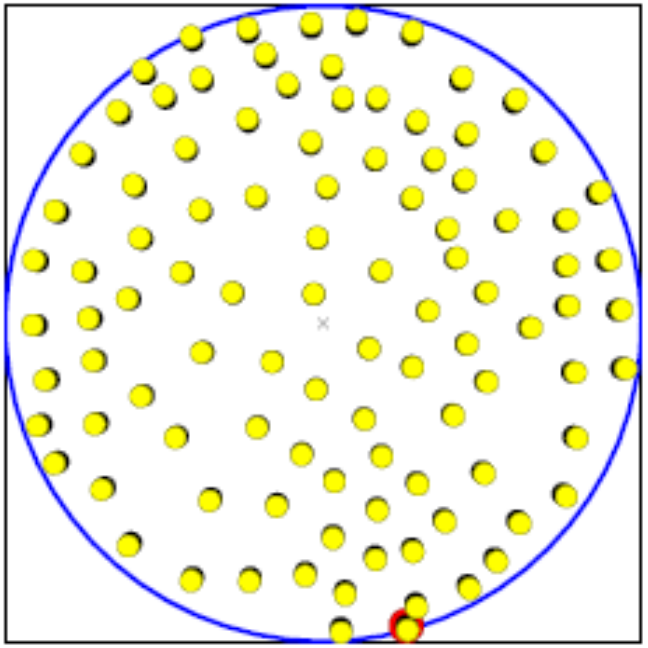
c) removal_NEW



DoE in hyperspheres - results

- spreading of the designs for comparable results:

$$X_{spread} = \frac{X}{\max \left\| X_i \right\|_2}; \qquad i = 1,\ldots,n$$

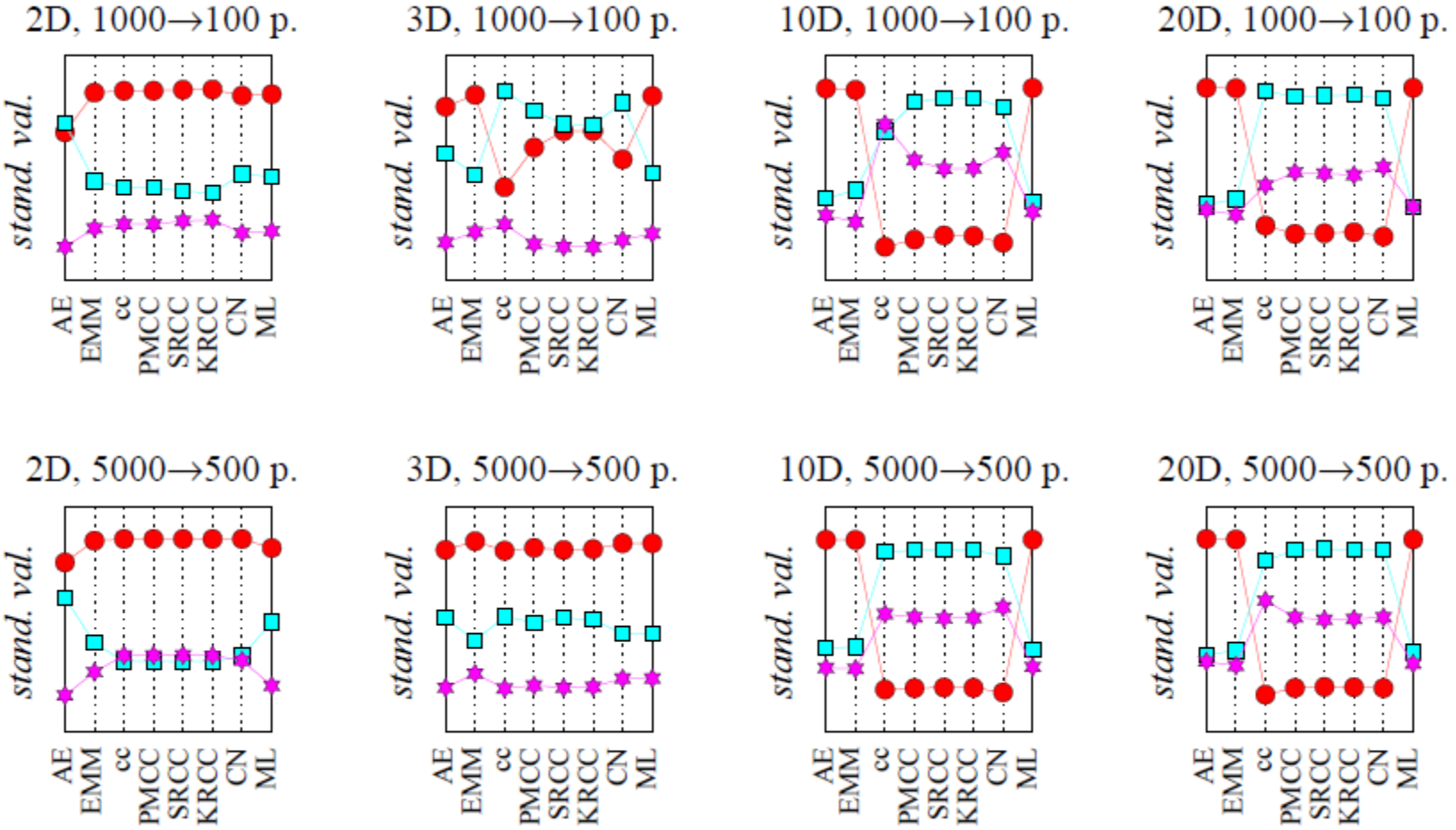


DoE in hyperspheres - results

- criteria for evaluation of the design quality:
  - criterion Audze-Eglais (AE)
  - Euclidean Maximin distance (EMM)
  - the largest element of correlation matrix (cc)
  - Pearson correlation coefficient (PMCC)
  - Spearman correlation coefficient (SRCC)
  - Kendall correlation coefficient (KRCC)
  - condition number (CN)
  - ML2 dicrepancy (ML)

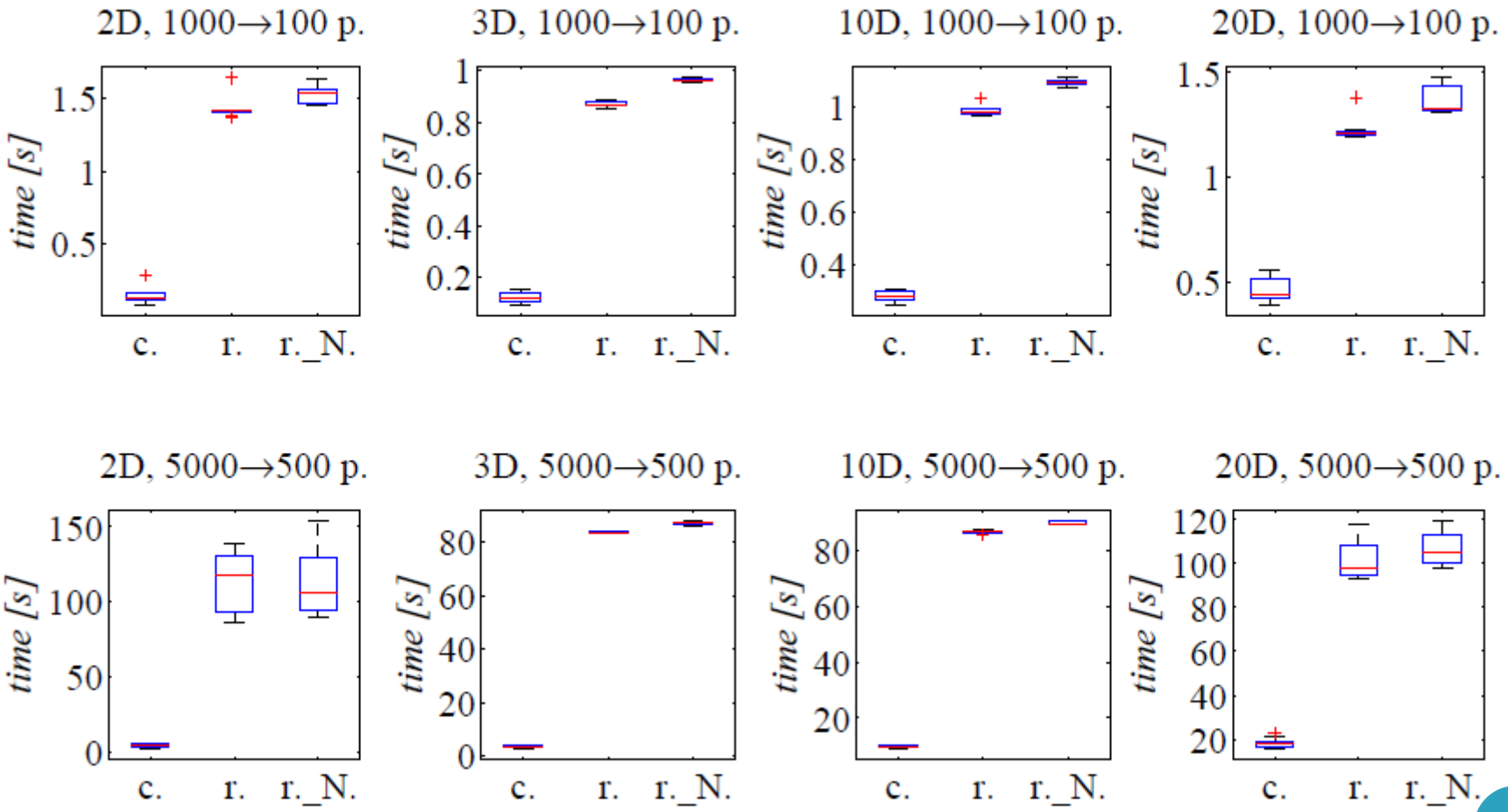
DoE in hyperspheres - results

- quality:



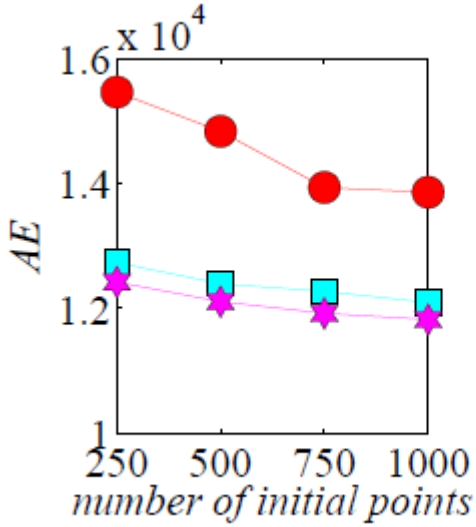
DoE in hyperspheres - results

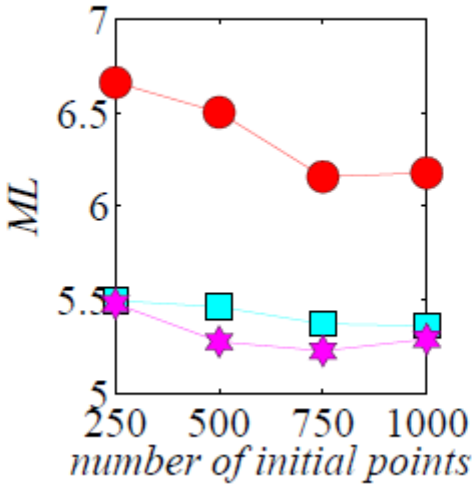
- time demands:

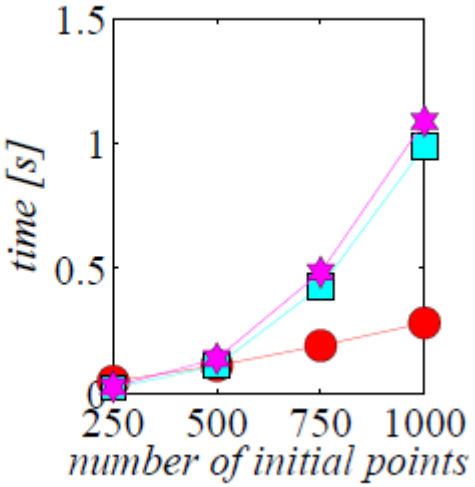


DoE in hyperspheres - results

- influence of the overcrowding level:







Thank you for your attention.