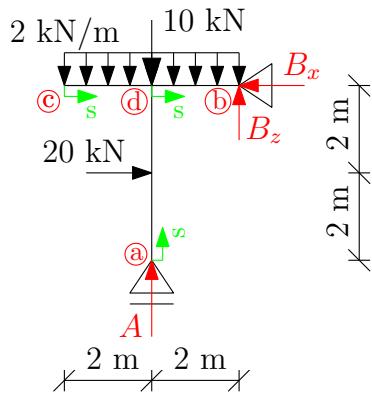


Příklady k procvičení 7: Lomené nosníky

Zadání: Vykreslete průběhy vnitřních sil N, V a M.

Příklad 7.1



1) Výpočet statické určitosti:

$$s = 3 - 2 - 1 = 0 \quad \rightarrow \quad \text{konstrukce je staticky určitá}$$

2) Výpočet reakcí:

$$\begin{aligned} \textcircled{b}: \quad & 10 \cdot 2 + 2 \cdot 4 \cdot 2 + 20 \cdot 2 - A \cdot 2 = 0, \quad \mathbf{A = 38 \text{ kN}} \\ \uparrow: \quad & -2 \cdot 4 - 10 + A + B_z = 0, \quad \mathbf{B_z = -20 \text{ kN}} \\ \rightarrow: \quad & -B_x + 20 = 0, \quad \mathbf{B_x = 20 \text{ kN}} \end{aligned}$$

kontrola: $\textcircled{a}: -20 \cdot 2 + B_z \cdot 2 + B_x \cdot 4 = 0$, O.K.

3) Výpočet důležitých hodnot:

interval (c,d):

$$\begin{array}{ll} N_{cd} = 0 \text{ kN} & N_{dc} = 0 \text{ kN} \\ V_{cd} = 0 \text{ kN} & V_{dc} = -2 \cdot 2 = -4 \text{ kN} \\ M_{cd} = 0 \text{ kNm} & M_{dc} = -2 \cdot 2 \cdot 1 = -4 \text{ kNm} \end{array}$$

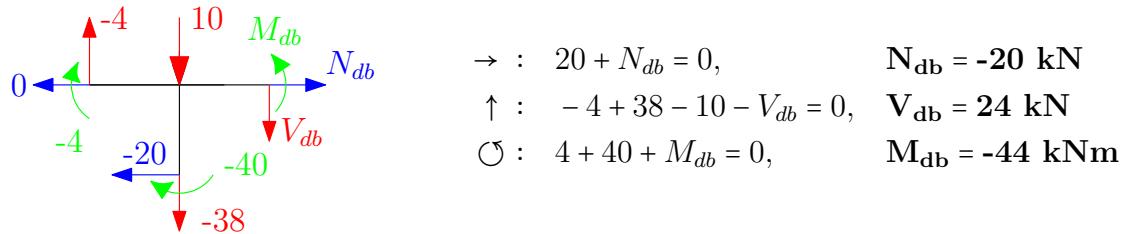
interval (a,e):

$$\begin{array}{ll} N_{ae} = -38 \text{ kN} & N_{ea} = -38 \text{ kN} \\ V_{ae} = 0 \text{ kN} & V_{ea} = 0 \text{ kN} \\ M_{ae} = 0 \text{ kNm} & M_{ea} = 0 \text{ kNm} \end{array}$$

interval (e,d):

$$\begin{array}{ll} N_{ed} = -38 \text{ kN} & N_{de} = -38 \text{ kN} \\ V_{ed} = -20 \text{ kN} & V_{de} = -20 \text{ kN} \\ M_{ed} = 0 \text{ kNm} & M_{de} = -20 \cdot 2 = -40 \text{ kNm} \end{array}$$

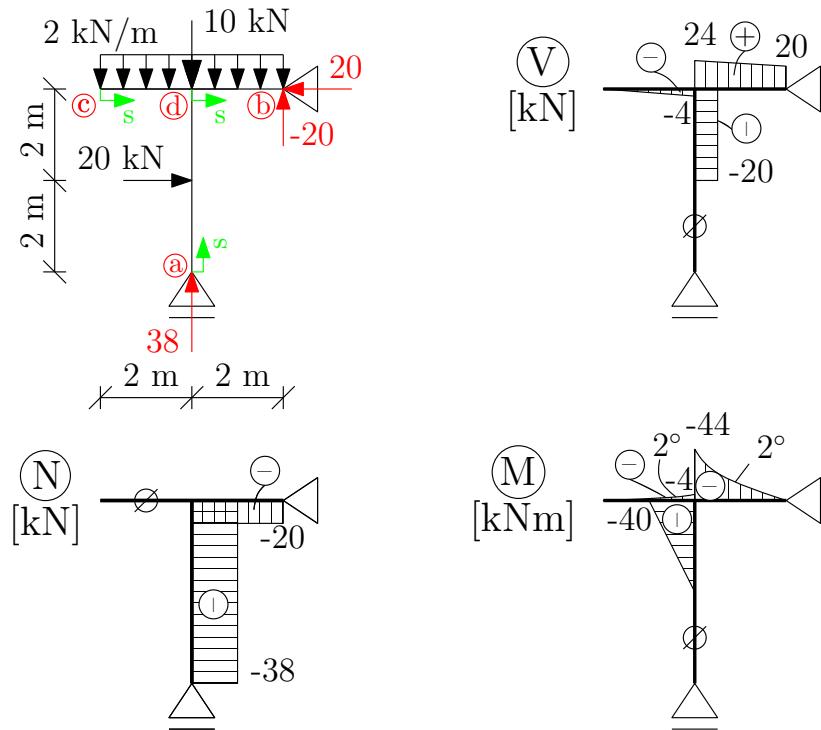
Styčník d:



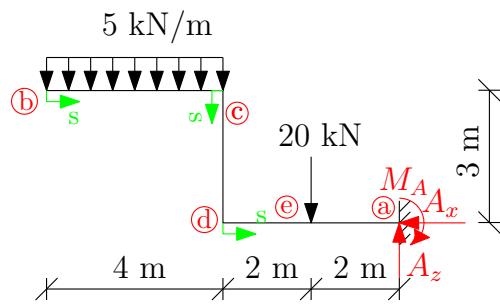
interval (d,b):

$$\begin{array}{ll} N_{db} = -20 \text{ kN} & N_{bd} = -20 \text{ kN} \\ V_{db} = 24 \text{ kN} & V_{bd} = 24 - 2 \cdot 2 = 20 \text{ kN} \\ M_{db} = -44 \text{ kNm} & M_{bd} = -44 + 24 \cdot 2 - 2 \cdot 2 \cdot 1 = 0 \text{ kNm} \end{array}$$

4) Vykreslení:



Příklad 7.2



1) Výpočet statické určitosti:

$$s = 3 - 3 = 0 \rightarrow \text{konstrukce je staticky určitá}$$

2) Výpočet reakcí:

$$\begin{aligned} \rightarrow : \quad & A_x = 0, & \mathbf{A}_x = 0 \text{ kN} \\ \uparrow : \quad & A_z - 5 \cdot 4 - 20 = 0, & \mathbf{A}_z = 40 \text{ kN} \\ \circlearrowleft a : \quad & -M_A + 20 \cdot 2 + 5 \cdot 4 \cdot (4+2) = 0, & \mathbf{M}_A = 160 \text{ kNm} \end{aligned}$$

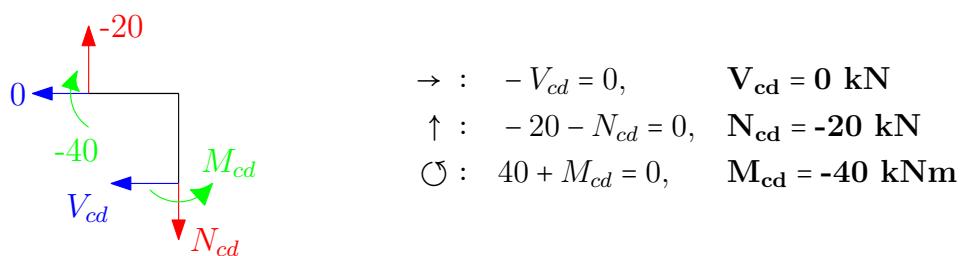
kontrola: $\circlearrowleft b : -20 \cdot 2 - 20 \cdot 6 + A_z \cdot 8 - M_A = 0$, O.K.

3) Výpočet důležitých hodnot:

interval (b,c):

$$\begin{aligned} N_{bc} &= 0 \text{ kN} & N_{cb} &= 0 \text{ kN} \\ V_{bc} &= 0 \text{ kN} & V_{cb} &= -5 \cdot 4 = -20 \text{ kN} \\ M_{bc} &= 0 \text{ kNm} & M_{cb} &= -5 \cdot 4 \cdot \frac{4}{2} = -40 \text{ kNm} \end{aligned}$$

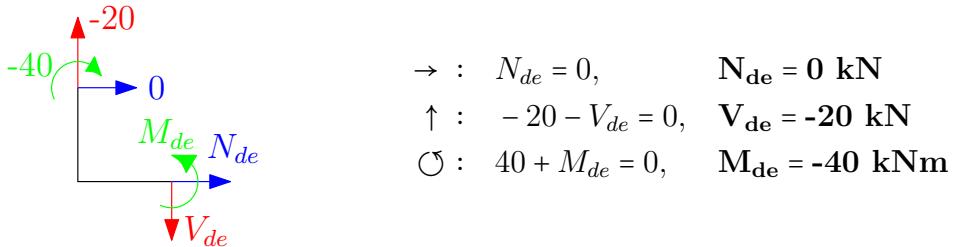
Styčník c:



interval (c,d):

$$\begin{array}{ll} N_{cd} = -20 \text{ kN} & N_{dc} = -20 \text{ kN} \\ V_{cd} = 0 \text{ kN} & V_{dc} = 0 \text{ kN} \\ M_{cd} = -40 \text{ kNm} & M_{dc} = -40 \text{ kNm} \end{array}$$

Styčník d:



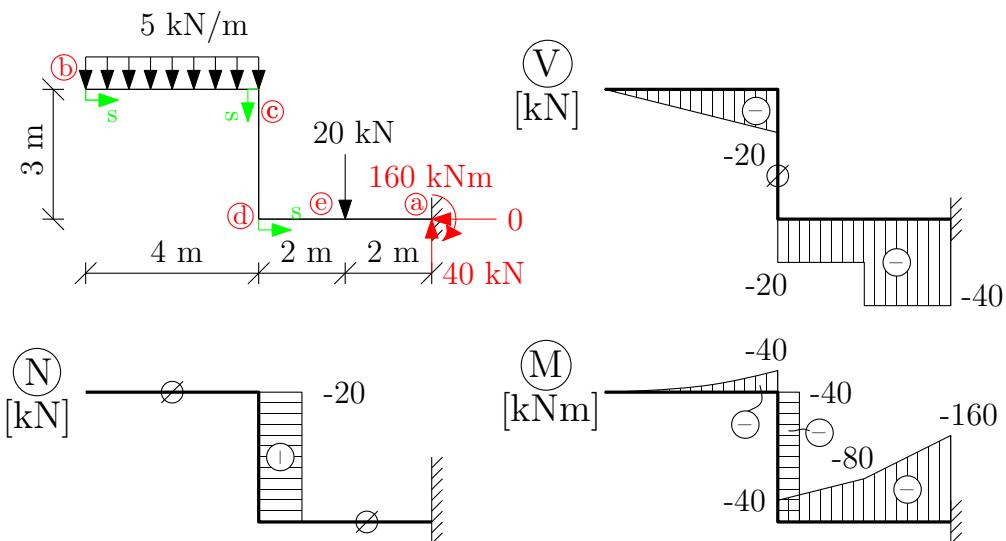
interval (d,e):

$$\begin{array}{ll} N_{de} = 0 \text{ kN} & N_{ed} = 0 \text{ kN} \\ V_{de} = -20 \text{ kN} & V_{ed} = -20 \text{ kN} \\ M_{de} = -40 \text{ kNm} & M_{ed} = -40 - 20 \cdot 2 = -80 \text{ kNm} \end{array}$$

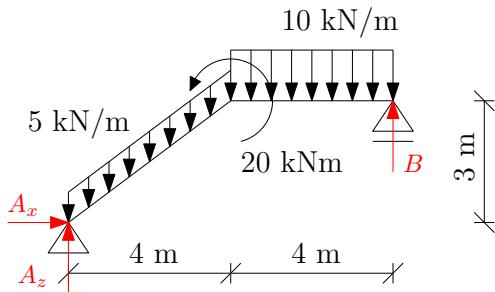
interval (e,a):

$$\begin{array}{ll} N_{ea} = 0 \text{ kN} & N_{ae} = 0 \text{ kN} \\ V_{ea} = -20 - 20 = -40 \text{ kN} & V_{ae} = -40 \text{ kN} \\ M_{ea} = -80 \text{ kNm} & M_{ae} = -80 - 40 \cdot 2 = -160 \text{ kNm} \end{array}$$

4) Vykreslení:



Příklad 7.3



1) Výpočet statické určitosti:

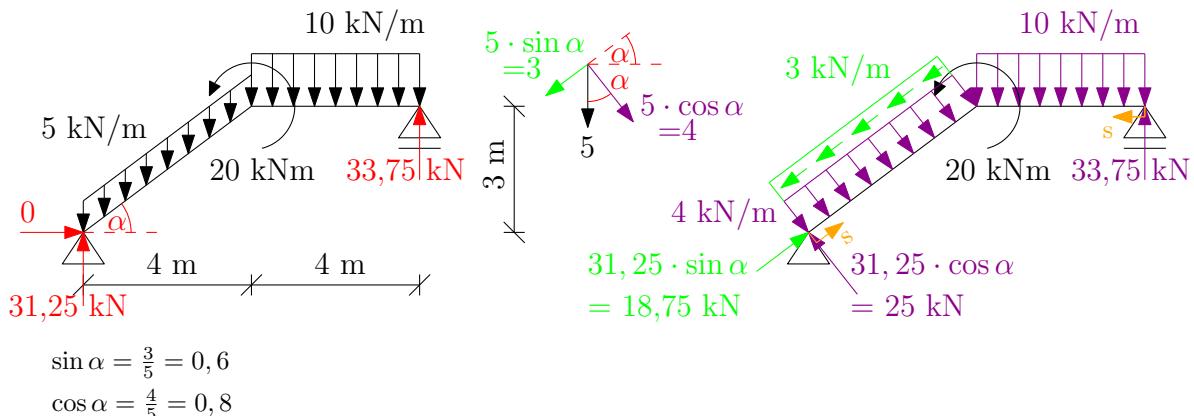
$$s = 3 - 2 - 1 = 0 \rightarrow \text{konstrukce je staticky určitá}$$

2) Výpočet reakcí:

$$\begin{aligned} \rightarrow : A_x &= 0, & \mathbf{A}_x &= 0 \text{ kN} \\ \circlearrowleft a : -20 - 5 \cdot 5 \cdot 2 - 10 \cdot 4 \cdot (4+2) + B \cdot 8 &= 0, & B &= 33,75 \text{ kN} \\ \uparrow : A_z + B - 5 \cdot 5 - 10 \cdot 4 &= 0, & \mathbf{A}_z &= 31,25 \text{ kN} \end{aligned}$$

$$\text{kontrola: } \circlearrowleft b : -20 - A \cdot 8 + 5 \cdot 5 \cdot 6 + 10 \cdot 4 \cdot 2 = 0, \text{ O.K.}$$

Rozkreslení:



3) Výpočet důležitých hodnot:

interval (a,c):

$$\begin{aligned} N_{ac} &= -18,75 \text{ kN} & N_{ca} &= -18,75 + 3 \cdot 5 = -3,75 \text{ kN} \\ V_{ac} &= 25 \text{ kN} & V_{ca} &= 25 - 4 \cdot 5 = 5 \text{ kN} \\ M_{ac} &= 0 \text{ kNm} & M_{ca} &= 25 \cdot 5 - 4 \cdot 5 \cdot \frac{5}{2} = 75 \text{ kNm} \end{aligned}$$

interval (b,c):

$$N_{bc} = 0 \text{ kN}$$

$$V_{bc} = -33,75 \text{ kN}$$

$$N_{cb} = 0 \text{ kN}$$

$$V_{cb} = -33,75 + 10 \cdot 4 = 6,25 \text{ kN}$$

$$M_{bc} = 0 \text{ kNm}$$

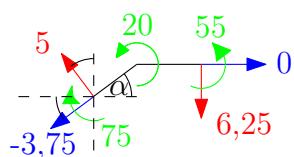
$$M_{cb} = 33,75 \cdot 4 - 10 \cdot 4 \cdot \frac{4}{2} = 55 \text{ kNm}$$

extrém na intervalu (c,b):

$$x_{ext} = \frac{6,25}{10} = 0,625 \text{ m}$$

$$M_{ext} = 55 + 6,25 \cdot 0,625 = 56,953 \text{ kNm}$$

Rovnováha na styčníku c:



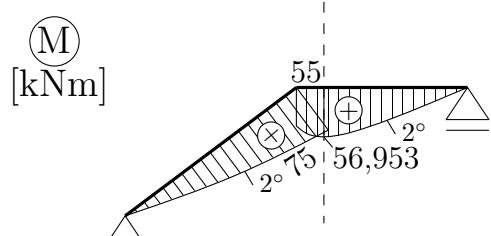
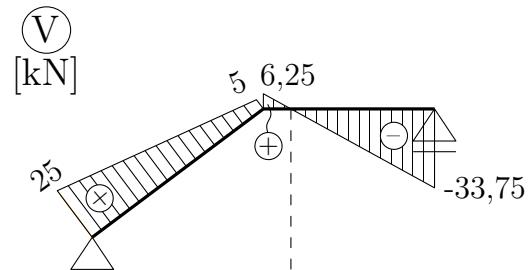
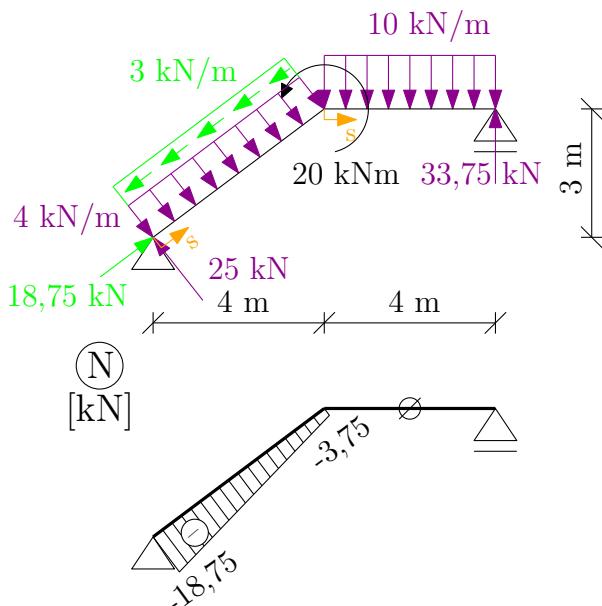
$$\rightarrow : 0 - (-3,75) \cdot 0,8 - 5 \cdot 0,6 = 0, \quad \text{O.K.}$$

$$\uparrow : -6,25 - (-3,75) \cdot 0,6 + 5 \cdot 0,8 = 0, \quad \text{O.K.}$$

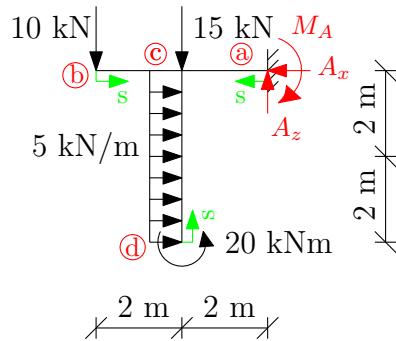
$$\circlearrowleft : 55 + 20 - 75 = 0, \quad \text{O.K.}$$

$$\cos \alpha = 0,8 \\ \sin \alpha = 0,6$$

4) Vykreslení:



Příklad 7.4



1) Výpočet statické určitosti:

$$s = 3 - 3 = 0 \rightarrow \text{konstrukce je staticky určitá}$$

2) Výpočet reakcí:

$$\rightarrow : 5 \cdot 4 - A_x = 0, \quad A_x = 20 \text{ kN}$$

$$\uparrow : A_z - 10 - 15 = 0, \quad A_z = 25 \text{ kN}$$

$$\circlearrowleft a : 20 + 10 \cdot 4 + 15 \cdot 2 + 5 \cdot 4 \cdot 2 - M_A = 0, \quad M_A = 130 \text{ kNm}$$

kontrola: $\circlearrowleft b : -15 \cdot 2 + 5 \cdot 4 \cdot 2 + 20 - M_A + A_z \cdot 4 = 0, \text{ O.K.}$

3) Výpočet důležitých hodnot:

interval (b,c):

$$N_{bc} = 0 \text{ kN} \quad N_{cb} = 0 \text{ kN}$$

$$V_{bc} = -10 \text{ kN} \quad V_{cb} = -10 \text{ kN}$$

$$M_{bc} = 0 \text{ kNm} \quad M_{cb} = -10 \cdot 2 = -20 \text{ kNm}$$

interval (d,c):

$$N_{dc} = 0 \text{ kN} \quad N_{cd} = 0 \text{ kN}$$

$$V_{dc} = 0 \text{ kN} \quad V_{cd} = -5 \cdot 4 = -20 \text{ kN}$$

$$M_{dc} = -20 \text{ kNm} \quad M_{cd} = -20 - 5 \cdot 4 \cdot \frac{4}{2} = -60 \text{ kNm}$$

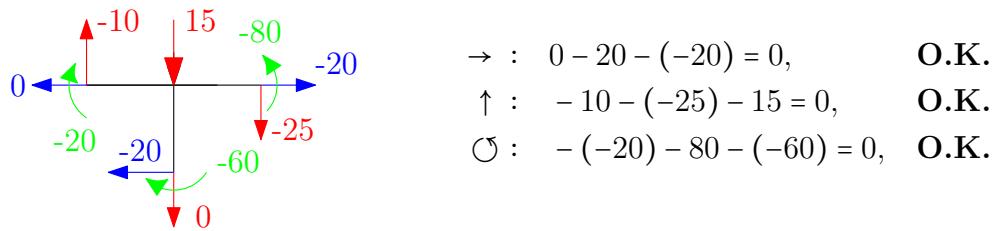
interval (a,c):

$$N_{ac} = -20 \text{ kN} \quad N_{ca} = -20 \text{ kN}$$

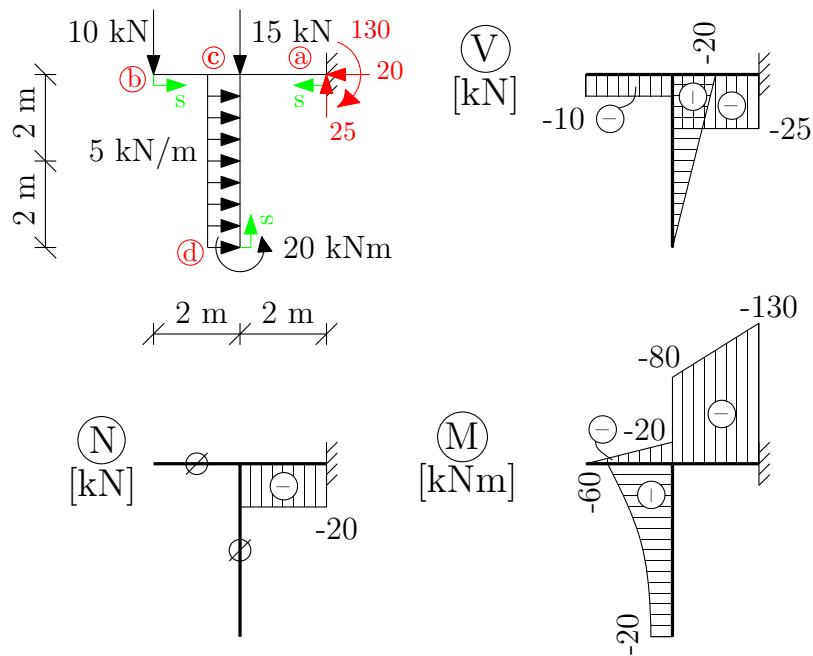
$$V_{ac} = -25 \text{ kN} \quad V_{ca} = -25 \text{ kN}$$

$$M_{ac} = -130 \text{ kNm} \quad M_{ca} = -130 + 25 \cdot 2 = -80 \text{ kNm}$$

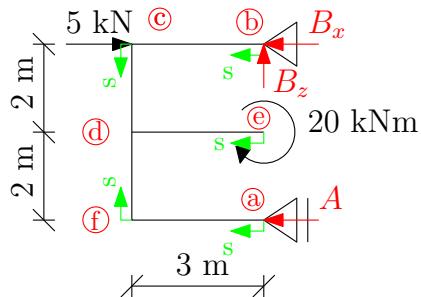
Rovnováha na styčníku c:



4) Vykreslení:



Příklad 7.5



1) Výpočet statické určitosti:

$$s = 3 - 3 = 0 \quad \rightarrow \quad \text{konstrukce je staticky určitá}$$

2) Výpočet reakcí:

$$\begin{array}{ll} \uparrow : & B_z = 0, \quad \mathbf{B}_z = \mathbf{0} \text{ kN} \\ \circlearrowleft b : & -20 - A \cdot 4 = 0, \quad \mathbf{A} = -5 \text{ kN} \\ \rightarrow : & 5 - B_x - A = 0, \quad \mathbf{B}_x = 10 \text{ kN} \end{array}$$

kontrola: $\circlearrowleft e : B_x \cdot 5 - 2 \cdot 2 - 20 - A \cdot 2 = 0$, O.K.

3) Výpočet důležitých hodnot:

interval (b,c):

$$\begin{array}{ll} N_{bc} = -10 \text{ kN} & N_{cb} = -10 \text{ kN} \\ V_{bc} = 0 \text{ kN} & V_{cb} = 0 \text{ kN} \\ M_{bc} = 0 \text{ kNm} & M_{cb} = 0 \text{ kNm} \end{array}$$

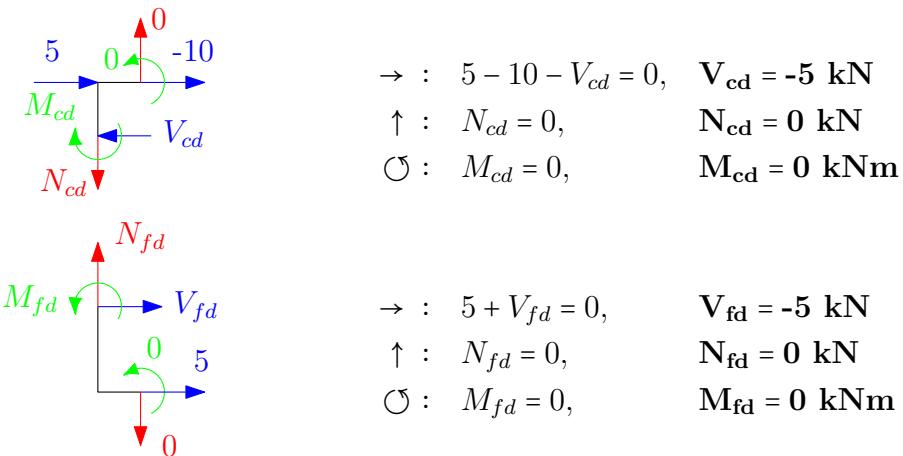
interval (e,d):

$$\begin{array}{ll} N_{ed} = 0 \text{ kN} & N_{de} = 0 \text{ kN} \\ V_{ed} = 0 \text{ kN} & V_{de} = 0 \text{ kN} \\ M_{ed} = -20 \text{ kNm} & M_{de} = -20 \text{ kNm} \end{array}$$

interval (a,f):

$$\begin{array}{ll} N_{af} = 5 \text{ kN} & N_{fa} = 5 \text{ kN} \\ V_{af} = 0 \text{ kN} & V_{fa} = 0 \text{ kN} \\ M_{af} = 0 \text{ kNm} & M_{fa} = 0 \text{ kNm} \end{array}$$

Styčníky c a f:



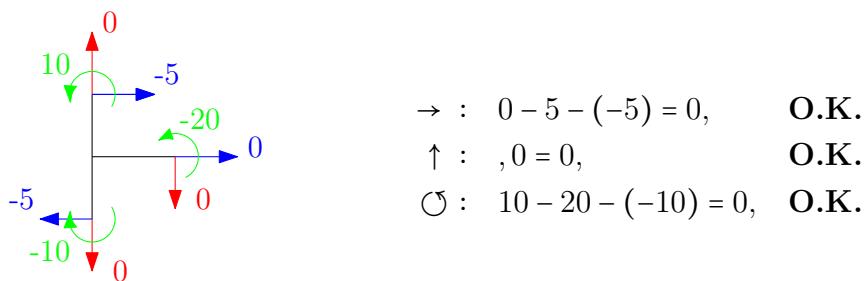
interval (c,d):

$$\begin{array}{ll} N_{cd} = 0 \text{ kN} & N_{dc} = 0 \text{ kN} \\ V_{cd} = -5 \text{ kN} & V_{dc} = -5 \text{ kN} \\ M_{cd} = 0 \text{ kNm} & M_{dc} = 5 \cdot 2 = 10 \text{ kNm} \end{array}$$

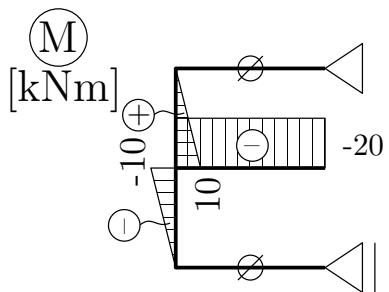
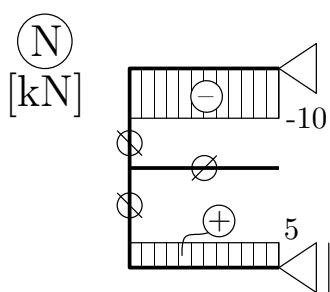
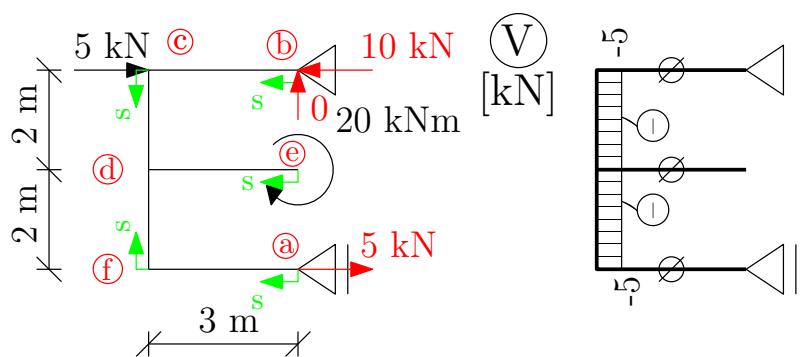
interval (f,d):

$$\begin{array}{ll} N_{fd} = 0 \text{ kN} & N_{df} = 0 \text{ kN} \\ V_{fd} = -5 \text{ kN} & V_{df} = -5 \text{ kN} \\ M_{fd} = 0 \text{ kNm} & M_{df} = -5 \cdot 2 = -10 \text{ kNm} \end{array}$$

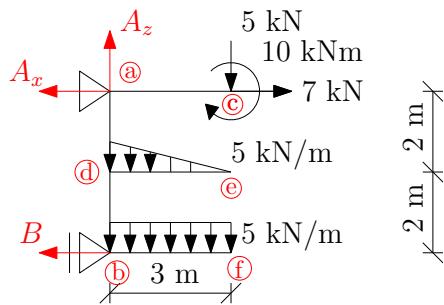
Rovnováha na styčníku d:



4) Vykreslení:



Příklad 7.6



1) Výpočet statické určitosti:

$$s = 3 - 3 = 0 \rightarrow \text{konstrukce je staticky určitá}$$

2) Výpočet reakcí:

$$\begin{aligned} \uparrow : A_z - 5 - 7,5 - 15 &= 0, & A_z &= 27,5 \text{ kN} \\ \textcircled{a} : -5 \cdot 3 - 10 - 7,5 \cdot 1 - 15 \cdot 1,5 - B_x \cdot 4 &= 0, & B_x &= -13,75 \text{ kN} \\ \rightarrow : 7 - A_x - B_x &= 0, & A_x &= 20,75 \text{ kN} \end{aligned}$$

kontrola: $\textcircled{b} : -15 \cdot 1,5 - 7,5 \cdot 1 + A_x \cdot 4 - 10 - 5 \cdot 3 - 7 \cdot 4 = 0$, O.K.

3) Výpočet důležitých hodnot:

interval (c,a):

$$\begin{array}{ll} N_{ca} = 7 \text{ kN} & N_{ac} = 7 \text{ kN} \\ V_{ca} = 5 \text{ kN} & V_{ac} = 5 \text{ kN} \\ M_{ca} = -10 \text{ kNm} & M_{ac} = -10 - 5 \cdot 3 = 15 \text{ kNm} \end{array}$$

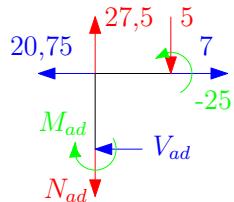
interval (e,d):

$$\begin{array}{ll} N_{ed} = 0 \text{ kN} & N_{de} = 0 \text{ kN} \\ V_{ed} = 0 \text{ kN} & V_{de} = \frac{5 \cdot 3}{2} = 7,5 \text{ kN} \\ M_{ed} = 0 \text{ kNm} & M_{de} = -\frac{5 \cdot 3}{2} \cdot 1 = -7,5 \text{ kNm} \end{array}$$

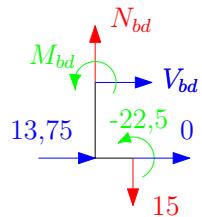
interval (f,b):

$$\begin{array}{ll} N_{fb} = 0 \text{ kN} & N_{bf} = 0 \text{ kN} \\ V_{fb} = 0 \text{ kN} & V_{bf} = 5 \cdot 3 = 15 \text{ kN} \\ M_{fb} = 0 \text{ kNm} & M_{bf} = -5 \cdot 3 \cdot 1,5 = -22,5 \text{ kNm} \end{array}$$

Styčníky a a b:



$$\begin{aligned} \rightarrow : & 7 - V_{ad} - 20,75 = 0, & \mathbf{V}_{ad} = -13,75 \text{ kN} \\ \uparrow : & -N_{ad} - 5 + 27,5 = 0, & \mathbf{N}_{ad} = 22,5 \text{ kN} \\ \circlearrowleft : & -25 - M_{ad} = 0, & \mathbf{M}_{ad} = -25 \text{ kNm} \end{aligned}$$



$$\begin{aligned} \rightarrow : & 13,75 + V_{bd} = 0, & \mathbf{V}_{bd} = -13,75 \text{ kN} \\ \uparrow : & -15 + N_{bd} = 0, & \mathbf{N}_{bd} = 15 \text{ kN} \\ \circlearrowright : & M_{bd} - 22,5 = 0, & \mathbf{M}_{bd} = 22,5 \text{ kNm} \end{aligned}$$

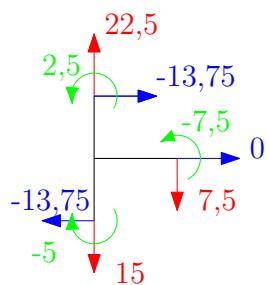
interval (a,d):

$$\begin{array}{ll} N_{ad} = 22,5 \text{ kN} & N_{da} = 22,5 \text{ kN} \\ V_{ad} = -13,75 \text{ kN} & V_{da} = -13,75 \text{ kN} \\ M_{ad} = -25 \text{ kNm} & M_{da} = -25 - (-13,75) \cdot 2 = 2,5 \text{ kNm} \end{array}$$

interval (b,d):

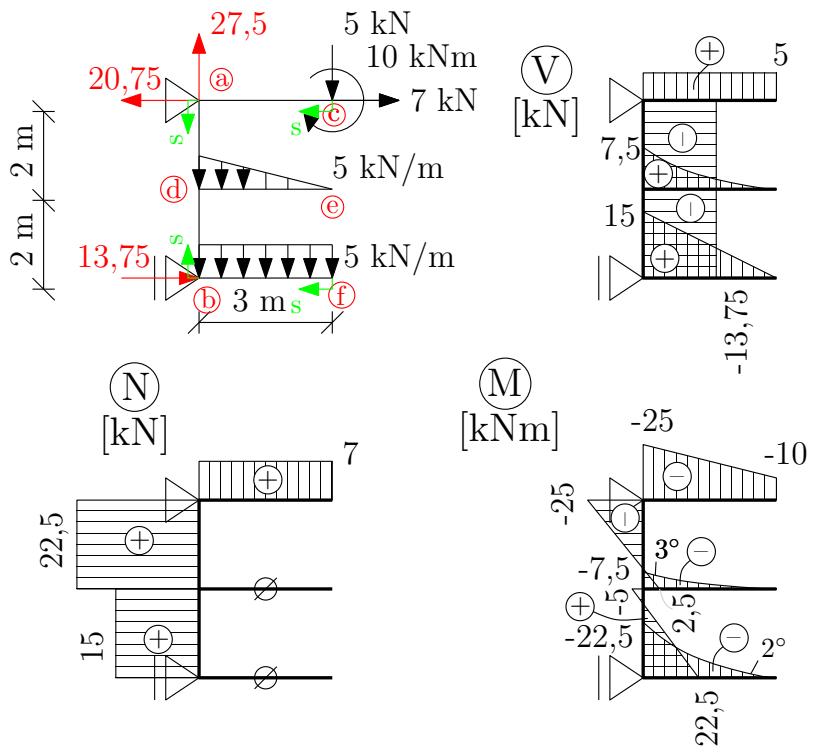
$$\begin{array}{ll} N_{bd} = 15 \text{ kN} & N_{db} = 15 \text{ kN} \\ V_{bd} = -13,75 \text{ kN} & V_{db} = -13,75 \text{ kN} \\ M_{bd} = 22,5 \text{ kNm} & M_{db} = 22,5 - 13,75 \cdot 2 = -5 \text{ kNm} \end{array}$$

Rovnováha na styčníku d:

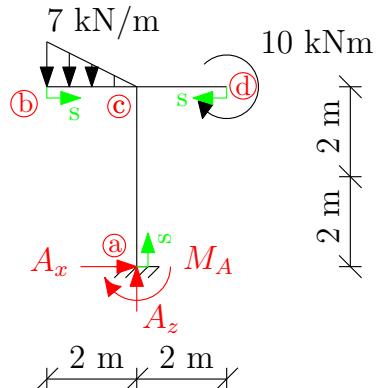


$$\begin{aligned} \rightarrow : & 0 - 13,75 - (-13,75) = 0, & \mathbf{O.K.} \\ \uparrow : & 22,5 - 7,5 - 15 = 0, & \mathbf{O.K.} \\ \circlearrowleft : & 2,5 - 7,5 - (-5) = 0, & \mathbf{O.K.} \end{aligned}$$

4) Vykreslení:



Příklad 7.7



1) Výpočet statické určitosti:

$$s = 3 - 3 = 0 \rightarrow \text{konstrukce je staticky určitá}$$

2) Výpočet reakcí:

$$\rightarrow : A_x = 0, \quad \mathbf{A}_x = 0 \text{ kN}$$

$$\uparrow : A_z - 7 \cdot 2 \cdot \frac{1}{2} = 0, \quad \mathbf{A}_z = 7 \text{ kN}$$

$$\circlearrowleft a : M_A - 10 + 7 \cdot \frac{2}{3} \cdot 2 = 0, \quad M_A = 0,667 \text{ kNm}$$

kontrola: $\circlearrowleft b: M_A + A_x \cdot 4 + A_z \cdot 2 - 10 - \frac{7 \cdot 2}{2} \cdot \frac{2}{3} = 0$, O.K.

3) Výpočet důležitých hodnot:

interval (b,c):

$$\begin{array}{ll} N_{bc} = 0 \text{ kN} & N_{cb} = 0 \text{ kN} \\ V_{bc} = 0 \text{ kN} & V_{cb} = \frac{-7 \cdot 2}{2} = -7 \text{ kN} \\ M_{bc} = 0 \text{ kNm} & M_{cb} = \frac{-7 \cdot 2}{2} \cdot \frac{4}{3} = -9,333 \text{ kNm} \end{array}$$

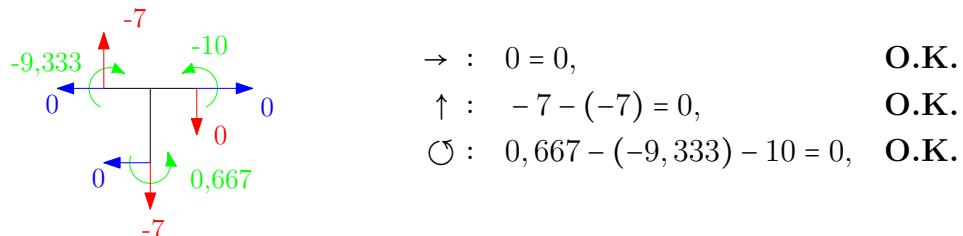
interval (d,c):

$$\begin{array}{ll} N_{dc} = 0 \text{ kN} & N_{cd} = 0 \text{ kN} \\ V_{dc} = 0 \text{ kN} & V_{cd} = 0 \text{ kN} \\ M_{dc} = -10 \text{ kNm} & M_{cd} = -10 \text{ kNm} \end{array}$$

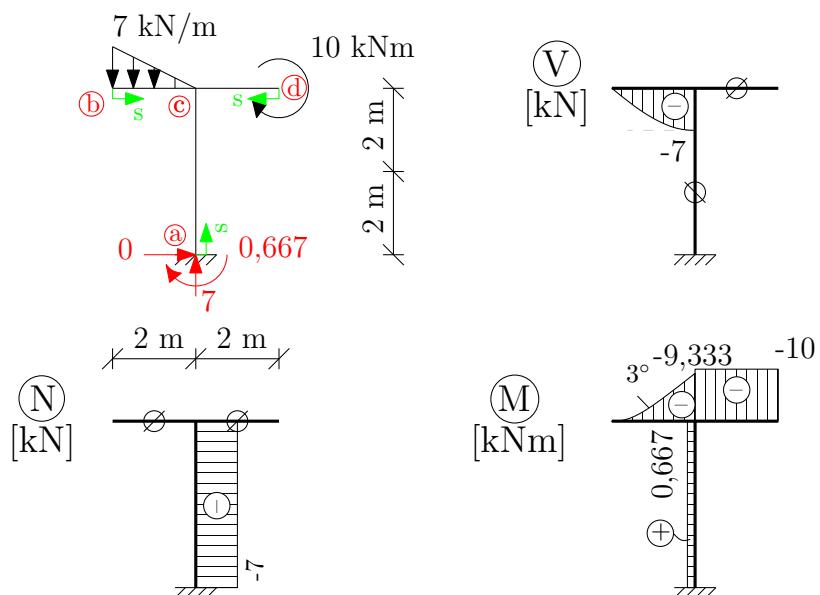
interval (a,c):

$$\begin{array}{ll} N_{ac} = -7 \text{ kN} & N_{ca} = -7 \text{ kN} \\ V_{ac} = 0 \text{ kN} & V_{ca} = 0 \text{ kN} \\ M_{ac} = 0,667 \text{ kNm} & M_{ca} = 0,667 \text{ kNm} \end{array}$$

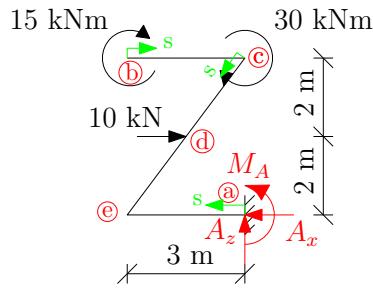
Rovnováha na styčníku:



4) Vykreslení:



Příklad 7.8



1) Výpočet statické určitosti:

$$s = 3 - 3 = 0 \rightarrow \text{konstrukce je staticky určitá}$$

2) Výpočet reakcí:

$$\rightarrow : -A_x + 10 = 0, \quad A_x = 10 \text{ kN}$$

$$\uparrow : A_z = 0, \quad A_z = 0 \text{ kN}$$

$$\circlearrowleft a : M_A - 10 \cdot 2 - 15 - 30 = 0, \quad M_A = 65 \text{ kNm}$$

$$\text{kontrola: } \circlearrowleft b : M_A - 15 - 30 + 10 \cdot 2 - A_x \cdot 4 + A_z \cdot 3 = 0, \quad \text{O.K.}$$

3) Výpočet důležitých hodnot:

interval (b,c):

$$N_{bc} = 0 \text{ kN} \quad N_{cb} = 0 \text{ kN}$$

$$V_{bc} = 0 \text{ kN} \quad V_{cb} = 0 \text{ kN}$$

$$M_{bc} = 15 \text{ kNm} \quad M_{cb} = 15 \text{ kNm}$$

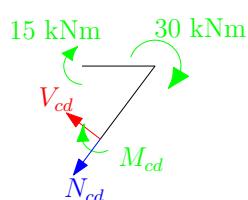
interval (a,e):

$$N_{ae} = -10 \text{ kN} \quad N_{ea} = -10 \text{ kN}$$

$$V_{ae} = 0 \text{ kN} \quad V_{ea} = 0 \text{ kN}$$

$$M_{ae} = 65 \text{ kNm} \quad M_{ea} = 65 \text{ kNm}$$

Styčník c:



$$\begin{aligned} \swarrow &: N_{cd} = 0, & N_{cd} &= 0 \text{ kN} \\ \nwarrow &: -V_{cd} = 0, & V_{cd} &= 0 \text{ kN} \\ \circlearrowleft &: -M_{cd} - 30 - 15 = 0, & M_{cd} &= -45 \text{ kNm} \end{aligned}$$

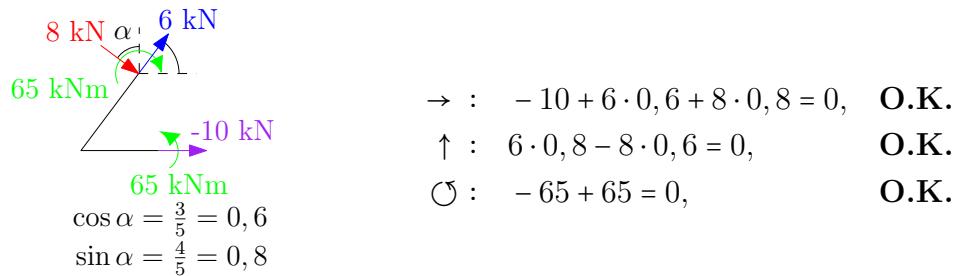
interval (c,d):

$$\begin{array}{ll} N_{cd} = 0 \text{ kN} & N_{dc} = 0 \text{ kN} \\ V_{cd} = 0 \text{ kN} & V_{dc} = 0 \text{ kN} \\ M_{cd} = -45 \text{ kNm} & M_{dc} = -45 \text{ kNm} \end{array}$$

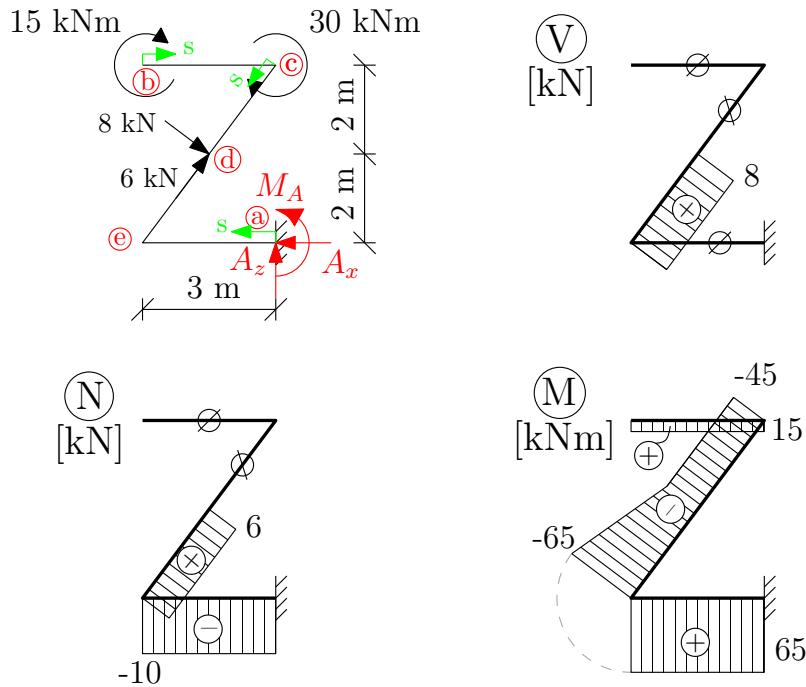
interval (d,e):

$$\begin{array}{ll} N_{cd} = 10 \cdot 0,6 = 6 \text{ kN} & N_{dc} = 6 \text{ kN} \\ V_{cd} = 10 \cdot 0,8 = 8 \text{ kN} & V_{dc} = 8 \text{ kN} \\ M_{cd} = -45 \text{ kNm} & M_{dc} = -45 - 8 \cdot 2,5 = -65 \text{ kNm} \end{array}$$

Rovnováha na styčníku:



4) Vykreslení:



Prosba V případě, že v textu objevíte nějakou chybu nebo budete mít námět na jeho vylepšení, ozvěte se prosím na adela.pospisilova@fsv.cvut.cz.