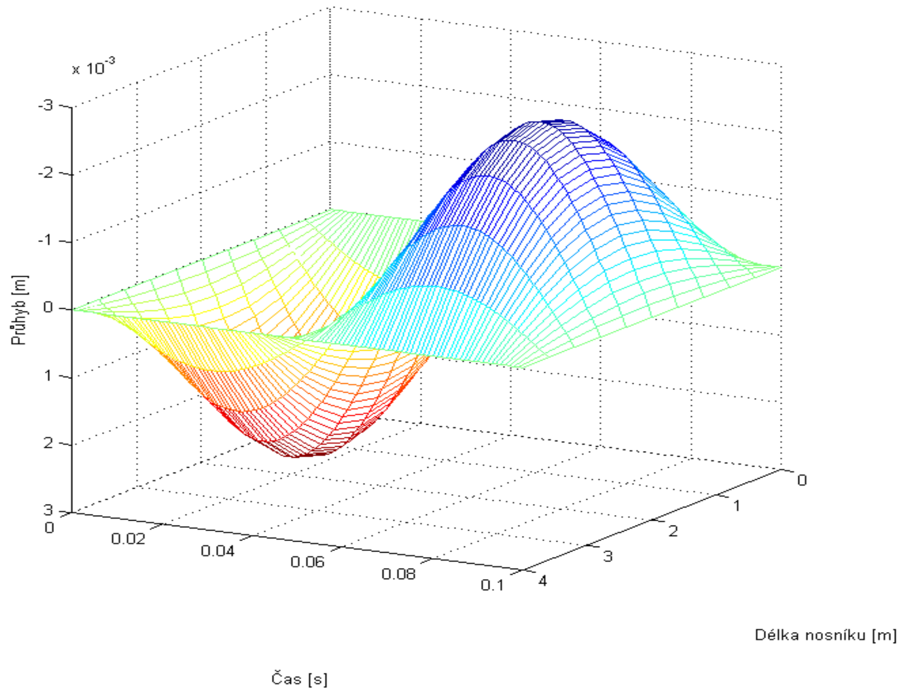
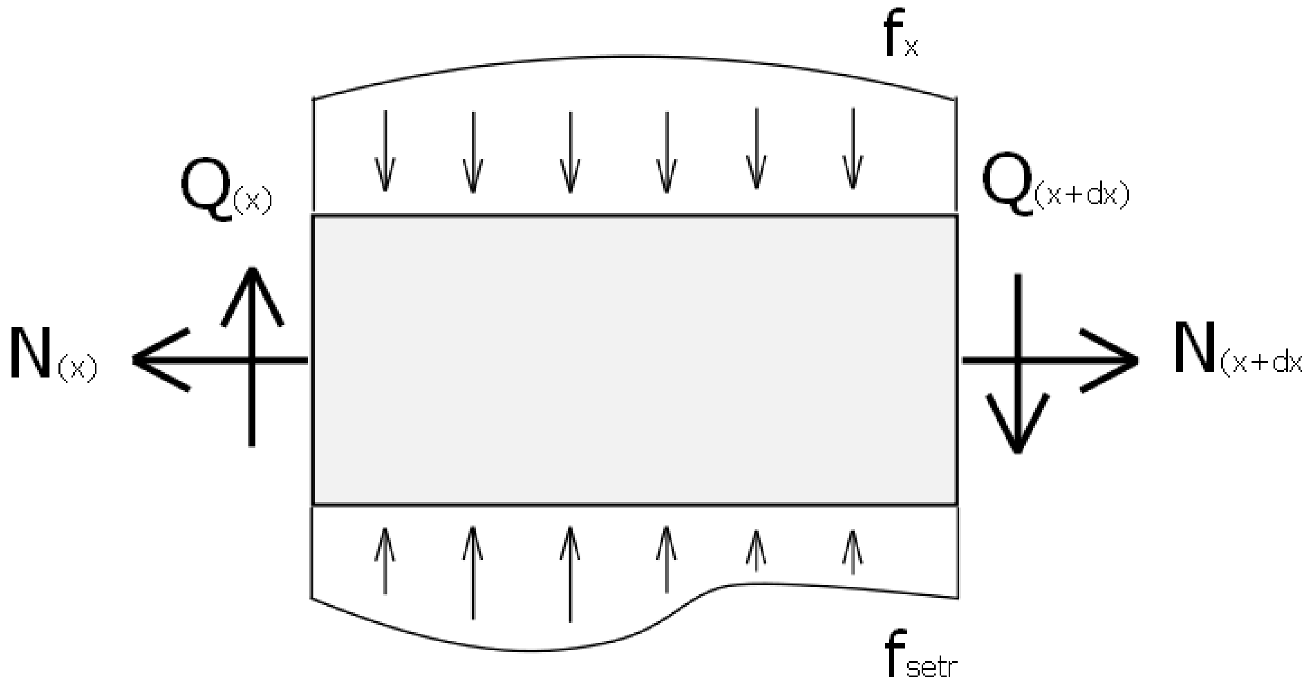
Kmitání nosníku a numerické řešení

semestrální práce k předmětu PRPE





$$Q_{(x+\Delta x,t)} - Q_{(x,t)} + f_x(x,t)\cdot \Delta x - f_{set}(x,t)\cdot \Delta x = 0$$

$$\vec{F}_{akt} = m\cdot \vec{a}$$

$$\vec{F}_{akt} + \vec{F}_{set} = 0$$

$$\frac{Q_{(x+\Delta x,t)} - Q_{(x,t)}}{\Delta x} + f_x(x,t) - \rho\cdot A\cdot a(x,t) = 0$$

$$\vec{F}_{akt} + m\cdot \vec{a} = 0$$

$$\frac{\partial Q_{(x,t)}}{\partial x} + f_x(x,t) - \mu\cdot a(x,t) = 0$$

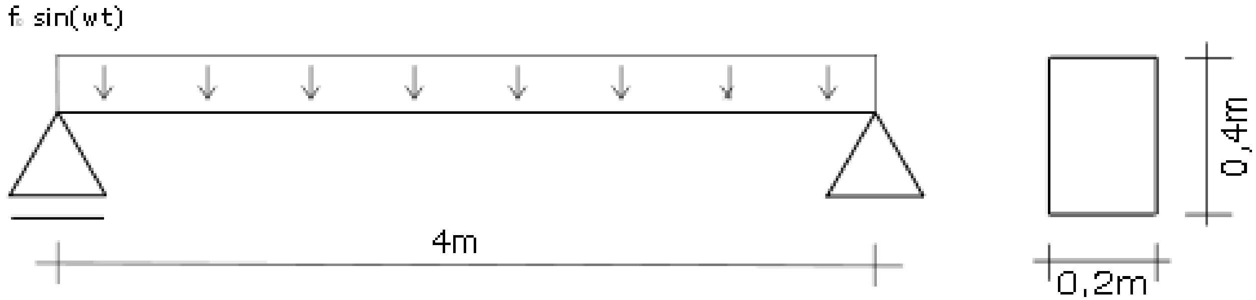
$$\frac{\partial Q_{(x,t)}}{\partial x} + f_x(x,t) - \mu \cdot a(x,t) = 0$$

$$\frac{\partial Q_{(x,t)}}{\partial x} = \frac{\partial^2}{\partial x^2}\left(-E \cdot I \frac{\partial^2 v_{(x,t)}}{\partial x^2}\right)$$

$$a(x,t) = \frac{\partial^2 v_{(x,t)}}{\partial t^2}$$

$$\mu = \rho \cdot A$$

$$-E \cdot I \frac{\partial^4 v_{(x,t)}}{\partial x^4} + f_{(x,t)} - \mu \frac{\partial^2 v_{(x,t)}}{\partial t^2} = 0$$



$$f_{(x,t)} = f_0 \cdot \sin(\omega \cdot t)$$

$$v_{(x,t)} = v_{(x)} \cdot \sin(\omega \cdot t)$$

$$\sin(\omega \cdot t) \cdot \left(-E \cdot I \frac{d^4 v_{(x)}}{d x^4} + f_{(x)} + \mu \cdot \omega^2 \cdot v_{(x)}\right) = 0$$

$$\frac{d^4 v_{(x)}}{d x^4} - \frac{\mu \cdot \omega^2}{E \cdot I} \cdot v_{(x)} = \frac{f_0}{E \cdot I}$$

Homogenní řešení získáme nalezením kořenů charakteristické rovnice

$$\lambda^4 - \alpha^4 = 0 \qquad \text{,kde} \qquad \alpha^4 = \frac{\omega^2 \mu}{E \cdot I}$$

$$(\lambda_1 \; \lambda_2 \; \lambda_3 \; \lambda_4) = (\alpha \; -\alpha \; i\alpha \; -i\alpha)$$

$$v_0 = C_1 \exp(\alpha x) + C_2 \exp(-\alpha x) + C_3 \exp(i\alpha x) + C_4 \exp(-i\alpha x)$$

$$v_0 = C_1 \cosh(\alpha x) + C_2 \sinh(\alpha x) + C_3 \cos(\alpha x) + C_4 \sin(\alpha x)$$

Partikulární řešení lze odhadnout

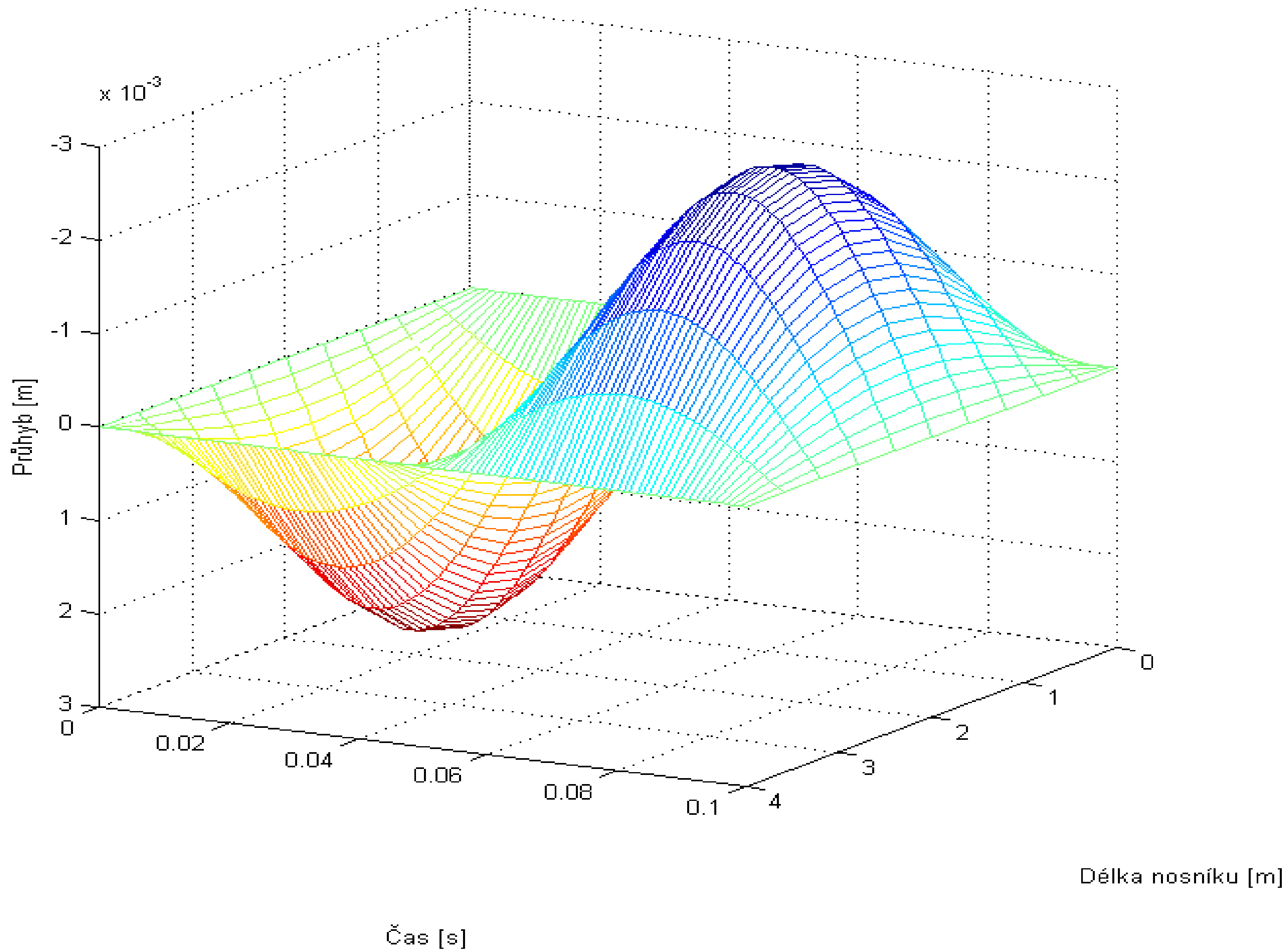
$$v_P = \frac{-f_0}{\mu \omega^2}$$

$$v_{(x)}=C_1\cosh(\alpha x)+C_2\sinh(\alpha x)+C_3\cos(\alpha x)+C_4\sin(\alpha x)-\frac{f_0}{\mu\omega^2}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ \cosh(\alpha l) & \sinh(\alpha l) & \cos(\alpha l) & \sin(\alpha l) \\ \cosh(\alpha l) & \sinh(\alpha l) & -\cos(\alpha l) & -\sin(\alpha l) \end{pmatrix}\cdot\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix}=\begin{pmatrix} \frac{f_0}{\mu\omega^2} \\ 0 \\ \frac{f_0}{\mu\omega^2} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix}=\begin{pmatrix} \frac{f_0}{2\mu\omega^2} \\ \frac{-f_0\cdot\tanh(\frac{\alpha l}{2})}{2\mu\omega^2} \\ \frac{f_0}{2\mu\omega^2} \\ \frac{f_0\cdot\sin^2(\frac{\alpha l}{2})}{\mu\omega^2\cdot\sin(\alpha l)} \end{pmatrix}$$

Vykreslení průběhu pro ilustrační příklad s fyzikálními hodnotami E= 30Gpa, f_0= 10kN, ρ= 2400kg/m³



$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ \cosh(\alpha l) & \sinh(\alpha l) & \cos(\alpha l) & \sin(\alpha l) \\ \cosh(\alpha l) & \sinh(\alpha l) & -\cos(\alpha l) & -\sin(\alpha l) \end{vmatrix} = -4 \cdot \sin(\alpha l) \cdot \sinh(\alpha l)$$

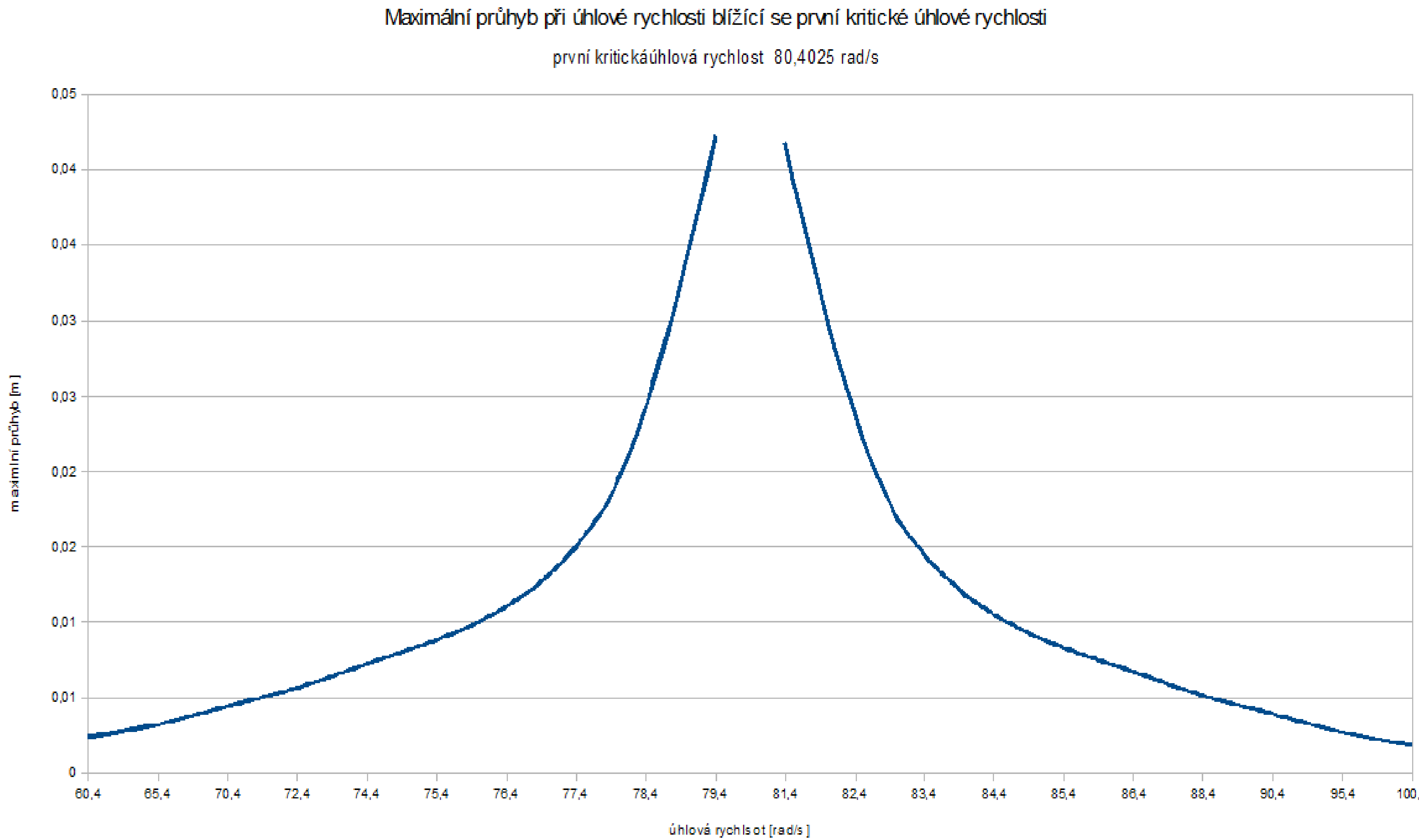
$$-4 \cdot \sin(\alpha l) \cdot \sinh(\alpha l) = 0$$

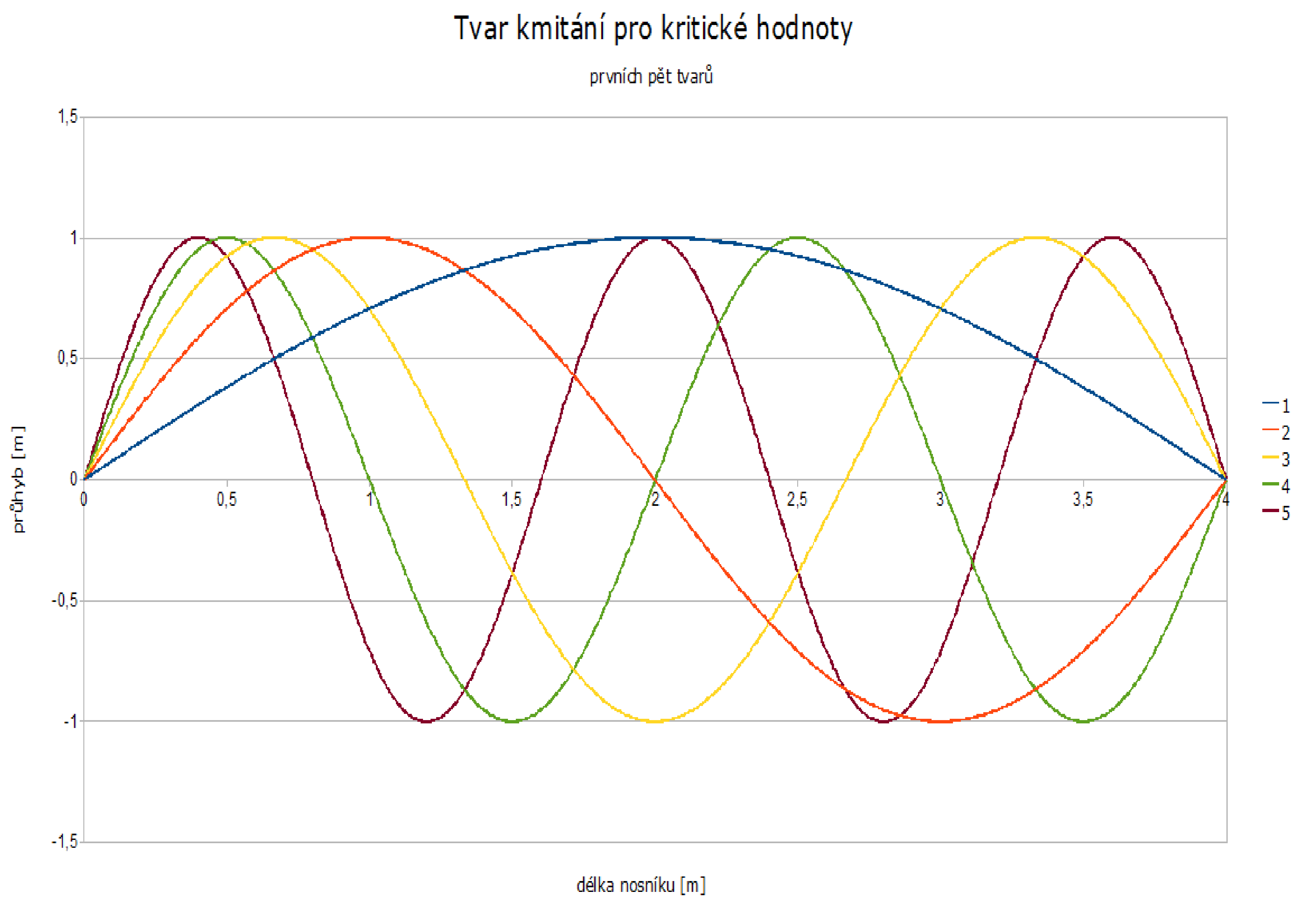
$$\alpha l = k \cdot \pi ; \quad k \in \mathbb{Z}$$

$$\sqrt[4]{\frac{\omega^2 \mu}{E \cdot I}} \cdot l = k \cdot \pi$$

$$f_{crit} = k^2 \cdot \pi \cdot \sqrt{\frac{E \cdot I}{4 \mu \cdot l^4}}$$

k	1	2	3	4	5
f_{crit}	12,80Hz	51,19Hz	115,17Hz	204,74Hz	319,91Hz




Tvar kmitání pro kritické hodnoty
prvních pět tvarů
1,5
1
0,5
0
-0,5
-1
-1,5
0
0,5
1
1,5
2
2,5
3
3,5
4
průhyb [m]
délka nosníku [m]
1
2
3
4
5

$$\frac{df_{(x)}}{dx} = \lim_{dx \to \infty} \frac{f_{(x+\frac{dx}{2})} - f_{(x-\frac{dx}{2})}}{dx}$$

$$\frac{df_{(x)}}{dx} \approx \frac{f_{(x+\frac{dx}{2})} - f_{(x-\frac{dx}{2})}}{dx}$$

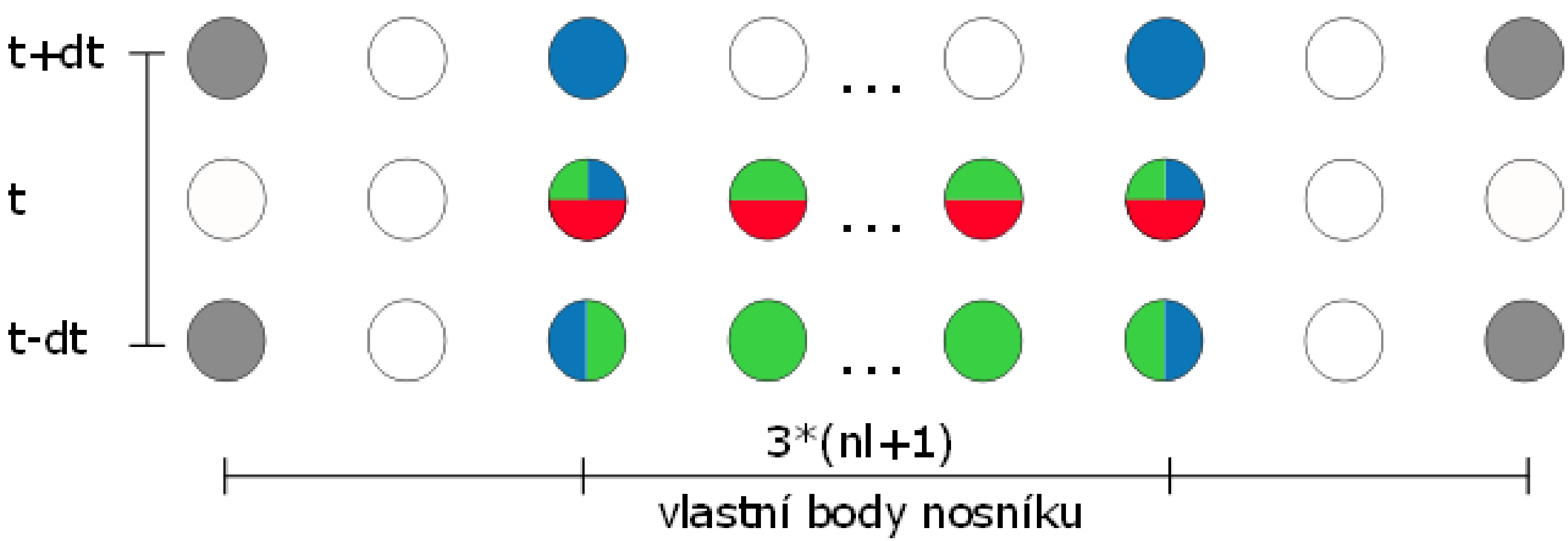
$$\frac{d^2 f_{(x)}}{dx^2} \approx \frac{f'_{(x+\frac{dx}{2})} - f'_{(x-\frac{dx}{2})}}{dx} \approx \frac{\frac{f_{(x+dx)} - f_{(x)}}{dx} - \frac{f_{(x-dx)} - f_{(x)}}{dx}}{dx}$$

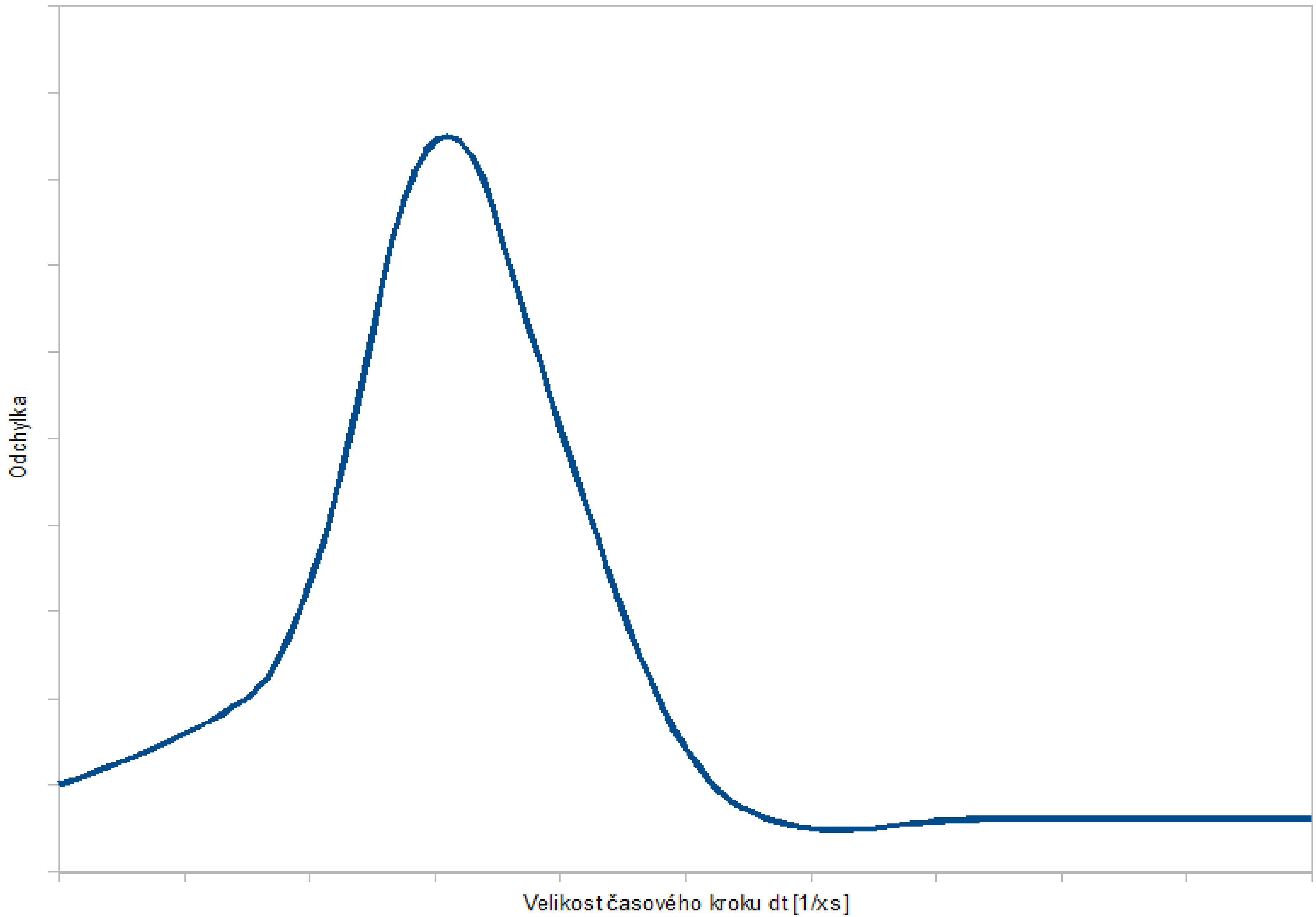
$$\frac{d^2 f_{(x)}}{dx^2} \approx \frac{f_{(x-dx)} - 2f_{(x)} + f_{(x+dx)}}{dx^2}$$

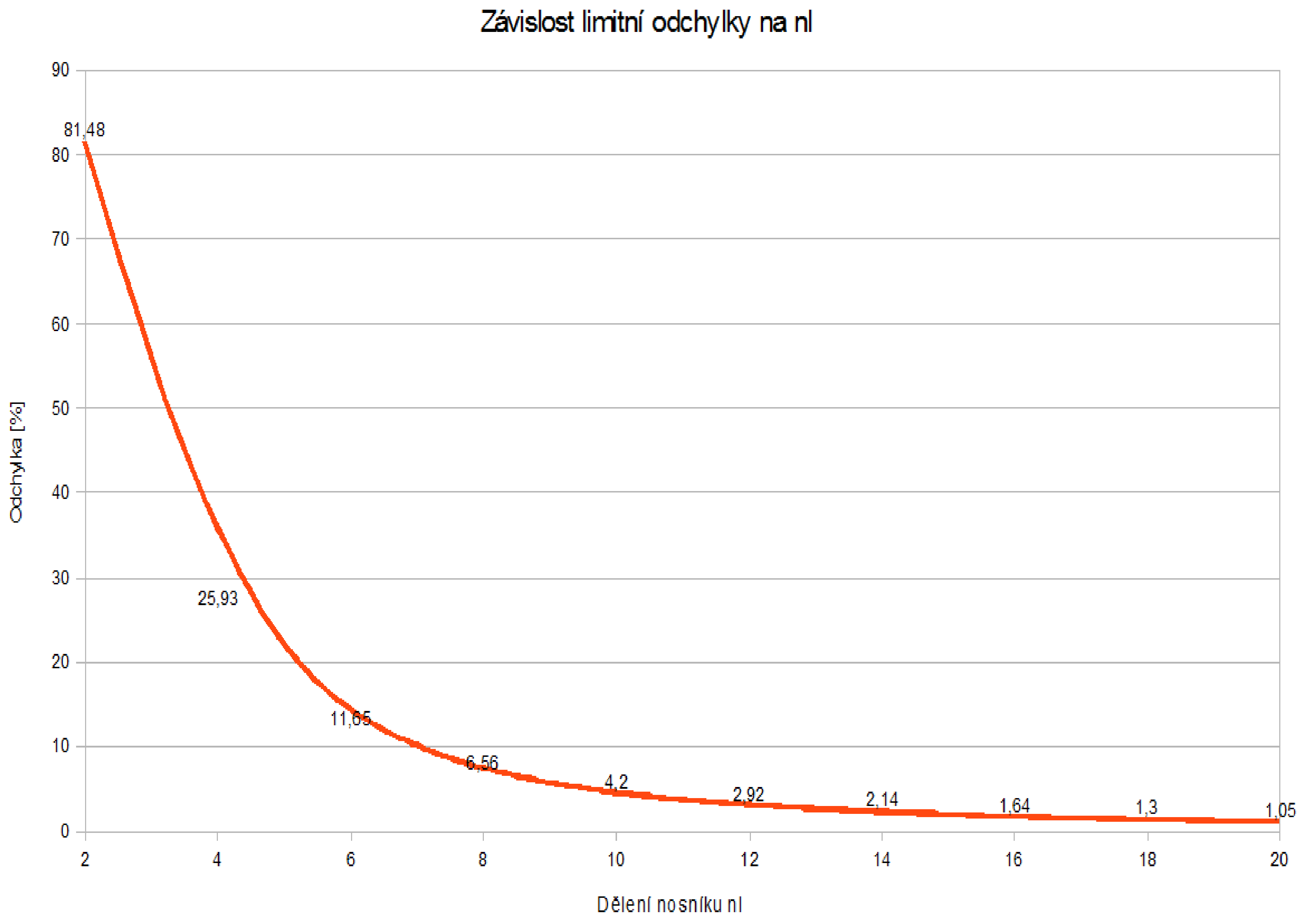
$$\frac{d^4 f_{(x)}}{dx^4} \approx \frac{f_{(x-2\mathrm{dx})} - 4f_{(x-dx)} + 6f_{(x)} - 4f_{(x+dx)} + f_{(x+2\mathrm{dx})}}{dx^4}$$

$$E \cdot I \frac{v_{(x-2\mathrm{dx},t)} - 4v_{(x-dx,t)} + 6v_{(x,t)} - 4v_{(x+dx,t)} + v_{(x+2\mathrm{dx},t)}}{dx^4}$$

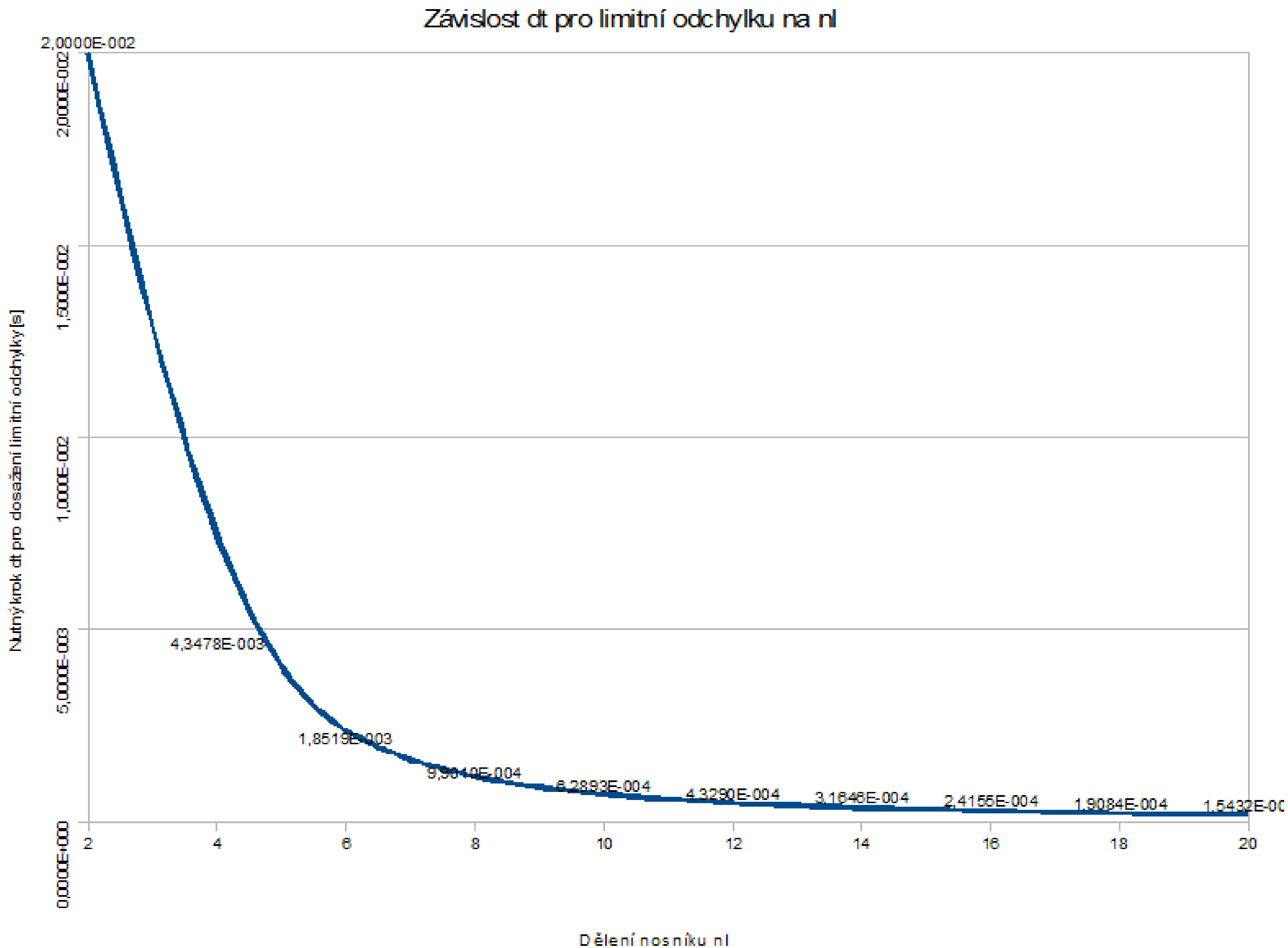
$$+ \mu \frac{v_{(x,t-dt)} - 2v_{(x,t)} + v_{(x,t+dt)}}{dt^2} = f_{(x,t)}$$



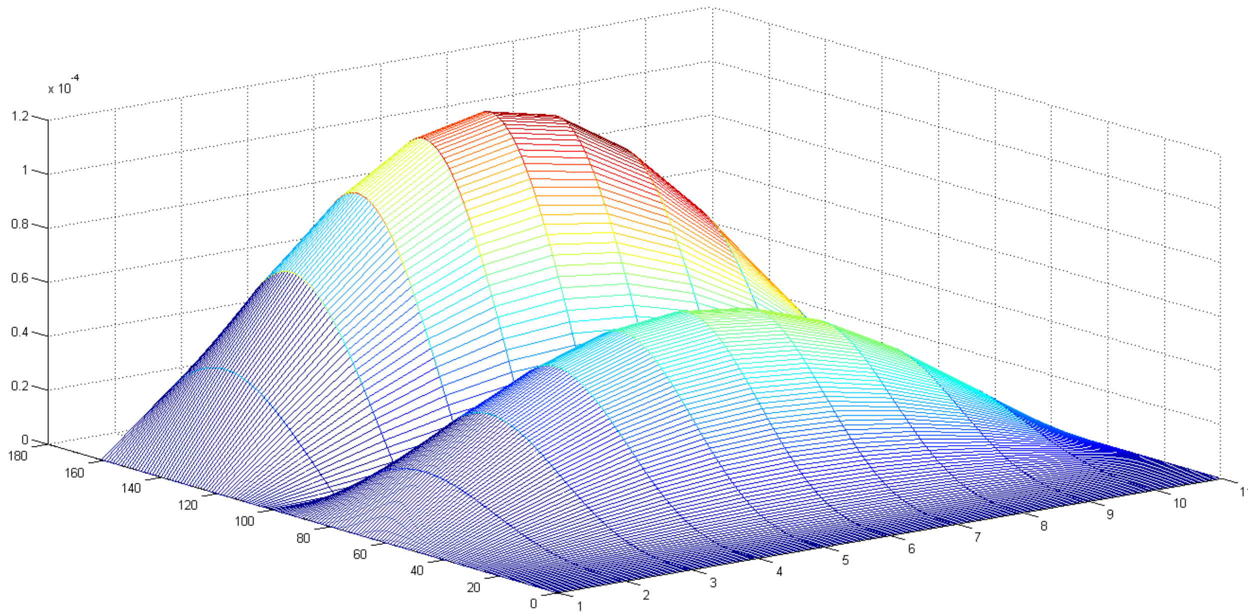

Odchylka
Velikost časového kroku dt [1/xs]

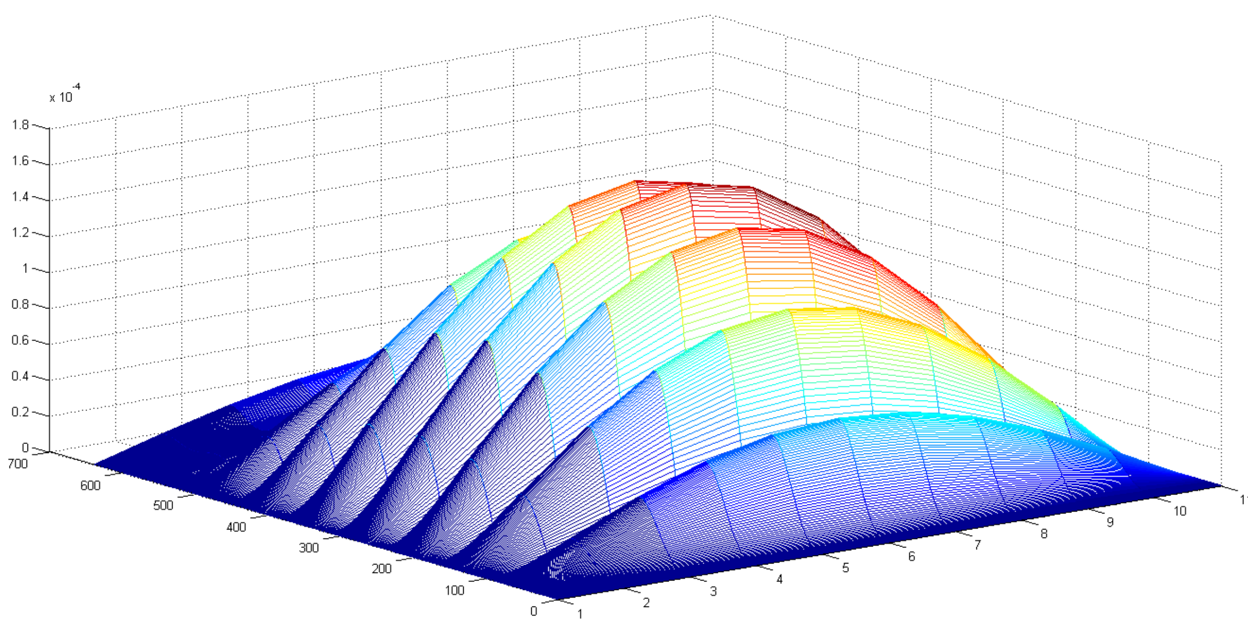


Závislost limitní odchylky na nl
Odchylka [%]
90
80
70
60
50
40
30
20
10
0
2
4
6
8
10
12
14
16
18
20
81,48
25,93
11,65
6,56
4,2
2,92
2,14
1,64
1,3
1,05
Dělení nosníku nl



Závislost dt pro limitní odchylku na nl
Nutný krok dt pro dosažení limitní odchylky [s]
Dělení nosníku nl
2,0000E-002
1,5000E-002
1,0000E-002
5,0000E-003
0,0000E+000
2
4
6
8
10
12
14
16
18
20
2,0000E-002
4,3478E-003
1,8519E-003
9,9010E-004
6,2893E-004
4,3290E-004
3,1646E-004
2,4155E-004
1,9084E-004


x 10^-4
1.2
1
0.8
0.6
0.4
0.2
0
180
160
140
120
100
80
60
40
20
0
1
2
3
4
5
6
7
8
9
10
11


x 10^-4
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
700
600
500
400
300
200
100
0
1
2
3
4
5
6
7
8
9
10
11

nl	2	4	6	8	10	12	14	16	18	20
nt	5	23	55	101	159	231	316	414	524	648
Limitní odchylka [%]	81,48	25,93	11,65	6,56	4,2	2,92	2,14	1,64	1,3	1,058
Maximální odchylka [%]	255,6	44,44	17,81	9,81	6,17	4,25	3,11	2,37	1,87	1,51