Semestrální práce z předmětu PRPE

Téma: Analýza LL modelu (Ladeveze-Lemaitre) pro deaktivaci poškození (nekonzistence při přechodu středního napětí přes nulu – opačný účinek, než by se dalo čekat)

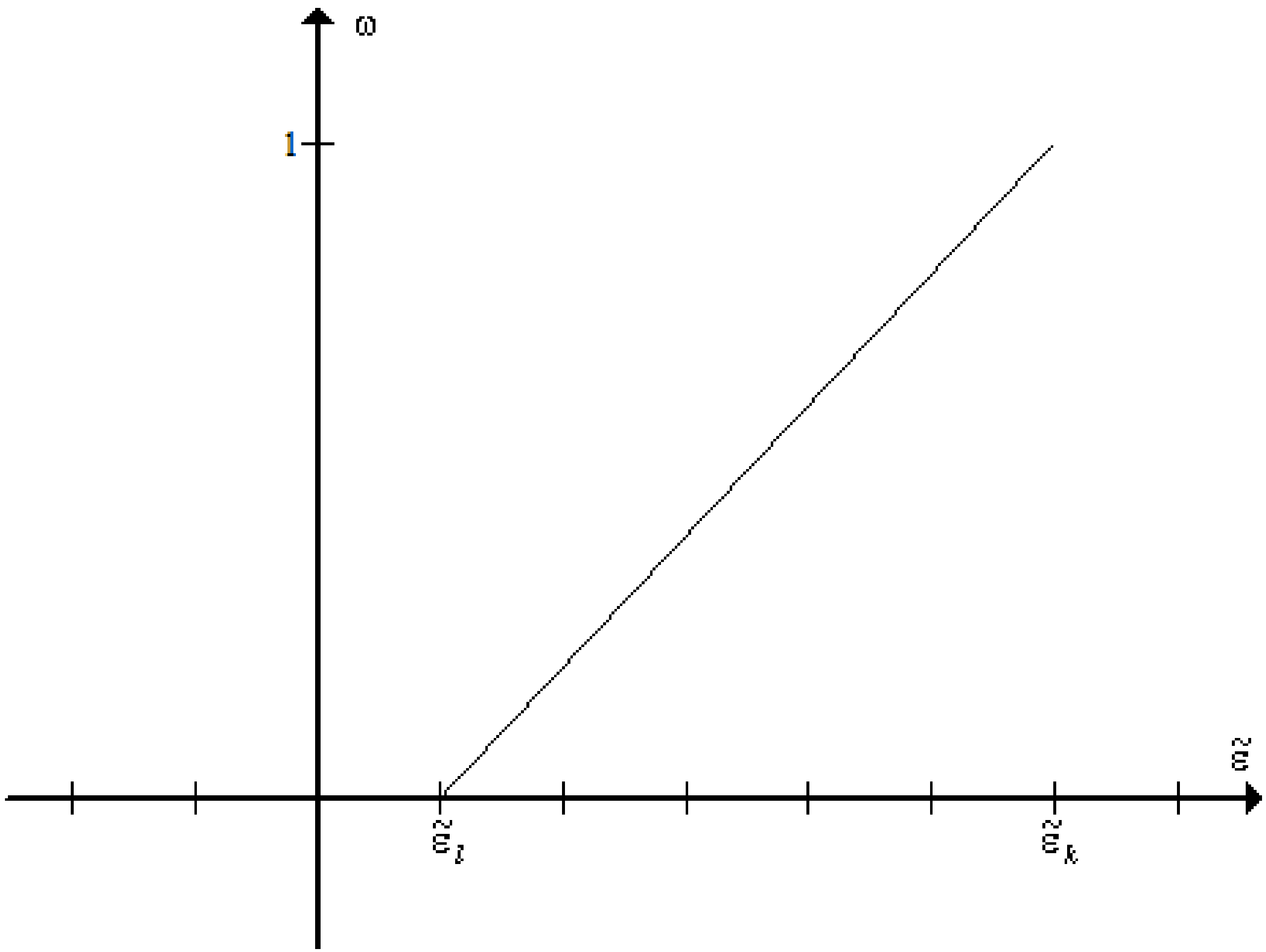
Vedoucí práce: prof. Ing. Milan Jirásek DrSc.
Vypracoval: Vladimír Vančík

Model izotropního poškození

$$\varepsilon = \frac{1}{1-\omega} C_e \sigma$$

$$\sigma = (1-\omega) D_e \varepsilon$$

Mazarsova deformace $\tilde{\varepsilon} = \sqrt{\langle \varepsilon_1 \rangle^2 + \langle \varepsilon_2 \rangle^2 + \langle \varepsilon_3 \rangle^2}$



ω
1
$\tilde{\omega}_l$
$\tilde{\omega}_k$
$\tilde{\omega}$

$$\tilde{\varepsilon} \in \left\langle \tilde{\varepsilon}_l , \tilde{\varepsilon}_k \right\rangle \Rightarrow \omega\left(\tilde{\varepsilon}\right) = k\left(\tilde{\varepsilon} - \tilde{\varepsilon}_l\right)$$

$$\omega\left(\tilde{\varepsilon}_k\right) = 1 \Rightarrow k\left(\tilde{\varepsilon}_k - \tilde{\varepsilon}_l\right) = 1 \Rightarrow k = \frac{1}{\tilde{\varepsilon}_k - \tilde{\varepsilon}_l}$$

$$\nu = 0{,}2 \qquad E = 20GPa \qquad \tilde{\varepsilon}_l = 10^{-4} \qquad \tilde{\varepsilon}_k = 8 \times 10^{-3}$$

$$k = \frac{1}{\tilde{\varepsilon}_k - \tilde{\varepsilon}_l} = \frac{1}{79} \times 10^4$$

$$\omega\left(\tilde{\varepsilon}\right) = \frac{10^4}{79} \times \left(\tilde{\varepsilon} - 10^{-4}\right)$$

Stanovení mezí pevnosti

$$\sigma_x = \left(1 - \omega\left(\tilde{\varepsilon}\right)\right) \times \frac{E}{(1-2\nu)\times(1+\nu)} \times \left((1-\nu)\varepsilon_x + \nu\varepsilon_y + \nu\varepsilon_z\right)$$

$$jednoos\acute{y}\,tah \Rightarrow \varepsilon_x > 0, \varepsilon_y = \varepsilon_z = -\nu\varepsilon_x \Rightarrow \tilde{\varepsilon} = \varepsilon_x, \omega\left(\tilde{\varepsilon}\right) = \omega(\varepsilon_x)$$

$$\sigma_x = E \times \left(1 - \omega(\varepsilon_x)\right) \times \varepsilon_x$$

$$\frac{d\sigma_x}{d\varepsilon_x} = E \times \left(\left(1 - \omega(\varepsilon_x)\right) - \omega'(\varepsilon_x)\varepsilon_x\right)$$

$$1 - k\varepsilon_x + k\varepsilon_l - k\varepsilon_x = 0$$

$$\varepsilon_x = \frac{1 + k\varepsilon_l}{2k} = 4 \times 10^{-3} \qquad \omega = 0{,}494$$

$$mez\,pevnosti\,\sigma_{x_{\max}} = 40{,}5\,MPa$$

$$\sigma_x = \left(1 - \omega\left(\tilde{\varepsilon}\right)\right) \times \frac{E}{(1-2\nu)\times(1+\nu)} \times \left((1-\nu)\varepsilon_x + \nu\varepsilon_y + \nu\varepsilon_z\right)$$

$$\mathit{jednoos\acute{y}\ tlak} \Rightarrow \varepsilon_x < 0, \varepsilon_y = \varepsilon_z = -\nu\varepsilon_x,\ \tilde{\varepsilon} = \sqrt{\varepsilon_y^2 + \varepsilon_z^2} = \sqrt{2\nu^2\varepsilon_x^2}$$

$$E \times \left(\left(1 - \omega(\varepsilon_x)\right) - \omega'(\varepsilon_x)\varepsilon_x\right) = 0$$

$$1 - k\sqrt{2}\,\nu\varepsilon_x + k\varepsilon_l - k\sqrt{2}\,\nu\varepsilon_x = 0$$

$$\varepsilon_x = \frac{1 + k\varepsilon_l}{2k\sqrt{2}\,\nu} = 0{,}0141$$

$$\mathit{mez\ pevnosti}\ \sigma_{x_{\min}} = 142{,}7\ MPa$$

$$\frac{1}{\sqrt{2}\,\nu} \approx 3{,}5$$

LL model

Střední napětí $\sigma_m = \frac{1}{3} \times \left(\sigma_1 + \sigma_2 + \sigma_3\right)$

$$\varepsilon_1 = \frac{1+\nu}{E(1-\omega)}\sigma_1 - \frac{3\nu}{E(1-\omega)}\sigma_m$$

$$\varepsilon_2 = \frac{1+\nu}{E(1-\omega)}\sigma_2 - \frac{3\nu}{E(1-\omega)}\sigma_m$$

$$\varepsilon_3 = \frac{1+\nu}{E(1-\omega)}\sigma_3 - \frac{3\nu}{E(1-\omega)}\sigma_m$$

$$\sigma_m > 0, \sigma_i > 0 \Rightarrow \varepsilon_i = \frac{1+\nu}{E(1-\omega)}\sigma_i - \frac{3\nu}{E(1-\omega)}\sigma_m$$

$$\sigma_m > 0, \sigma_i < 0 \Rightarrow \varepsilon_i = \frac{1+\nu}{E}\sigma_i - \frac{3\nu}{E(1-\omega)}\sigma_m$$

$$\sigma_m < 0, \sigma_i > 0 \Rightarrow \varepsilon_i = \frac{1+\nu}{E(1-\omega)}\sigma_i - \frac{3\nu}{E}\sigma_m$$

$$\sigma_m < 0, \sigma_i < 0 \Rightarrow \varepsilon_i = \frac{1+\nu}{E}\sigma_i - \frac{3\nu}{E}\sigma_m$$

Jednoosé namáhání

$\sigma_x = 15MPa$ překračujeme mez pevnosti

$\varepsilon_x = 7{,}17 \times 10^{-3}$ $\omega = 0{,}895$

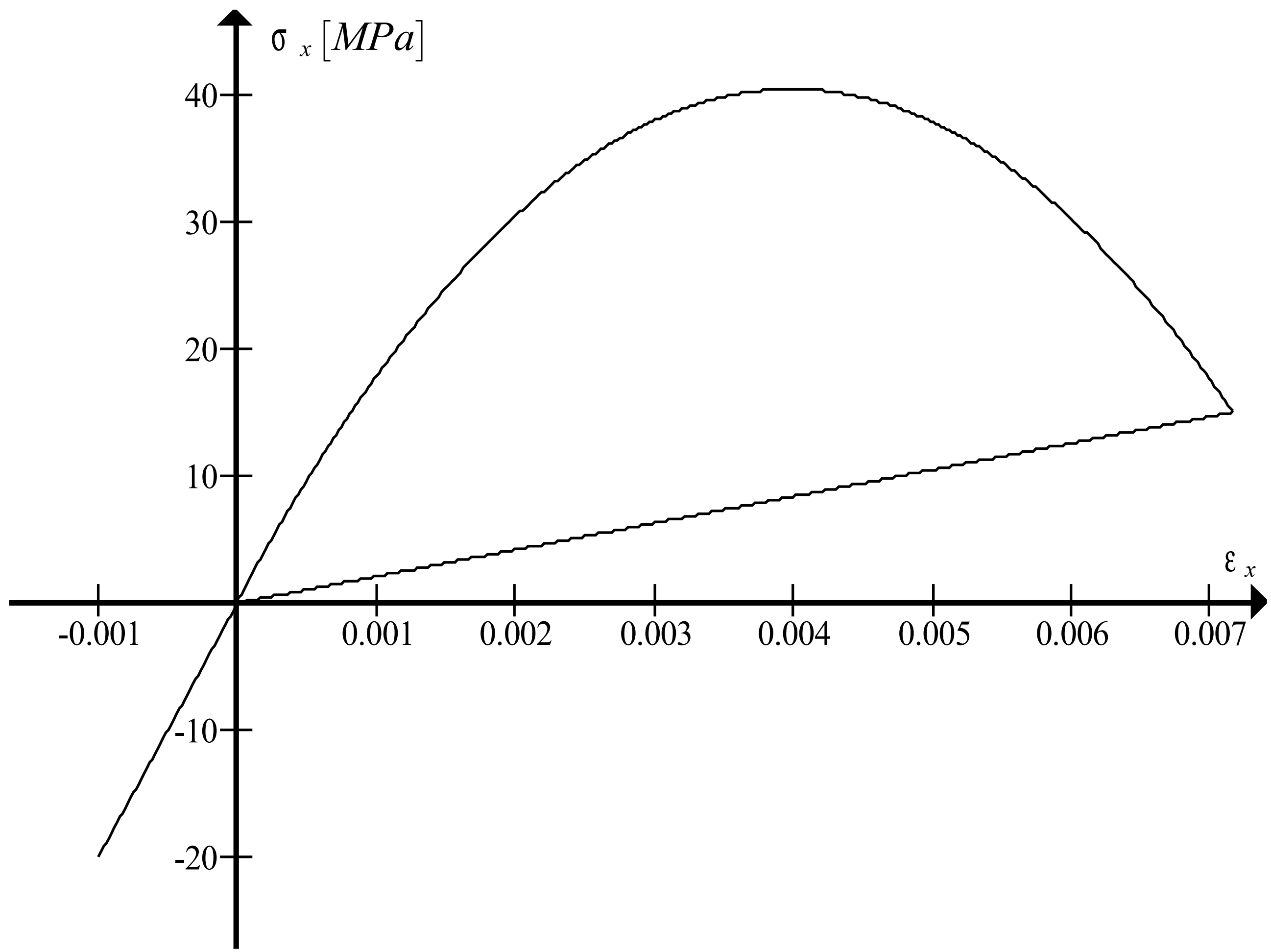
Odtěžování:

$$\sigma_x \in \left\langle 0{,}15 \right\rangle ; \varepsilon_x = \frac{1}{E(1-\omega)}\left((1+\nu)\sigma_x - 3\nu\sigma_m\right)$$

poškození je zapnuté u obou členů

$$\sigma_x \in \left\langle -20{,}0 \right\rangle ; \varepsilon_x = \frac{1}{E}\left((1+\nu)\sigma_x - 3\nu\sigma_m\right)$$

poškození je vypnuté u obou členů



σ x [MPa]
40
30
20
10
-10
-20
ε x
-0.001
0.001
0.002
0.003
0.004
0.005
0.006
0.007

Zatěžovací zkouška č.1

Smyková napětí předpokládáme nulová

$$\sigma_x = 5MPa, \sigma_y = 0, \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E\left(1-\omega\left(\varepsilon_x\right)\right)}\sigma_x$$

$$\varepsilon_x\left(1-\omega\left(\varepsilon_x\right)\right) - \frac{\sigma_x}{E} = 0$$

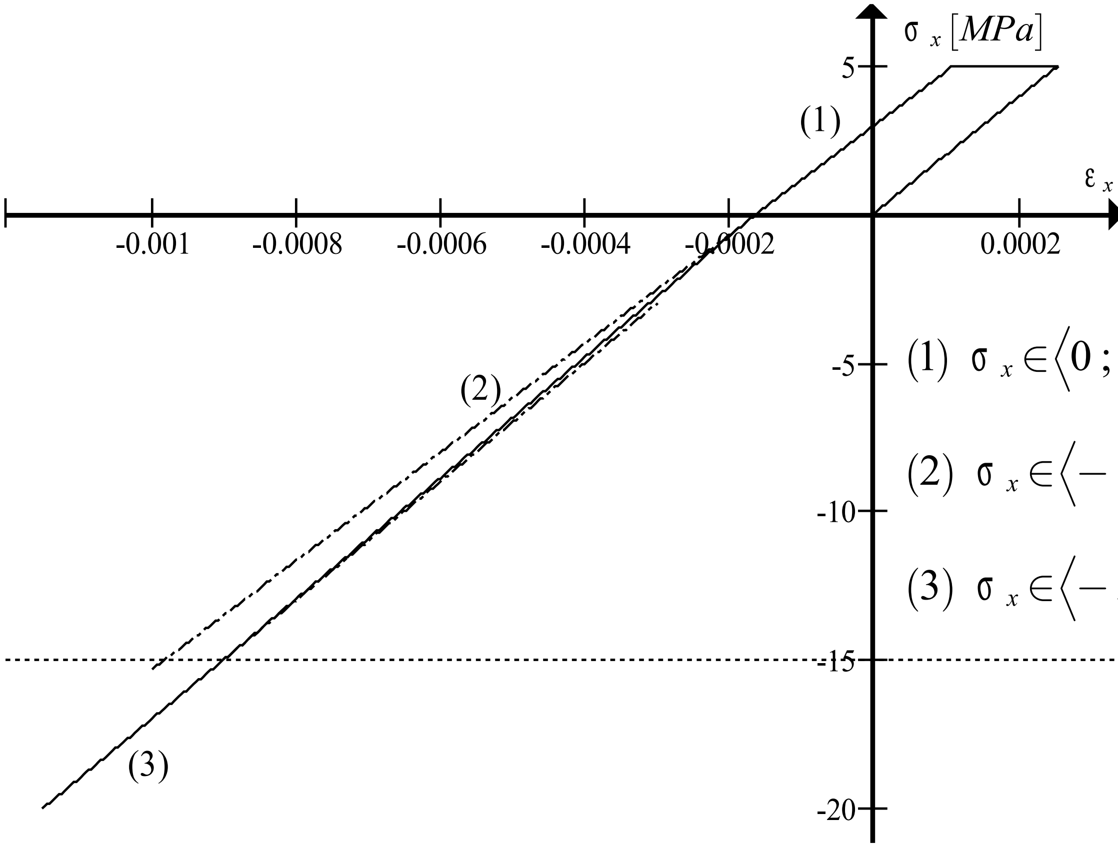
$$-k\varepsilon_x^2 + \left(k\varepsilon_l + 1\right)\varepsilon_x - \frac{\sigma_x}{E} = 0 \Rightarrow \varepsilon_x = 2{,}55\times 10^{-4}$$

$$\sigma_x = 5\,MPa, \sigma_y = 0, \sigma_z = 15MPa$$

$$\varepsilon_x = \frac{1}{E\left(1-\omega\left(\tilde{\varepsilon}\right)\right)}\left(\sigma_x - \nu\sigma_z\right) \qquad \Rightarrow \frac{\varepsilon_x}{\varepsilon_z} = \frac{1}{7} \Rightarrow \tilde{\varepsilon} = \sqrt{\varepsilon_x^2 + \left(7\varepsilon_x\right)^2} = \sqrt{50}\times\varepsilon_x$$

$$\varepsilon_z = \frac{1}{E\left(1-\omega\left(\tilde{\varepsilon}\right)\right)}\left(\sigma_z - \nu\sigma_x\right) \qquad \omega\left(\tilde{\varepsilon}\right) = k\left(\sqrt{50}\,\varepsilon_x - 10^{-4}\right)$$

$$\varepsilon_x = 1{,}0931\times 10^{-4} \qquad \omega = 0{,}0852$$



$$(1)\quad \sigma_x \in \langle 0\,;5\rangle \Rightarrow \varepsilon_x = \frac{1}{E(1-\omega)}\left((1+\nu)\sigma_x - 3\nu\sigma_m\right)$$

$$(2)\quad \sigma_x \in \langle -15\,;0\rangle \Rightarrow \varepsilon_x = \frac{1+\nu}{E}\sigma_x - \frac{3\nu}{E(1-\omega)}\sigma_m$$

$$(3)\quad \sigma_x \in \langle -20\,;-15\rangle \Rightarrow \varepsilon_x = \frac{1}{E}\left((1+\nu)\sigma_x - 3\nu\sigma_m\right)$$

Zatěžovací zkouška č.2

$$\sigma_y = 0, \sigma_x = \sigma_z > 0 \Rightarrow \varepsilon_x = \varepsilon_z > 0 \Rightarrow \omega = k\left(\sqrt{2\varepsilon_x^2} - \tilde{\varepsilon}_l\right)$$

$$\omega \to 1 \Rightarrow \varepsilon_x \to 5{,}66 \times 10^{-3}$$

$$\varepsilon_x = \varepsilon_z = 5 \times 10^{-3} \Rightarrow \omega = 0{,}8824$$

$$\sigma_x = \sigma_z = 14{,}7 MPa$$

$$\sigma_x \to -30 MPa$$

$$(1)\quad \sigma_x \in \left\langle 0 ; 14{,}7 \right\rangle \Rightarrow \varepsilon_x = \frac{1}{E(1-\omega)}\left((1+\nu)\sigma_x - 3\nu\sigma_m\right)$$

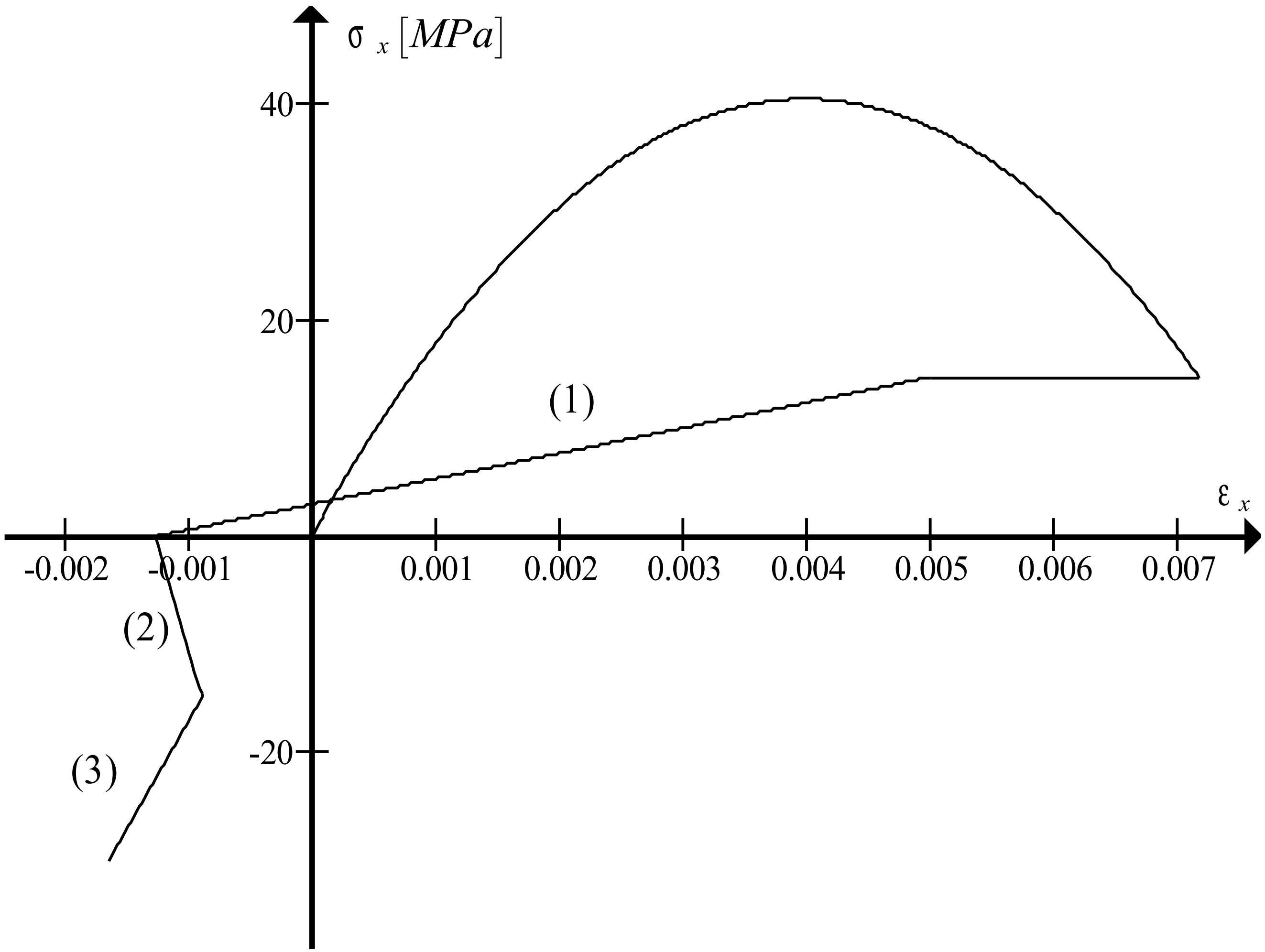
$$(2)\quad \sigma_x \in \left\langle -14{,}7 ; 0 \right\rangle \Rightarrow \varepsilon_x = \frac{1+\nu}{E}\sigma_x - \frac{3\nu}{E(1-\omega)}\sigma_m$$

$$(3)\quad \sigma_x \in \left\langle -30 ; -14{,}7 \right\rangle \Rightarrow \varepsilon_x = \frac{1}{E}\left((1+\nu)\sigma_x - 3\nu\sigma_m\right)$$

$$\sigma_x = 0 \Rightarrow \varepsilon_x = -1{,}25 \times 10^{-3}$$

$$\sigma_x = -14{,}7\, MPa \Rightarrow \varepsilon_x = -0{,}882 \times 10^{-3}$$

$$\sigma_x = -30\, MPa \Rightarrow \varepsilon_x = -1{,}647 \times 10^{-3}$$



σ x [MPa]
40
20
-20
(1)
(2)
(3)
ε x
-0.002
-0.001
0.001
0.002
0.003
0.004
0.005
0.006
0.007

Analýza vlastních čísel

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{pmatrix} = \frac{1}{E} \begin{pmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$$

$$\lambda_1 = 1 + \nu \qquad \lambda_2 = 1 + \nu \qquad \lambda_3 = 1 - 2\nu$$

$$\lambda_{\min} = 1 - 2\nu \qquad \lambda_{\max} = \frac{1 + \nu}{1 - \omega}$$

$$a)\ \sigma_x > 0, \sigma_y < 0, \sigma_z < 0, \sigma_m > 0$$

$$b)\ \sigma_x > 0, \sigma_y < 0, \sigma_z < 0, \sigma_m < 0$$

$$c)\ \sigma_x < 0, \sigma_y > 0, \sigma_z > 0, \sigma_m > 0$$

$$d)\ \sigma_x < 0, \sigma_y > 0, \sigma_z > 0, \sigma_m < 0$$

$$\sigma_x > 0, \sigma_y < 0, \sigma_z < 0, \sigma_m > 0$$

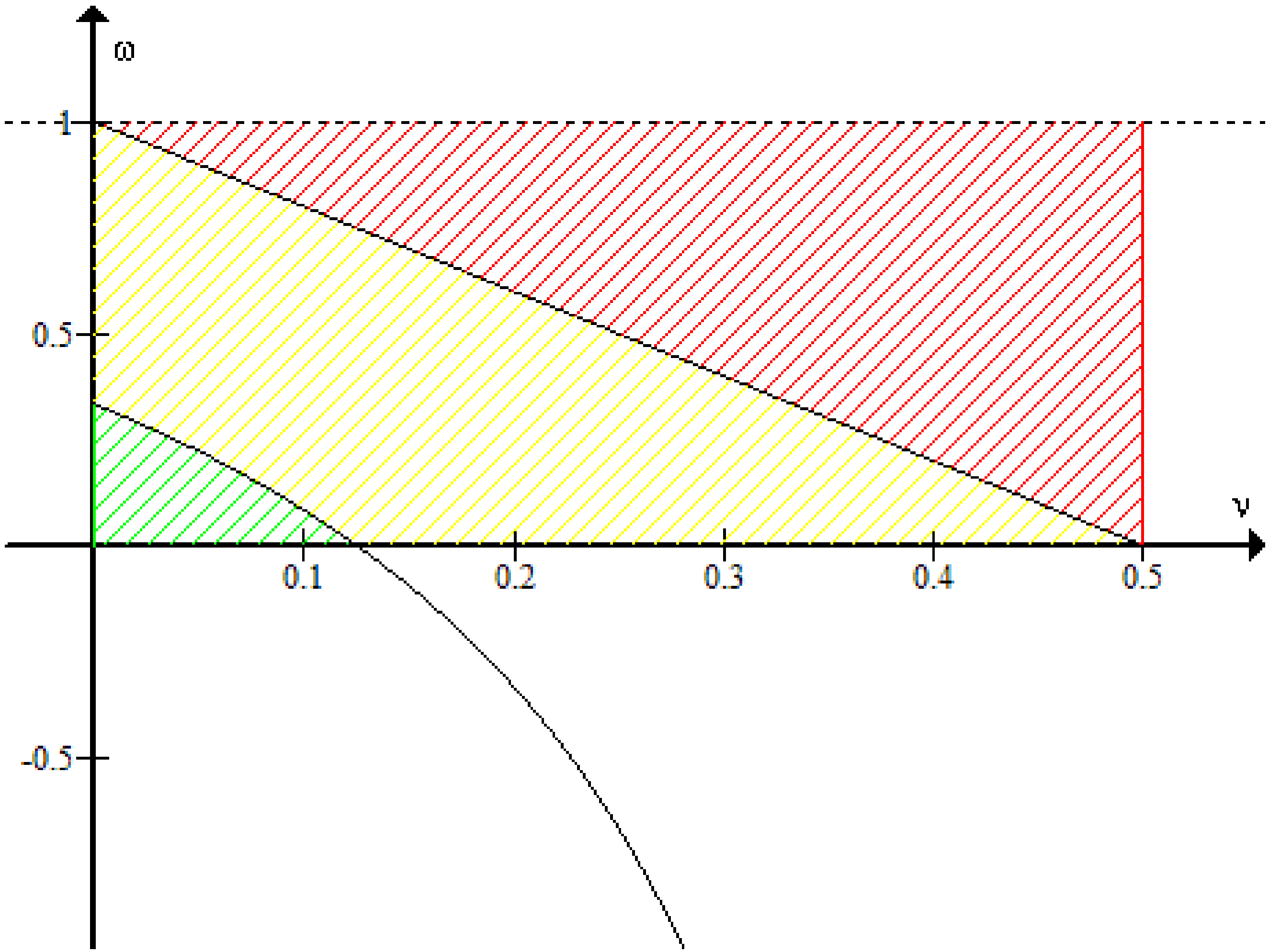
$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{pmatrix} = \frac{1}{E} \begin{pmatrix} \frac{1}{1-\omega} & \frac{-\nu}{1-\omega} & \frac{-\nu}{1-\omega} \\ \frac{-\nu}{1-\omega} & 1+\nu-\frac{\nu}{1-\omega} & \frac{-\nu}{1-\omega} \\ \frac{-\nu}{1-\omega} & \frac{-\nu}{1-\omega} & 1+\nu-\frac{\nu}{1-\omega} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$$

$$\lambda_1 = 1+\nu$$

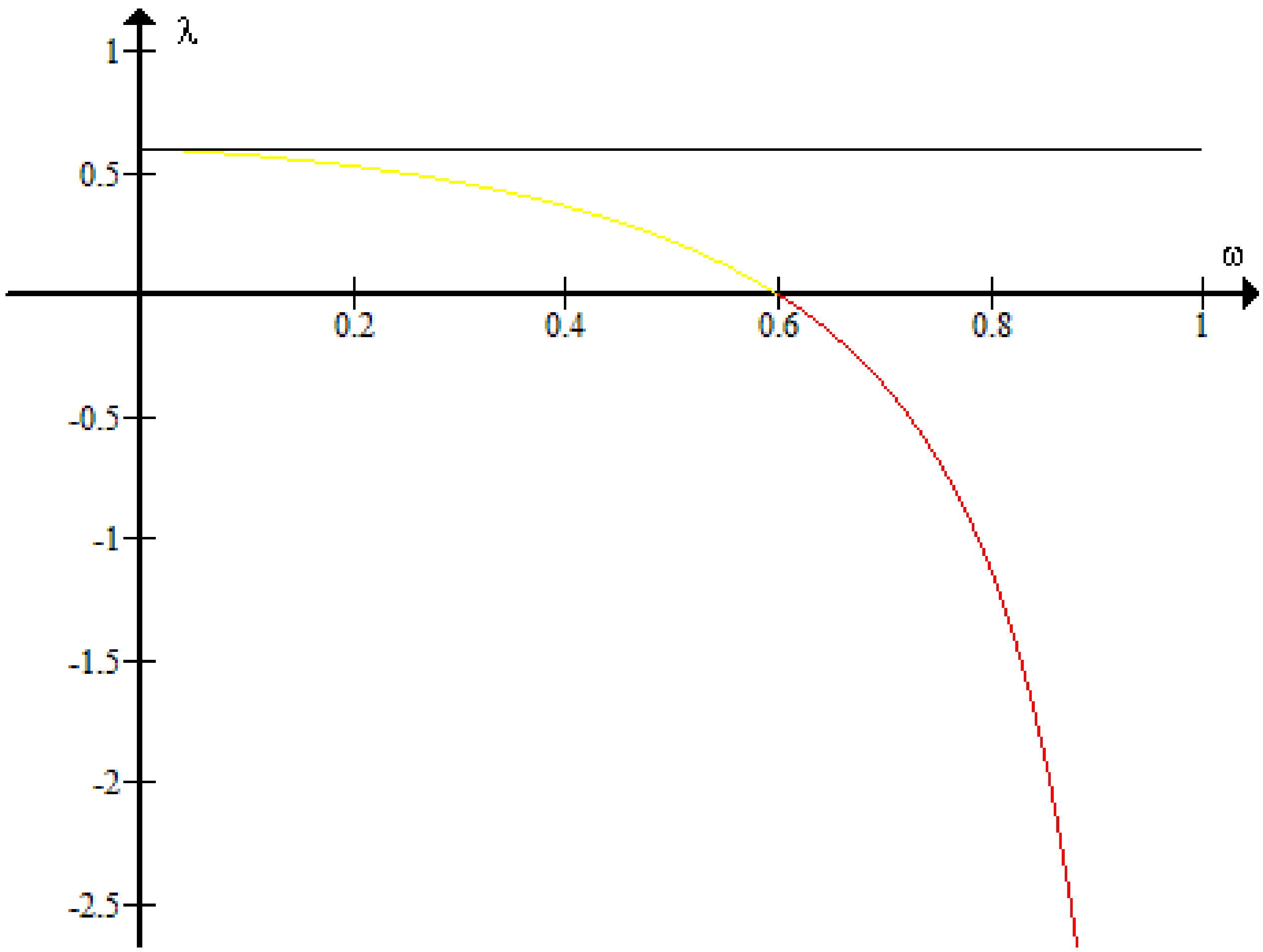
$$\lambda_2 = \frac{2-\nu-3\omega+\omega^2+\nu\omega^2-(-1+\omega)\sqrt{9\nu^2+2\nu\omega+2\nu^2\omega+\omega^2+2\nu\omega^2+\nu^2\omega^2}}{2\left(1-2\omega+\omega^2\right)}$$

$$\lambda_3 = \frac{2-\nu-3\omega+\omega^2+\nu\omega^2+(-1+\omega)\sqrt{9\nu^2+2\nu\omega+2\nu^2\omega+\omega^2+2\nu\omega^2+\nu^2\omega^2}}{2\left(1-2\omega+\omega^2\right)}$$

$$\lambda_3 < \lambda_{\min} \;\; pro \;\; \omega > \frac{8\nu-1}{6\nu-3} \qquad \lambda_3 < 0 \;\; pro \;\; \omega > 1-2\nu$$



ω
1
0.5
-0.5
ν
0.1
0.2
0.3
0.4
0.5


λ
1
0.5
-0.5
-1
-1.5
-2
-2.5
ω
0.2
0.4
0.6
0.8
1

$$\sigma_x > 0, \sigma_y < 0, \sigma_z < 0, \sigma_m < 0$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{pmatrix} = \frac{1}{E} \begin{pmatrix} \frac{1+\nu}{1-\omega} - \nu & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$$

$$\lambda_1 = 1 + \nu$$

$$\lambda_2 = \frac{-2 + \nu + \omega - 2\nu\omega - \sqrt{9\nu^2 + 2\nu\omega - 16\nu^2\omega + \omega^2 + 8\nu^2\omega^2}}{2(-1+\omega)}$$

$$\lambda_3 = \frac{-2 + \nu + \omega - 2\nu\omega + \sqrt{9\nu^2 + 2\nu\omega - 16\nu^2\omega + \omega^2 + 8\nu^2\omega^2}}{2(-1+\omega)}$$

$$\sigma_x < 0, \sigma_y > 0, \sigma_z > 0, \sigma_m > 0$$

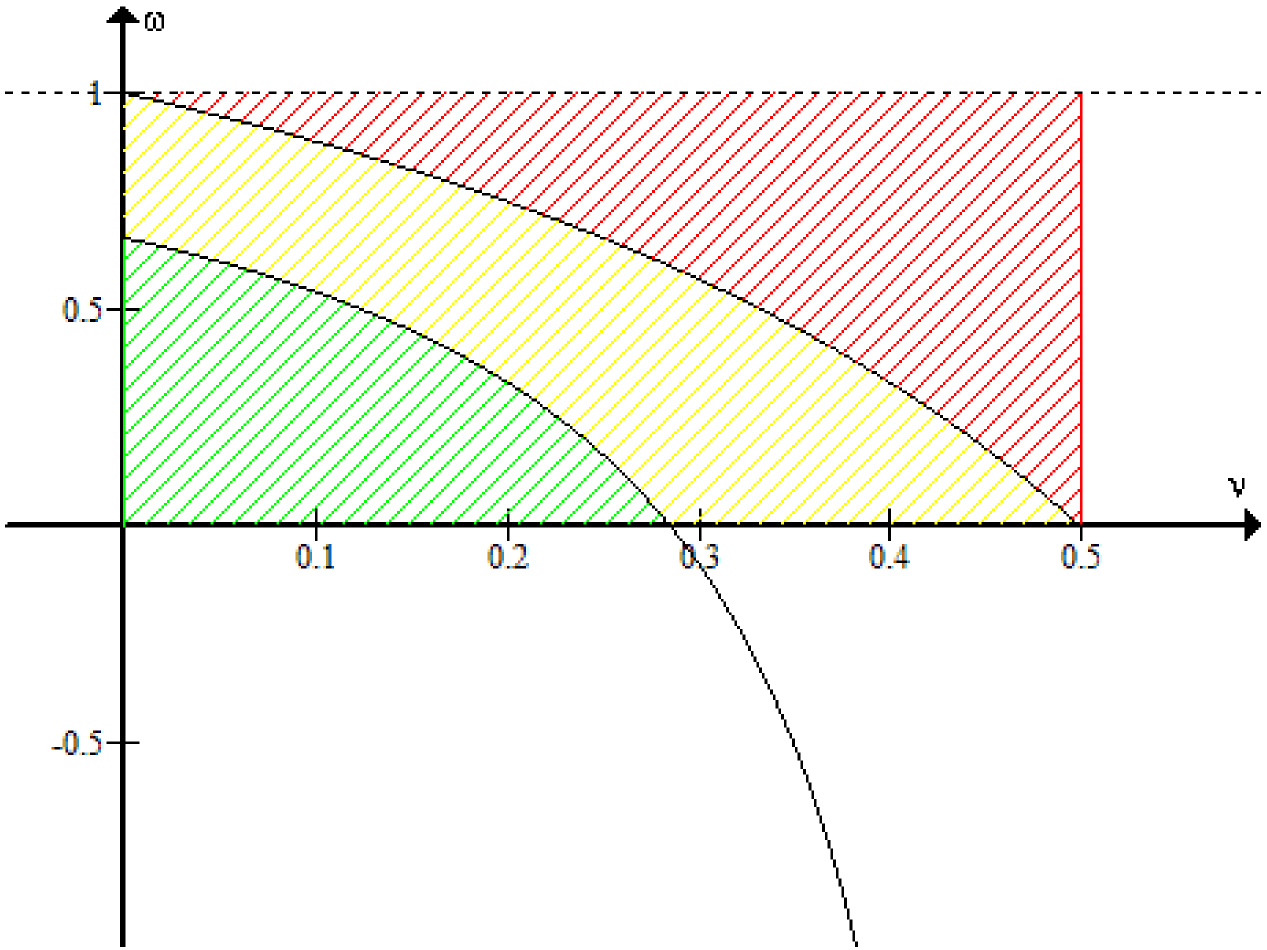
$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{pmatrix} = \frac{1}{E} \begin{pmatrix} 1+\nu-\frac{\nu}{1-\omega} & \frac{-\nu}{1-\omega} & \frac{-\nu}{1-\omega} \\ \frac{-\nu}{1-\omega} & \frac{1}{1-\omega} & \frac{-\nu}{1-\omega} \\ \frac{-\nu}{1-\omega} & \frac{-\nu}{1-\omega} & \frac{1}{1-\omega} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$$

$$\lambda_1 = \frac{-1-\nu}{-1+\omega}$$

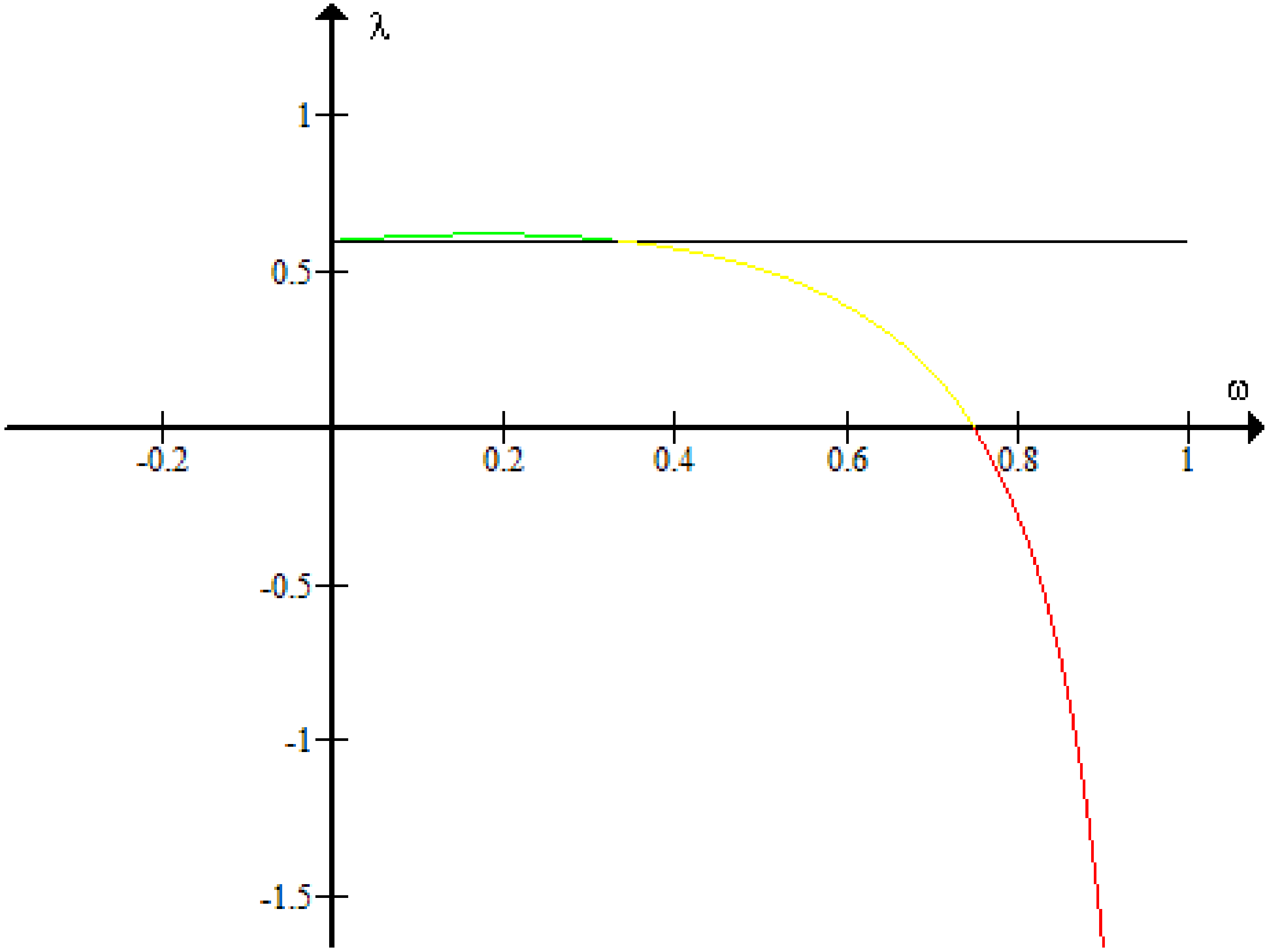
$$\lambda_2 = \frac{2-\nu-3\omega+\omega^2+\nu\omega^2-(-1+\omega)\sqrt{9\nu^2-2\nu\omega-2\nu^2\omega+\omega^2+2\nu\omega^2+\nu^2\omega^2}}{2\left(1-2\omega+\omega^2\right)}$$

$$\lambda_3 = \frac{2-\nu-3\omega+\omega^2+\nu\omega^2+(-1+\omega)\sqrt{9\nu^2-2\nu\omega-2\nu^2\omega+\omega^2+2\nu\omega^2+\nu^2\omega^2}}{2\left(1-2\omega+\omega^2\right)}$$

$$\lambda_3 < \lambda_{\min} \ \ pro \ \ \omega > \frac{7\nu-2}{6\nu-3} \qquad \lambda_3 < 0 \ \ pro \ \ \omega > \frac{2\nu-1}{\nu-1}$$



ω
1
0.5
-0.5
ν
0.1
0.2
0.3
0.4
0.5


λ
1
0.5
-0.5
-1
-1.5
ω
-0.2
0.2
0.4
0.6
0.8
1

$$\sigma_x < 0, \sigma_y > 0, \sigma_z > 0, \sigma_m < 0$$

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{pmatrix} = \frac{1}{E} \begin{pmatrix} 1 & -\nu & -\nu \\ -\nu & \frac{1+\nu}{1-\omega} - \nu & -\nu \\ -\nu & -\nu & \frac{1+\nu}{1-\omega} - \nu \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$$

$$\lambda_1 = \frac{-1-\nu}{-1+\omega}$$

$$\lambda_2 = \frac{-2+\nu+\omega-2\nu\omega-\sqrt{9\nu^2-2\nu\omega-20\nu^2\omega+\omega^2+4\nu\omega^2+12\nu^2\omega^2}}{2(-1+\omega)}$$

$$\lambda_3 = \frac{-2+\nu+\omega-2\nu\omega+\sqrt{9\nu^2-2\nu\omega-20\nu^2\omega+\omega^2+4\nu\omega^2+12\nu^2\omega^2}}{2(-1+\omega)}$$