Title (1): Overview of Micro-Elasticity theories with emphasis on strain gradient elasticity: Part I – Theoretical considerations

Abstract (1)

In terms of lattice theory, elasticity incorporates only the nearest neighbor interactions, and that's it. The theory does not have any intrinsic length scale and as a result a 2cm slab behaves like a 10μm film, and there is no difference among a microcrack and a geological fault. Moreover, classical elasticity implies that wave speed of plane shear and dilatational waves in an unbounded medium is independent of frequency, the same is predicted for Rayleigh waves, and surface SH waves are not predicted by elasticity. So the question is if one could develop an elasticity theory with intrinsic length scale or scales that can predict such phenomena. One way to include a length scale in an elasticity theory is by considering higher gradients of displacements (i.e. 2^{nd} , 3^{rd} and so forth). The fundamental idea of considering not only the first, but also the higher gradients of the displacement field in the expression for the strain energy function of an elastic solid, can be traced back to J. Bernoulli (1654-1705) and L. Euler (1707-1783) in connection with their work on beam theory. In elementary beam theory there are associated two sets of kinematical quantities (a deformation vector and a rotation vector) and two sets of surface loads (tractions and bending couples) with a section of the bar. In plate theory the situation is similar. Mindlin's pioneering work in 1964 on gradient linear elasticity is strongly influenced from structural mechanics. On the other hand Casal in 1961 was the first to see the connection between surface tension effects and the anisotropic second gradient theory. Mindlin following another path in 1968 was enforced to embark in third gradient of displacement and triple stresses to capture the surface energy property of new surfaces in solids. In this respect, for pedagogical reasons, we first present a technical beam bending theory with surface energy that contains two length scales. One length scale arises from Timoshenko's correction to account for shear strains and the other to account for surface effects. It is shown that the surface energy length scale of the theory is responsible for (i) a "stiffening" effect of the beam similar to that produced by the consideration of pre-tension, and (ii) a size effect exhibited by the flexural strength of beams, namely the dependence of the flexural strength on the inverse length of the beam for the same aspect ratio. Then, an overview of the various formats of general gradient elasticity theories is displayed focusing on Mindlin's second gradient theory. The latter is useful in the direction of development of an elasticity beam theory by generalizing the results of the technical theory presented previously. The principal difficulties indeed are to discover the practical significance of these generalized theories, to design experimental methods to explore their physical validity and to identify the length scale parameters. However, it is shown here that a Mindlin-Casal type first strain gradient elasticity constitutes the simplest, in energy consistent, non-local extension of Hooke's law that is able to capture the surface energy and used to predict interesting phenomena like size and dispersion effects and cusping of crack lips. So, this simplified theory is subsequently used to attack some basic elasticity problems, albeit of technological importance, that are presented in Part 2 of these series of lectures.

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Comment: A re-birth of non-local elasticity theory happened after the 90's on and a plethora of papers were published in scientific journals. One may find many of them in the paper by: Askes H.,Aifantis E.C., Gradient elasticity in statics and dynamics: An overview of formulations, length scale identification procedures, finite element implementations and new results, *International Journal of Solids and Structures* **48** (2011) 1962–1990. Another paper is by Lazar M., Polyzos D., On non-singular crack fields in Helmholtz type enriched elasticity theories, *International Journal of Solids and Structures* **62** (2015) 1–7. Furthermore, a hierarchy of simple gradient models has been also recently presented by Polizzotto C., A hierarchy of simplified constitutive models within isotropic strain gradient elasticity, European Journal of Mechanics A/Solids 61 (2017).

Suggested textbooks for reading:

- Any textbook on continuum mechanics
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- Timoshenko S.P. and Goodier J.N., Theory of Elasticity, McGraw-Hill. New York 1951.
- Vardoulakis I. & Sulem, J.. Bifurcation analysis in geomechanics. Blackie Academic & Professional, 1995.