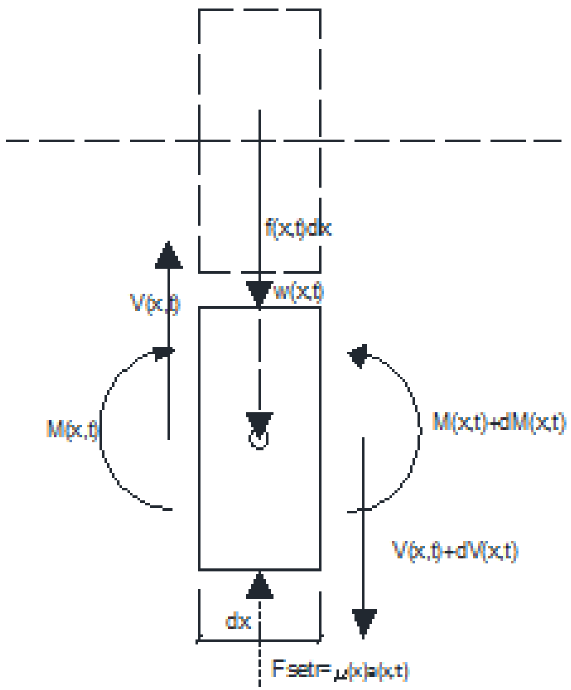
Vlastní kmitání nosníku

Semestrální práce z předmětu Pružnost a Pevnost

Zavedené předpoklady a principy

- Nosník je prizmatický – $\mu(x)=A\rho=konst.$
- Podélné přemístění je nulové
- Bernoulli-Navierova hypotéza
- Hookův zákon
- d'Alembertův princip



$$F_{setr} = \mu(x).a(x,t)dx = \rho A \frac{\partial^2 w(x,t)}{\partial t^2} dx \qquad ,\mu(x) = konst.$$

$$V(x,t) + \frac{\partial V(x,t)}{\partial x} dx - V(x,t) + f(x,t)dx = \rho A \frac{\partial^2 w(x,t)}{\partial t^2} dx$$

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t)$$

HOMOGENNÍ ŘEŠENÍ

$f(x,t) = 0$

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0$$

$$w(x,t) = w_0(x)\sin(\omega t)$$

Počáteční podmínka: $t = 0, w(x,0) = w_0(x)\sin(0) = 0$

$$EI\frac{\partial^4 w_0(x)}{\partial x^4}\sin(\omega t) - \rho A\,\omega^2 w_0(x)\sin(\omega t) = 0$$

Řešení hledáme ve tvaru:

$$w_0(x) = e^{\alpha x}$$

$$\sin(\omega t)\left(EI\alpha^4 e^{\alpha x} - \rho A\,\omega^2 e^{\alpha x}\right) = 0$$

$$\sin(\omega t)\, e^{\alpha x}(EI\alpha^4 - \rho A\, \omega^2) = 0$$

$$\alpha^4 = \frac{\rho A\, \omega^2}{EI}$$

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (1, -1, i, -i)\sqrt[4]{\frac{\rho A\, \omega^2}{EI}}$$

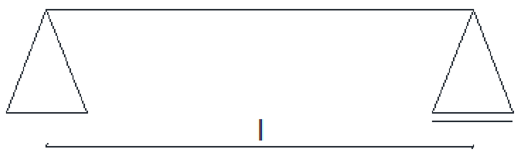
$$zavedeme\ substituci\ \lambda = l\sqrt[4]{\frac{\rho A\, \omega^2}{EI}}$$

$$w_0(x) = C_1 e^{\lambda x/l} + C_2 e^{-\lambda x/l} + C_3 e^{i\lambda x/l} + C_4 e^{-i\lambda x/l}$$

$$w_0(x) = C_1 \cosh\left(\frac{\lambda x}{l}\right) + C_2 \sinh\left(\frac{\lambda x}{l}\right) + C_3 \cos\left(\frac{\lambda x}{l}\right) + C_4 \sin\left(\frac{\lambda x}{l}\right)$$

$$w_0(x) = C_1 \cosh\left(\frac{\lambda x}{l}\right) + C_2 \sinh\left(\frac{\lambda x}{l}\right) + C_3 \cos\left(\frac{\lambda x}{l}\right) + C_4 \sin\left(\frac{\lambda x}{l}\right)$$

Př. :



$$w_{(0)} = 0 \qquad w''_{(0)} = 0$$

$$w_{(l)} = 0 \qquad w''_{(l)} = 0$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ cosh(\lambda) & \sinh(\lambda) & cos(\lambda) & \sin(\lambda) \\ \cosh(\lambda) & sinh(\lambda) & -cos(\lambda) & -\sin(\lambda) \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = 0$$

$$C_1 = 0$$

$$C_2 \sinh(\lambda) = 0$$

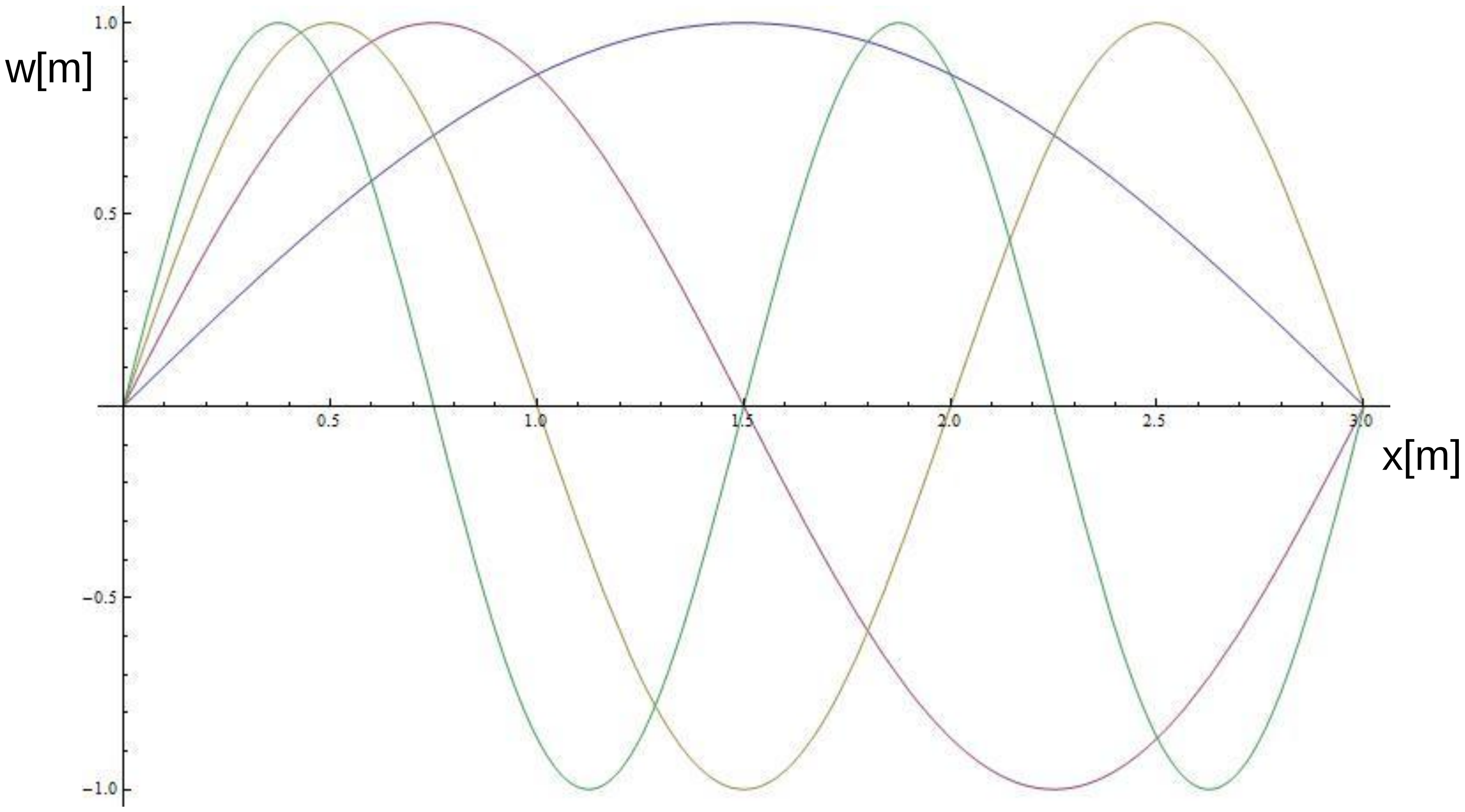
$$C_3 = 0$$

$$C_4 \sin(\lambda) = 0$$

$C_4 \sin(\lambda) = 0 \rightarrow \lambda = k\pi$, C_4 *není nula*

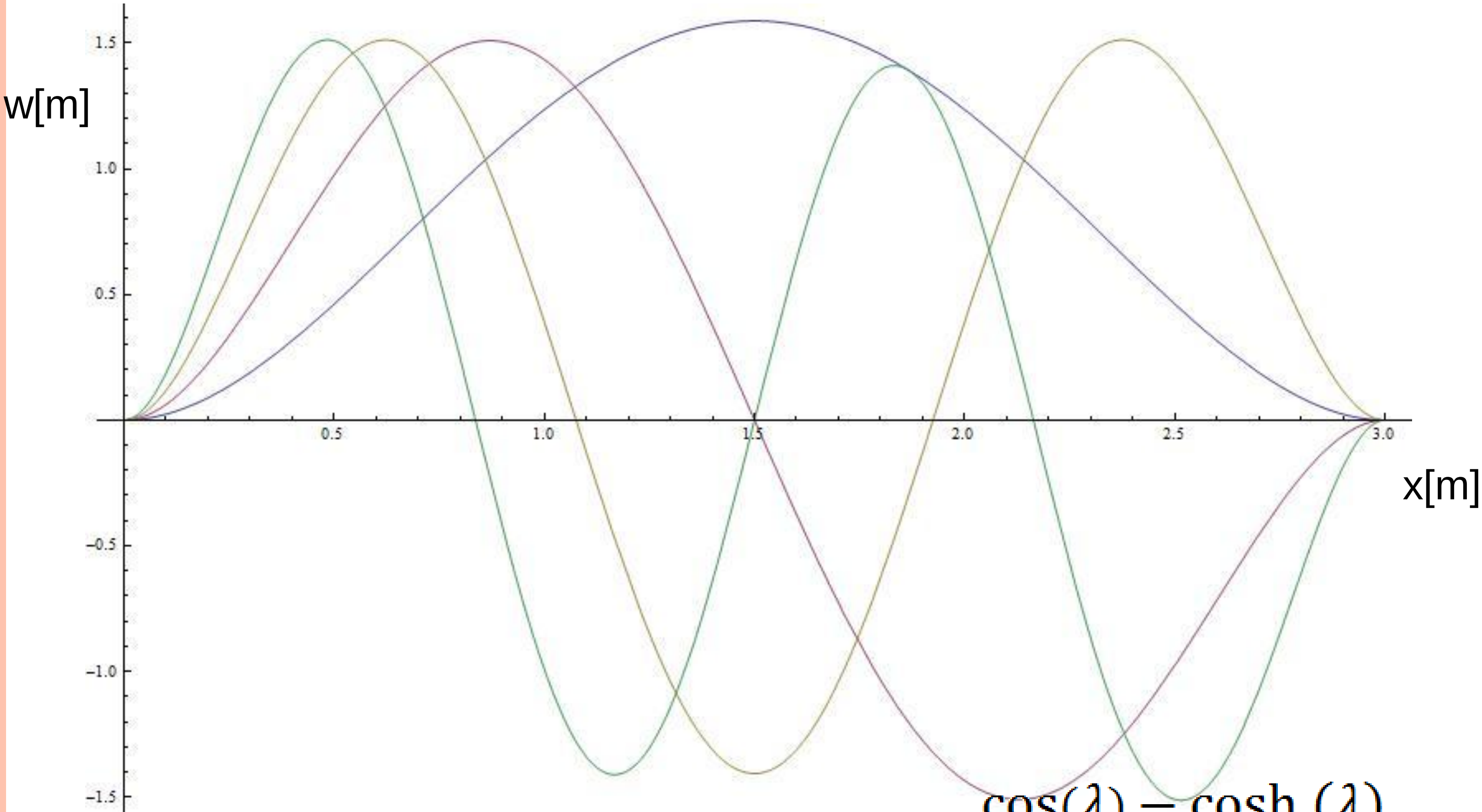
$$\omega_{krit} = \frac{k^2\pi^2}{l^2}\sqrt{\frac{EI}{\mu}}$$

Vlastní tvary průhybu pro oboustranně kloubově uložený nosník



$$w_0^k(x) = C_4 sin(\frac{\lambda_k x}{l}) \qquad \lambda_k = \pi, 2\pi, 3\pi$$

Vlastní tvary průhybu pro oboustranně vetknutý nosník



$$\cos(\lambda)\cosh(\lambda) - 1 = 0 \qquad g(\lambda) = \frac{\cos(\lambda) - \cosh(\lambda)}{\sinh(\lambda) - \sin(\lambda)}$$

$$w_0^v(x) = C_1\left(\cosh\left(\frac{\lambda x}{l}\right) + g(\lambda)\sinh\left(\frac{\lambda x}{l}\right) - \cos\left(\frac{\lambda x}{l}\right) - g(\lambda)\sin\left(\frac{\lambda x}{l}\right)\right)$$

Partikulární řešení

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\,\frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t) \qquad f(x,t) = f sin(\omega t)$$

Předpoklad : $w(x,t) = w(x)\sin\,(\omega t)$

Počáteční podmínka : $t = 0, w(x,0) = w(x)\sin(0) = 0$

Řešení lze odhadnout : $w_p(x) = -\dfrac{f}{\mu\omega^2}$

$$w(x) = C_1\cosh\left(\frac{\lambda x}{l}\right) + C_2\sinh\left(\frac{\lambda x}{l}\right) + C_3\cos\left(\frac{\lambda x}{l}\right) + C_4\sin\left(\frac{\lambda x}{l}\right) - \frac{f}{\mu\omega^2}$$

$$w_{(0)} = 0 \qquad w''_{(0)} = 0$$

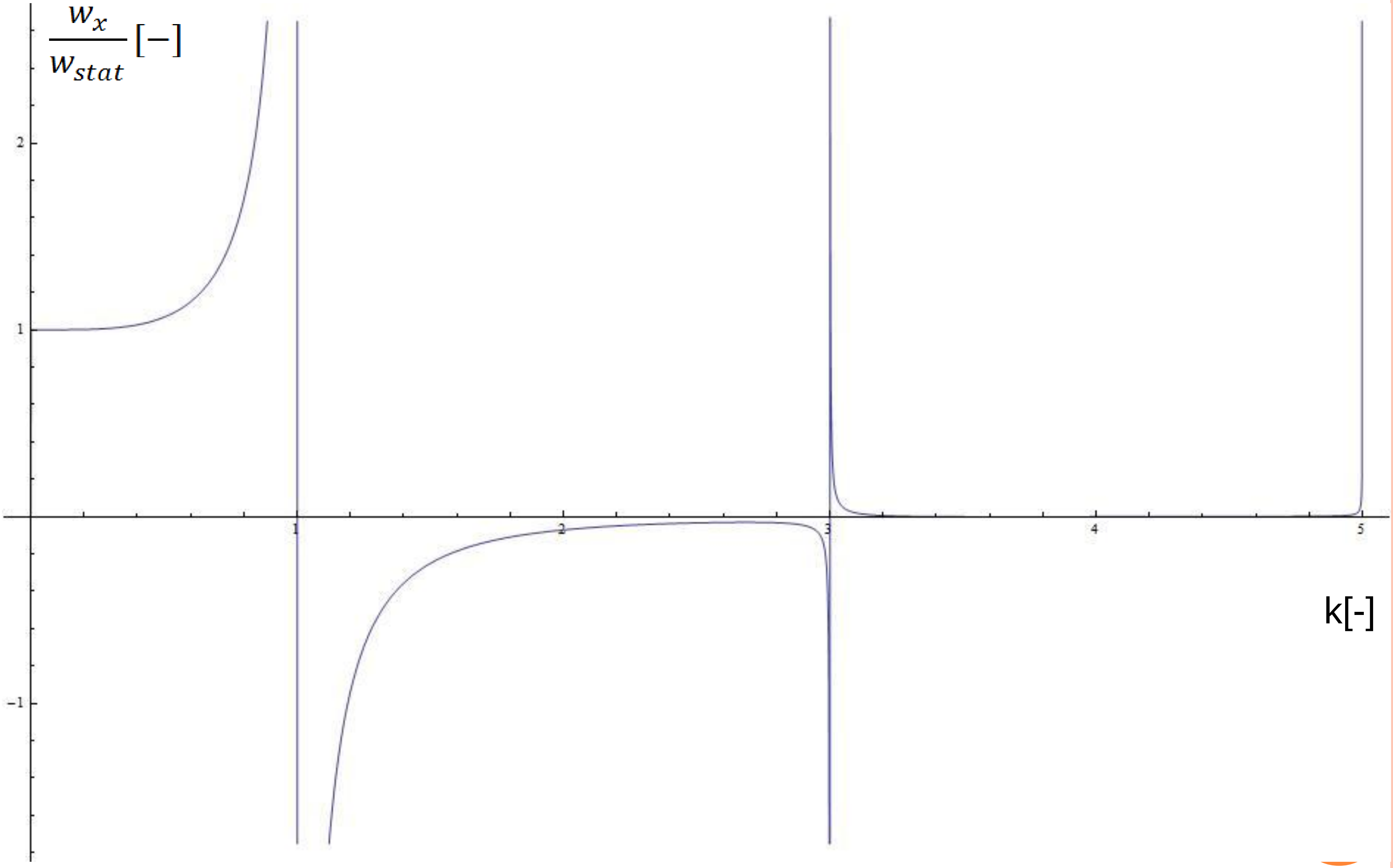
$$w_{(l)} = 0 \qquad w''_{(l)} = 0$$

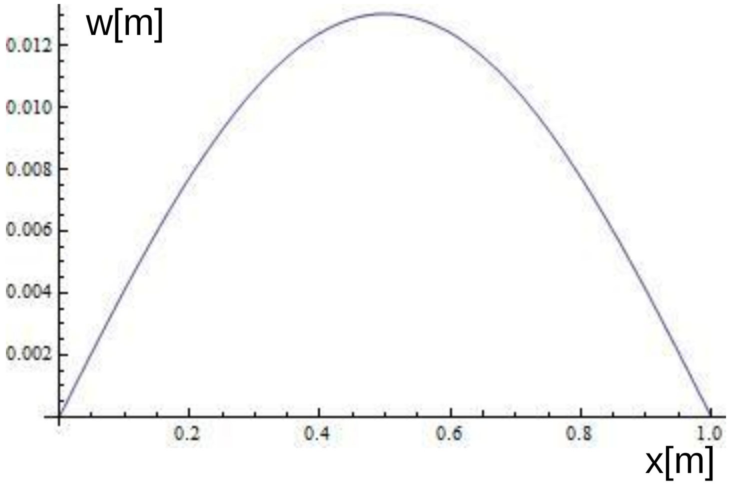
$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ cosh(\lambda) & \sinh(\lambda) & cos(\lambda) & \sin(\lambda) \\ \cosh(\lambda) & sinh(\lambda) & -cos(\lambda) & -\sin(\lambda) \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} \frac{f}{\mu\omega^2} \\ 0 \\ \frac{f}{\mu\omega^2} \\ 0 \end{pmatrix}$$

$$C_1 = C_3 = \frac{f}{2\mu\omega^2} \qquad\qquad C_2 = \frac{f}{2\mu\omega^2}\frac{1-\cosh(\lambda)}{\sinh(\lambda)}$$

$$C_4 = \frac{f}{2\mu\omega^2}\frac{1-\cos(\lambda)}{\sin(\lambda)}$$

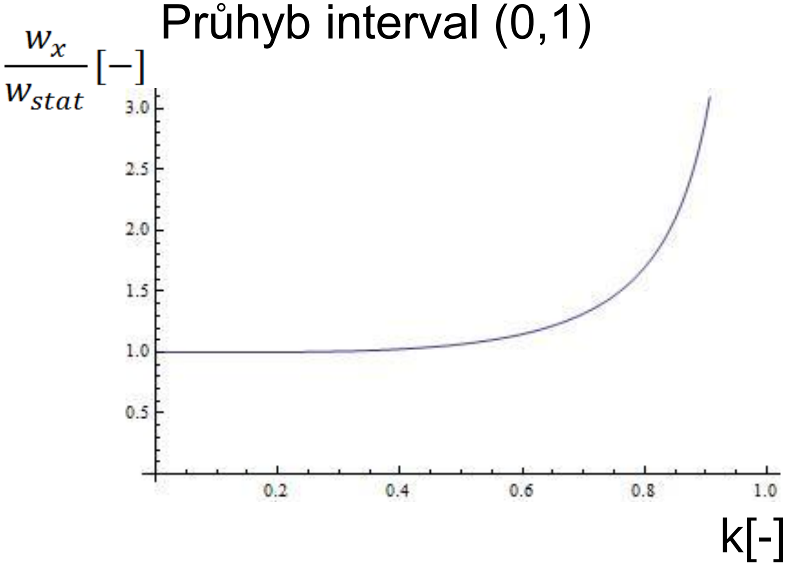
Průběh průhybu v polovině rozpětí

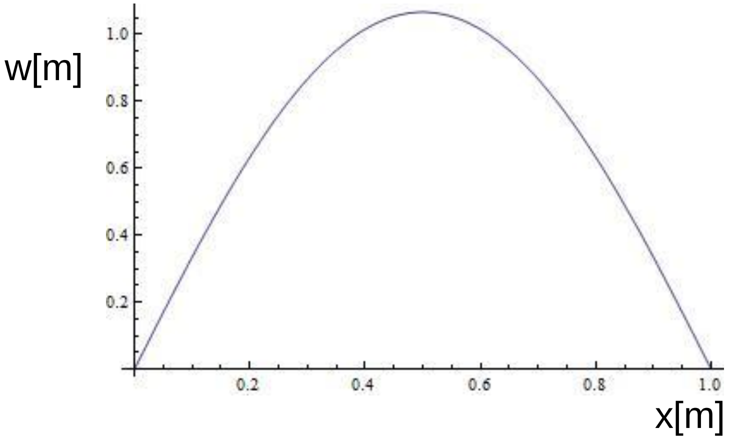




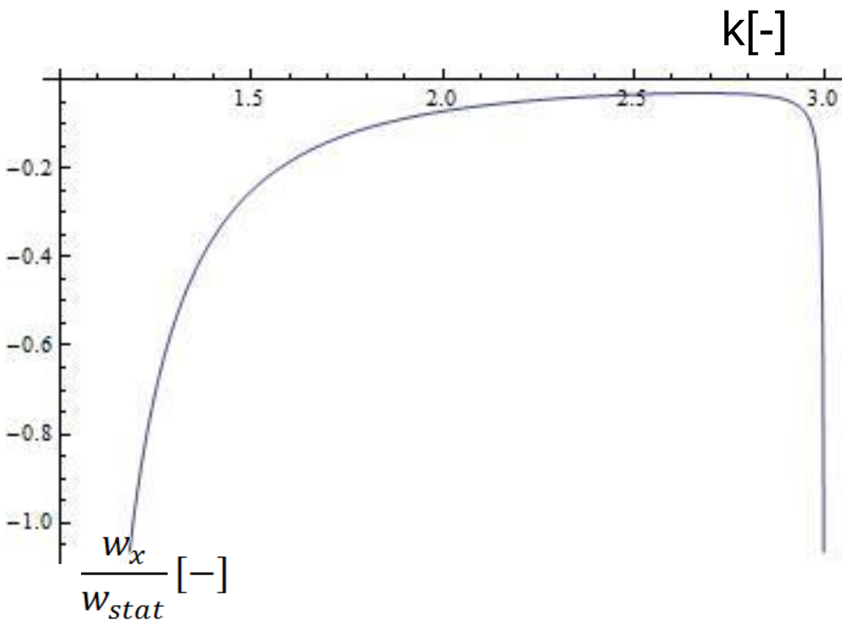
Průhyb od statického zatížení

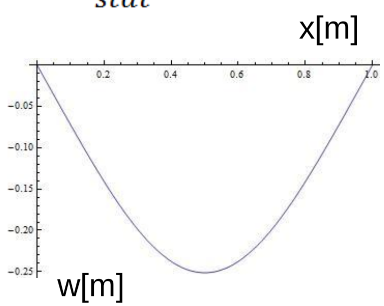
Průhyb interval (0,1)

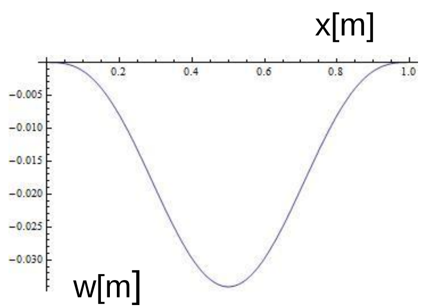




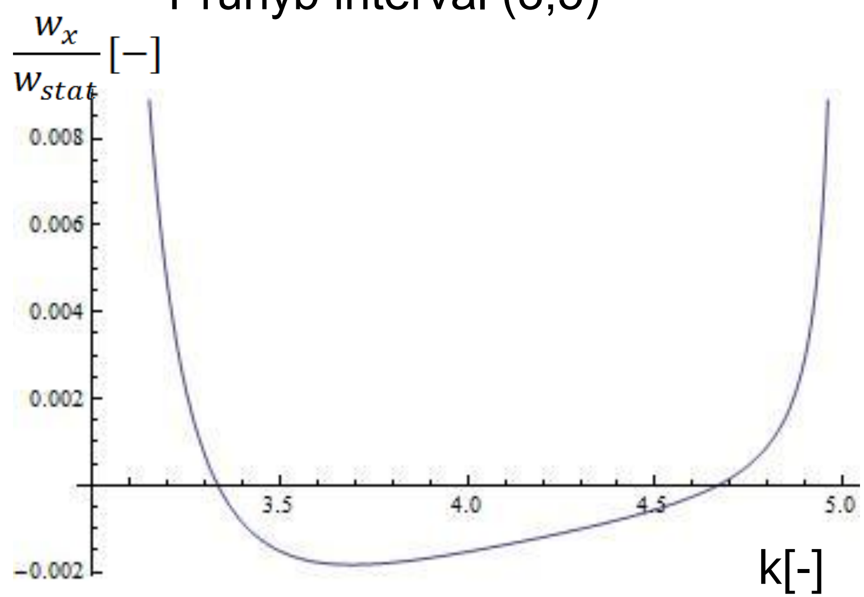
Průhyb interval (1,3)

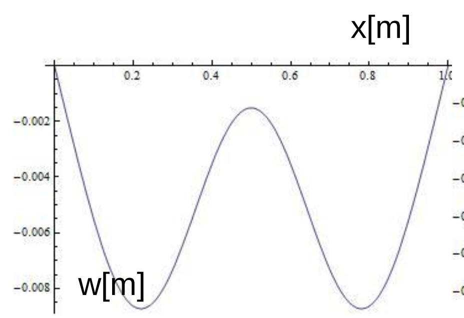


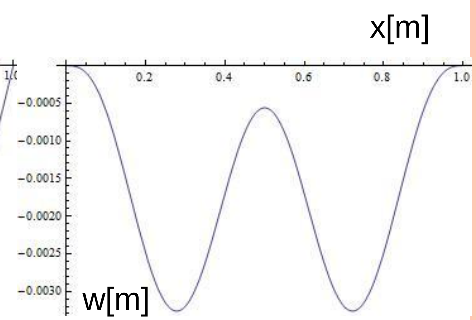




Průhyb interval (3,5)

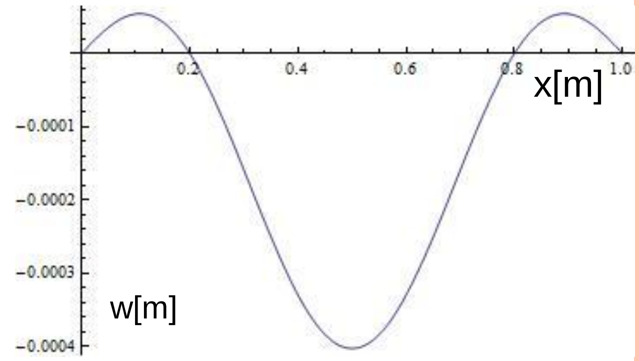




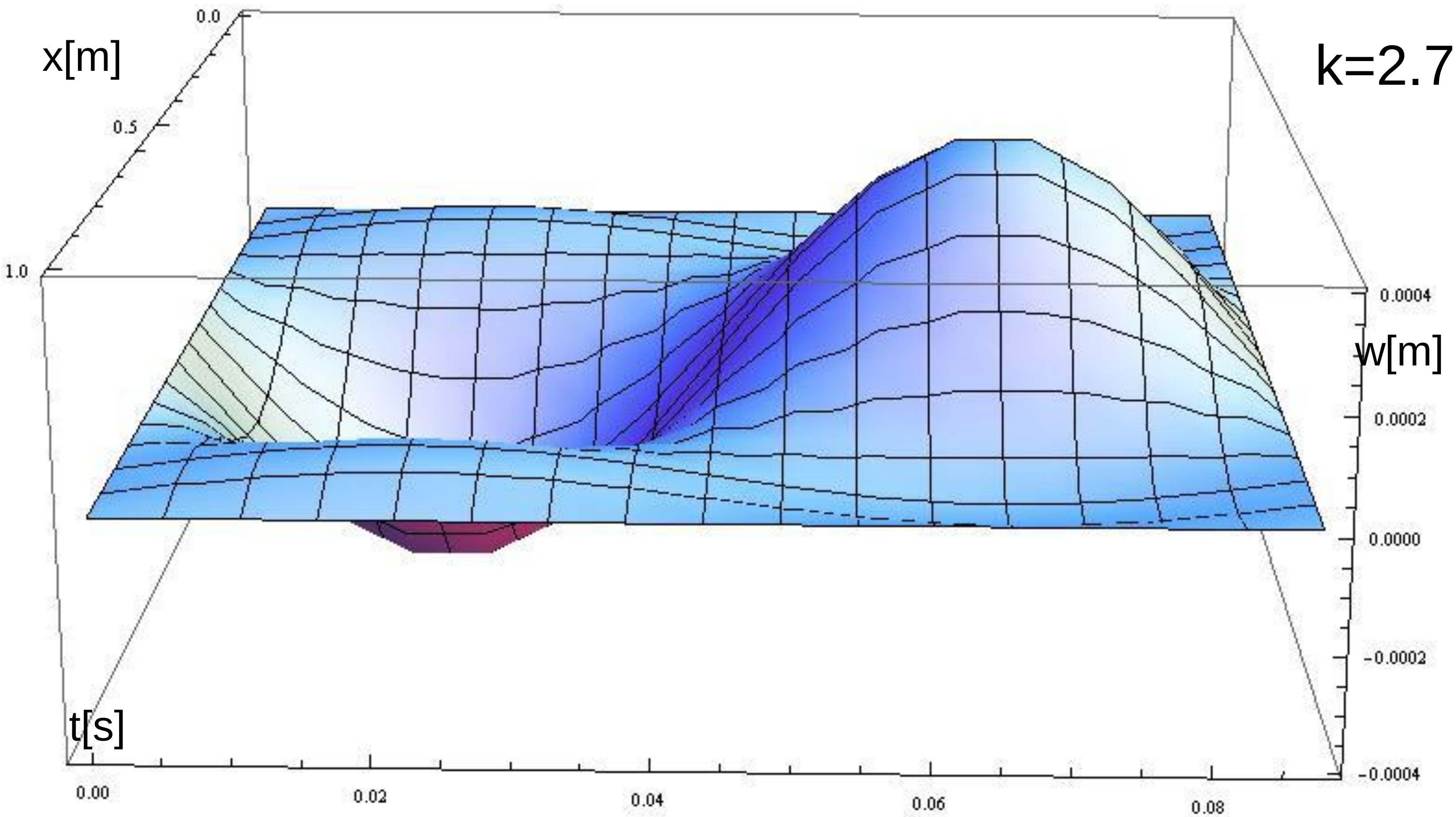


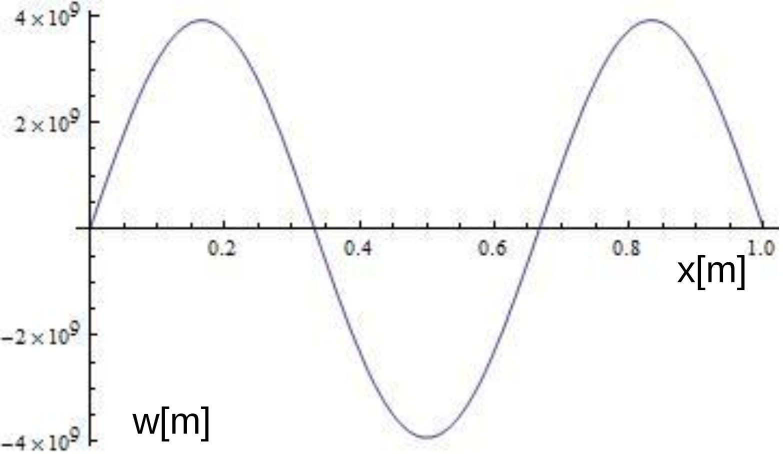
Vykreslení

- Průběh průhybu kloubově uloženého nosníku v závislosti na prostorové souřadnici x a času t

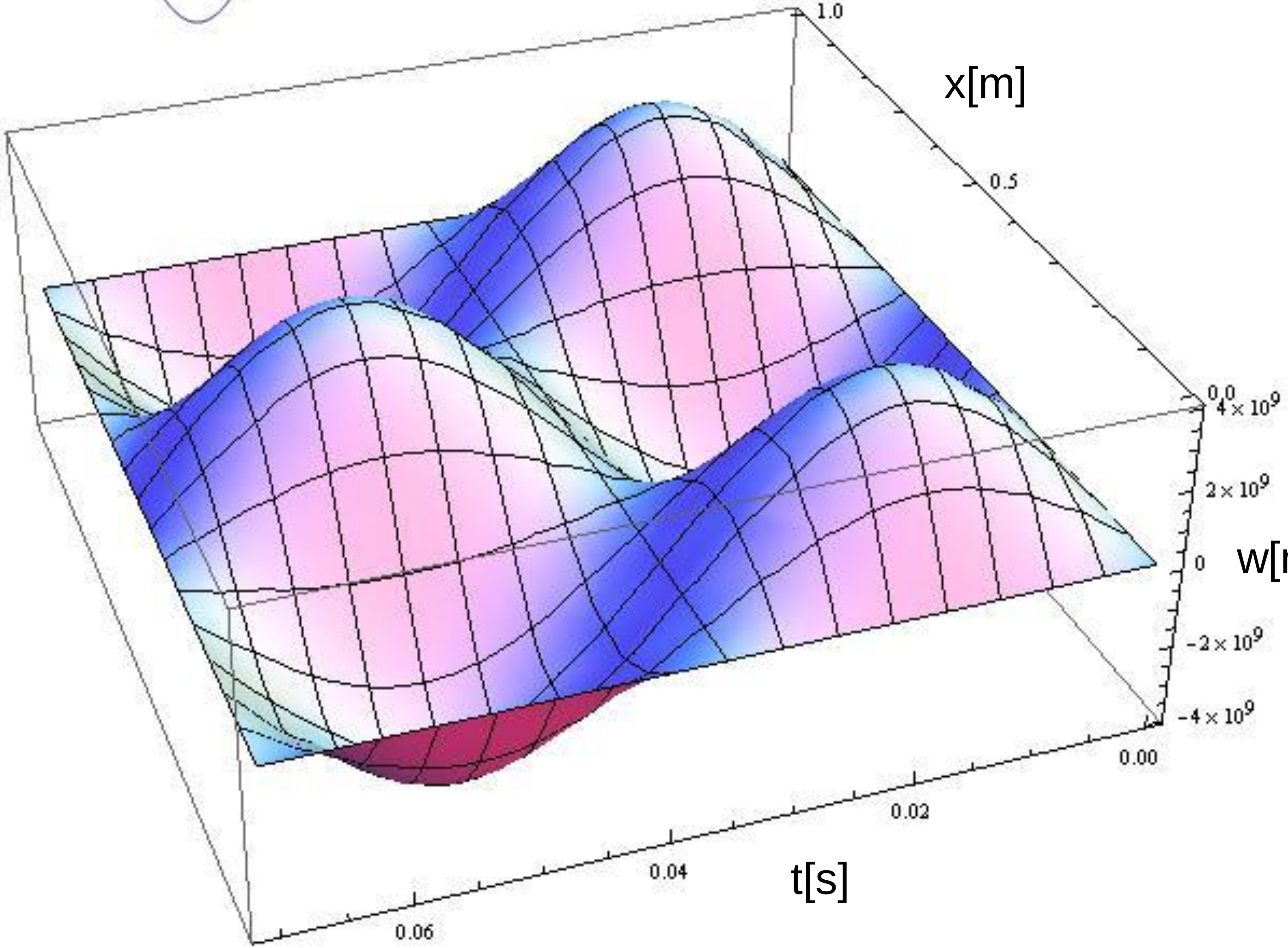


E=1Pa f=1N/m μ=1kg/m I=1m^4





k=2.99999999999



POUŽITÁ LITERATURA A PROGRAMY

- Dynamika stavebních konstrukcí – M.Baťa, V.Plachý, F. Trávníček
- Wolfram Mathematica 8.0