PROBABILISTIC ESTIMATION OF MATERIAL PARAMETERS BASED ON A SET OF EXPERIMENTAL CURVES

Eliška Janouchová*, *Anna Kučerová, Jan Sýkora

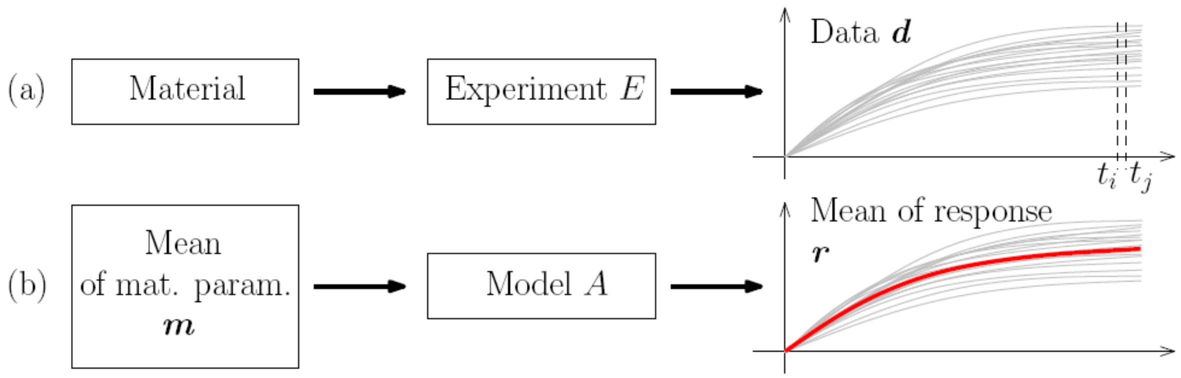
Solid Mechanics Seminar

Prague, Czech Republic

Parameter identification

Fitting the model response to experimental data (b)

- The most common approach of parameter estimation
- Parameter optimisation (ill-posed problem) – robust optimisation algorithms

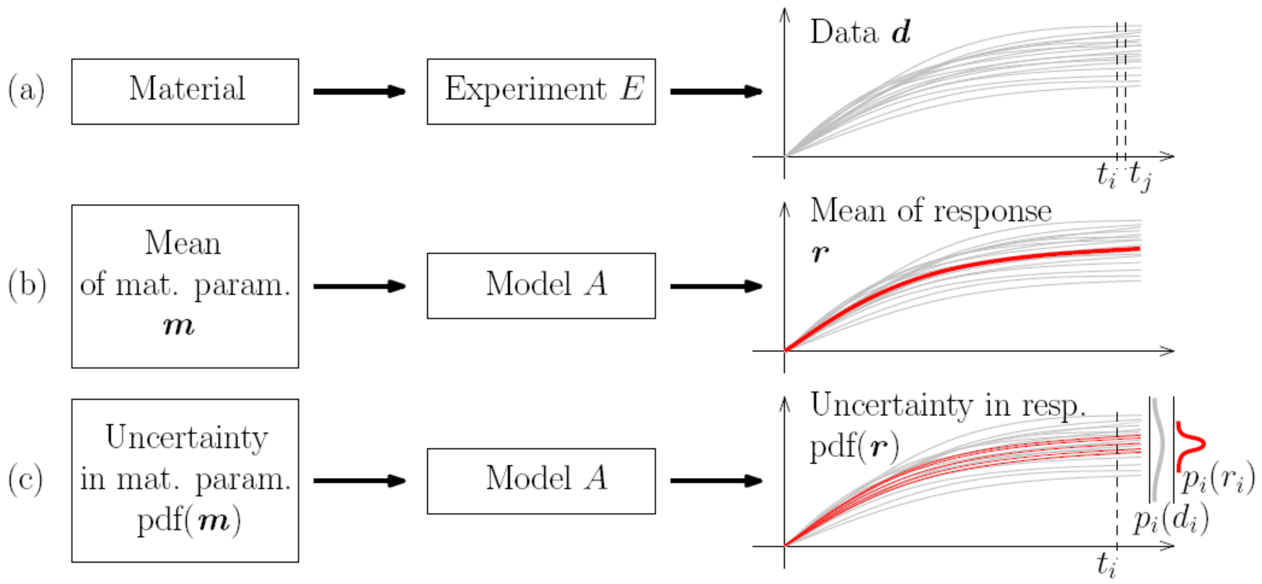


Parameter identification

Parameter identification in probabilistic setting (c)

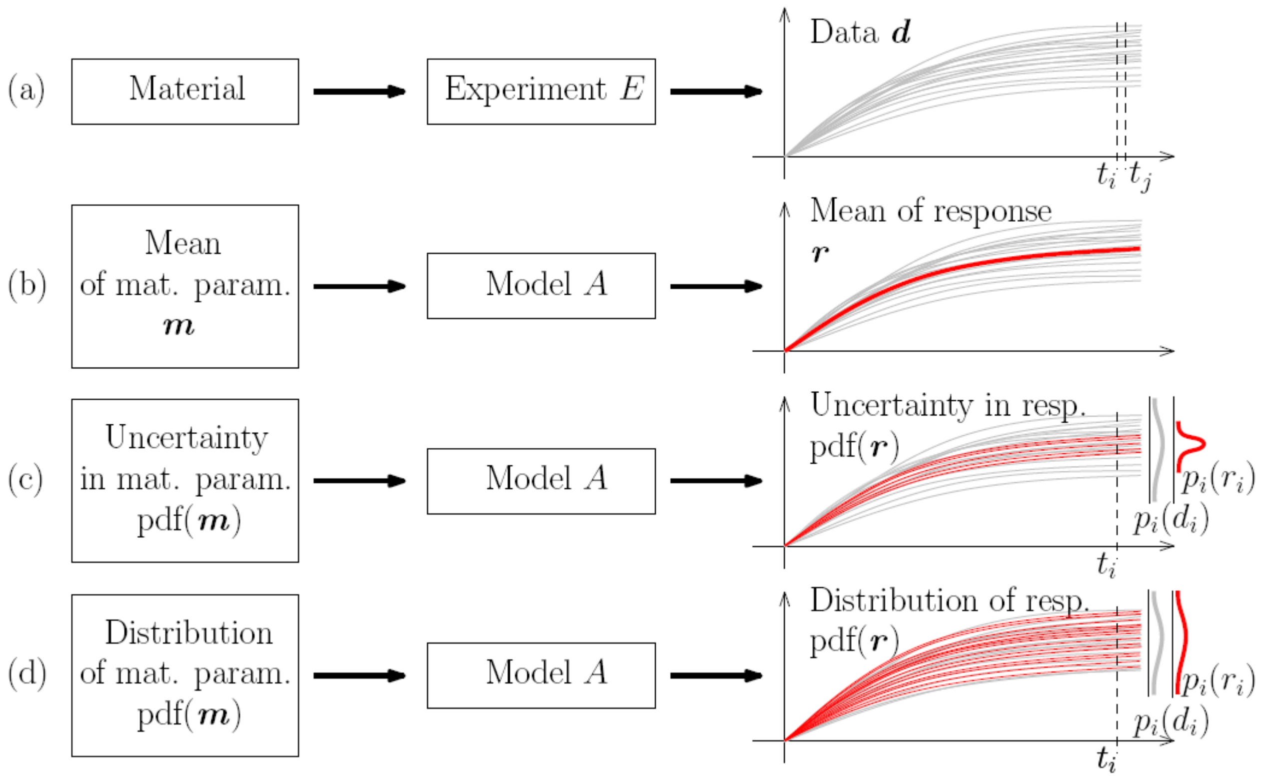
– Bayesian approach combining all available information

– Well-posed identification problem, probabilistic description of parameters



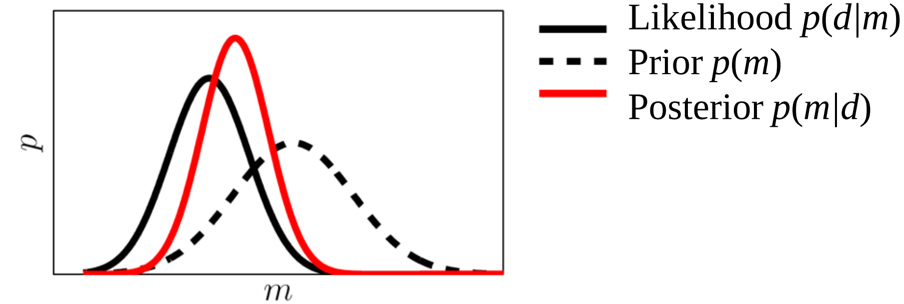
Parameter identification of heterogeneous materials (d)

– The probabilistic description reflecting the actual probability distribution of the parameters in the heterogeneous materials



Bayesian inference

Bayesian approach



Expert knowledge → A priori probability $p(\boldsymbol{m})$

Experimental data → Likelihood function $p(\boldsymbol{d}|\boldsymbol{m}) \approx L(\boldsymbol{d} - \boldsymbol{r}(\boldsymbol{m}))$

Bayes' rule

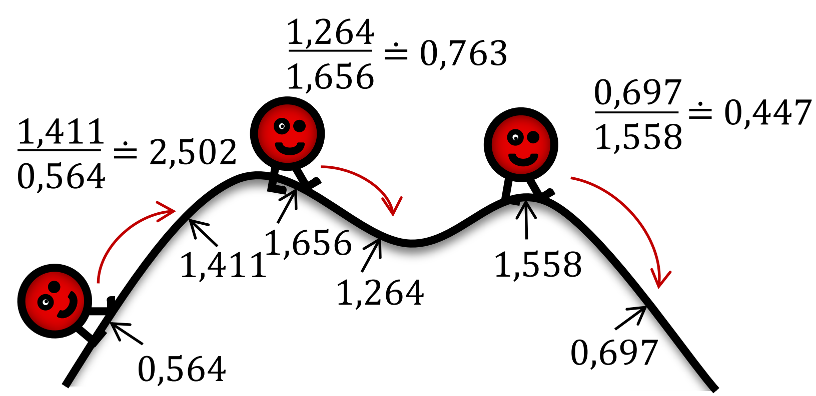
$$p(\boldsymbol{m}|\boldsymbol{d}) = \frac{p(\boldsymbol{d}|\boldsymbol{m})p(\boldsymbol{m})}{p(\boldsymbol{d})}$$

→ Posterior probability

Markov Chain Monte Carlo

- Metropolis algorithm
- Acceptance criterium of a new sample

$$w(X_s = Y_s|X_{s-1}) = \min\{1, \frac{p(Y_s|\boldsymbol{d})}{p(X_{s-1}|\boldsymbol{d})}\}$$



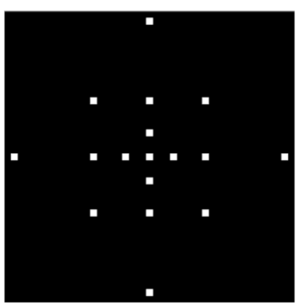
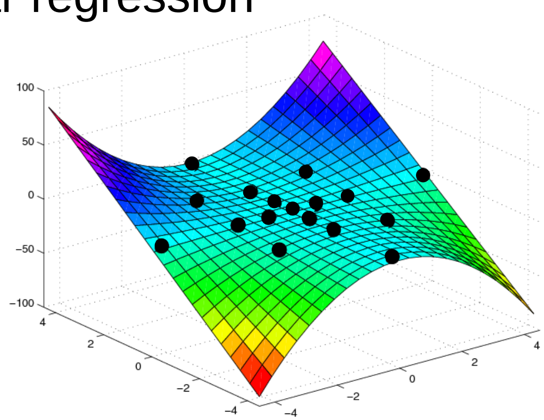
Approximation of a model response

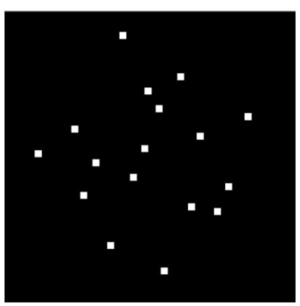
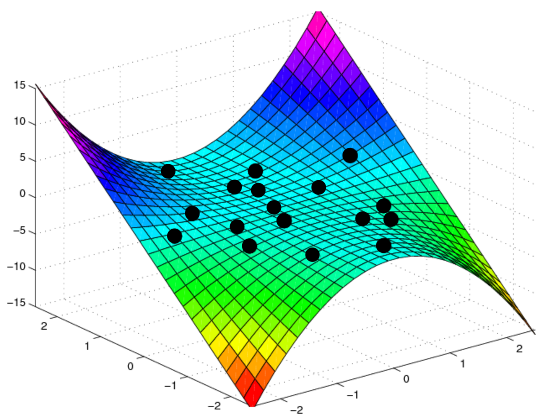
$$\tilde{\boldsymbol{r}}(\boldsymbol{m}(\boldsymbol{\xi})) = \sum_{\alpha} \boldsymbol{\beta}_\alpha \psi_\alpha(\boldsymbol{\xi})$$

- Respect to probability distribution of random variables
  - Hermite polynomials – Gaussian, Legendre polynomials – Uniform

Methods for construction of PCE-based approximation

- Stochastic Galerkin method
- Stochastic collocation method
- Linear regression

Global sensitivity analysis (SA)

- Dependence of the system response $\boldsymbol{r}$ on the system parameters $\boldsymbol{m}$
- Investigation of the system properties on the whole parameters' domain

Sampling-based SA

- Spearman's rank correlation coefficient (SRCC)
  - Nonlinear monotonic dependence
  - A large number of model simulations n

$$\rho_{m_i,r_j} = 1 - \frac{6\sum_{a=1}^{n}\left(\mathrm{rank}(m_{i,a}) - \mathrm{rank}(r_{j,a})\right)^2}{n(n^2-1)}$$

SA based on ANOVA decomposition

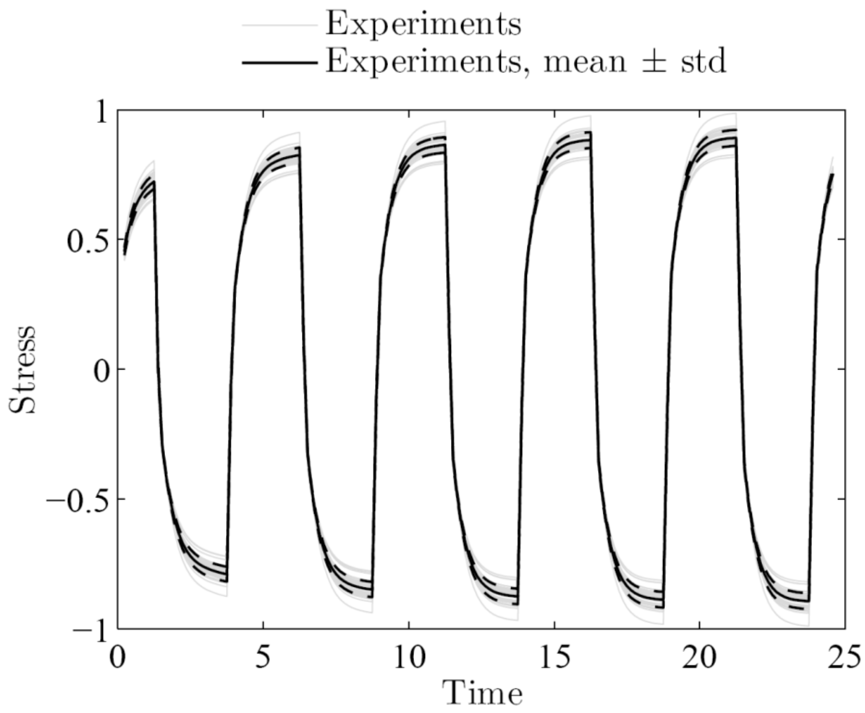
- Sobol' sensitivity indices
  - Nonmonotonic dependence
  - Analytical expression from PCE coefficients

$$S_{i_1,\ldots,i_s}^{\mathrm{PCE}} = \frac{\sum_{\alpha\in\mathcal{I}_{i_1,\ldots,i_s}} \beta_\alpha^2 \mathbb{E}[\psi_\alpha^2(\boldsymbol{\xi})]}{\sum_{\alpha=1}^{n_\beta} \beta_\alpha^2 \mathbb{E}[\psi_\alpha^2(\boldsymbol{\xi})]}, \text{ where } \mathcal{I}_{i_1,\ldots,i_s} = \{\alpha_k = 0 \Longleftrightarrow k \notin (i_1,\ldots,i_s), \forall k = 1,\ldots,n_\beta\}$$

Example

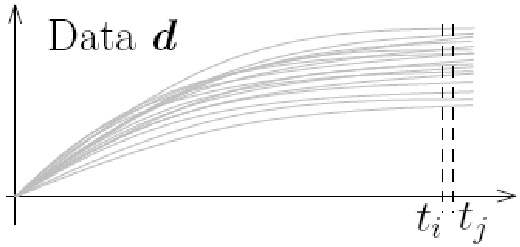
Cyclic loading test

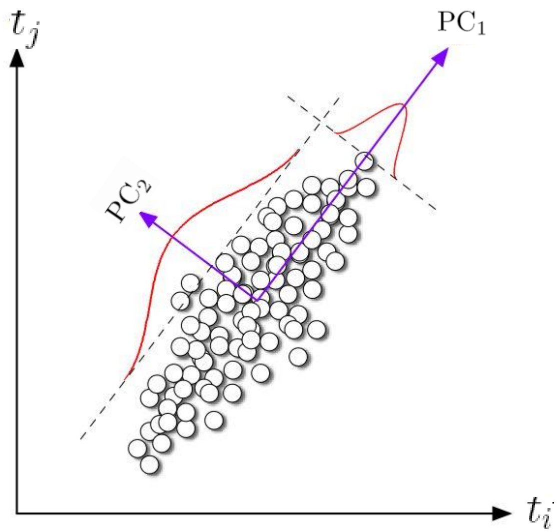
- Pseudo-experimental data set of 50 repetitions
- Model response is influenced by 6 uncertain parameters with lognormal prior
- Task: Identify the parameters' probability density functions



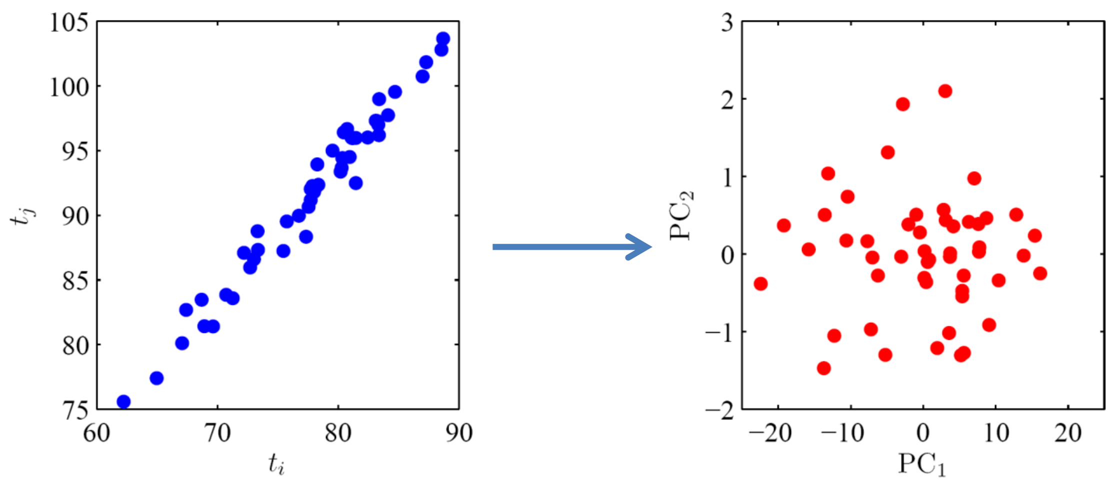
$$\text{Stress(Time)} = M(\boldsymbol{m})$$
$$\boldsymbol{m} = (m_1, m_2, m_3, m_4, m_5, m_6)$$

Principal component analysis



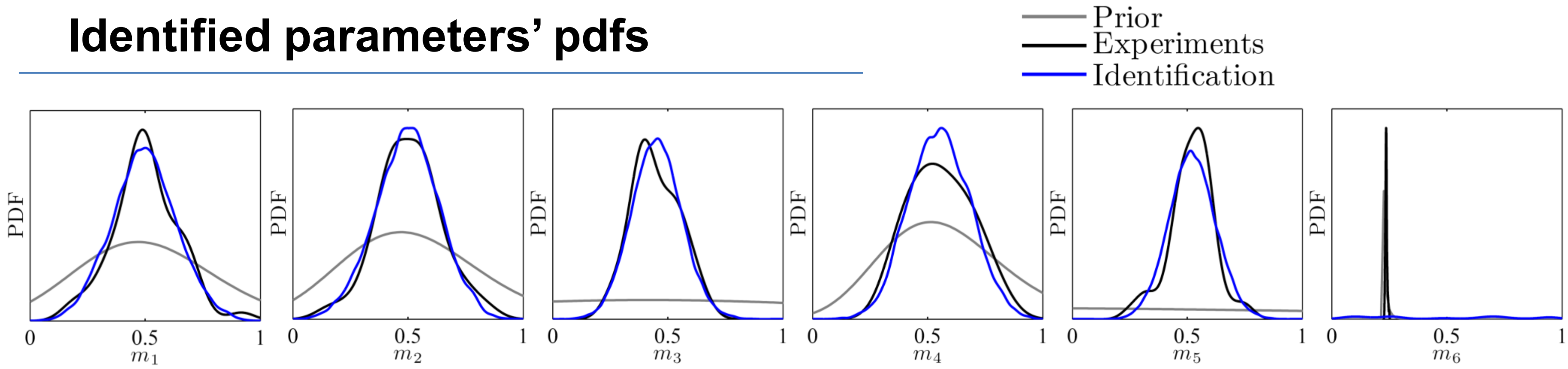


PC	VAR	VAR explained
1	9588.425	96.310
2	342.552	99.751
3	22.569	99.978
4	1.641	99.994
5	0.543	100.000
6	0.016	100.000
7	0.003	100.000
8	0.000	100.000
9	0.000	100.000
10	0.000	100.000

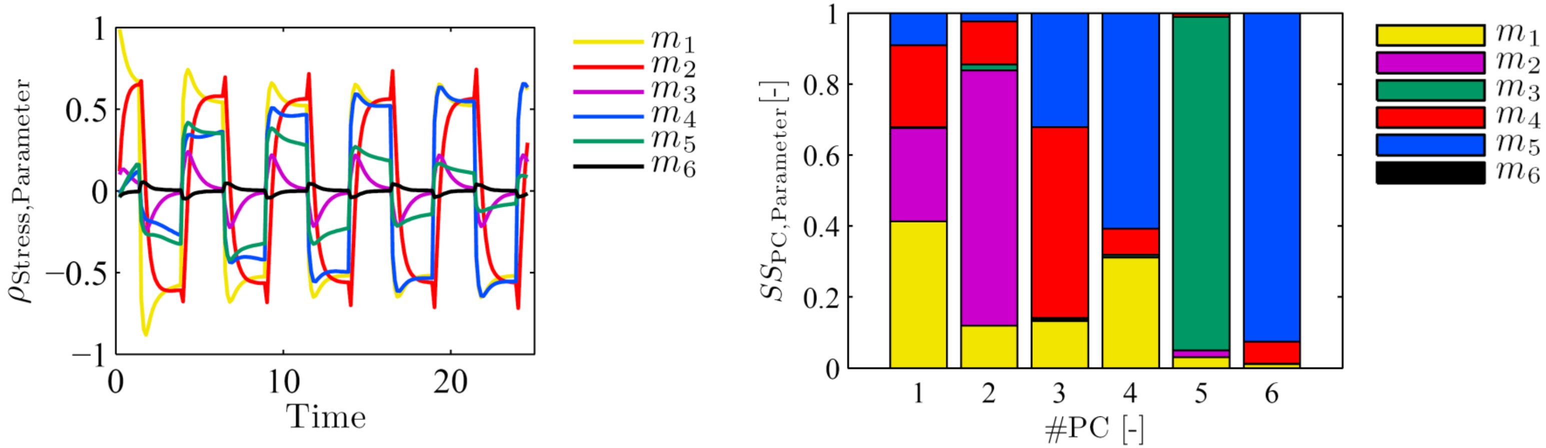


Results

Identified parameters' pdfs

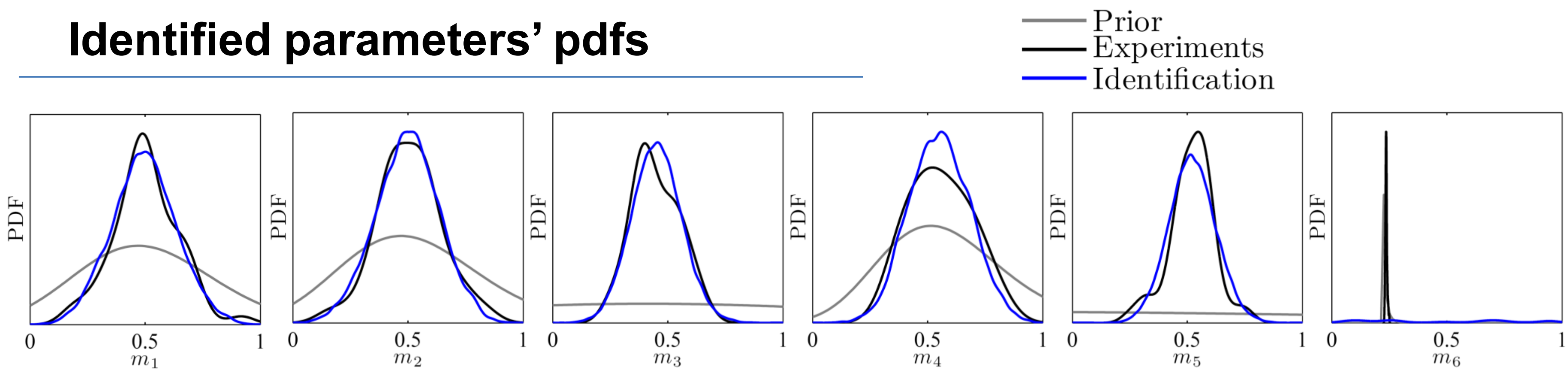


Sensitivity analysis

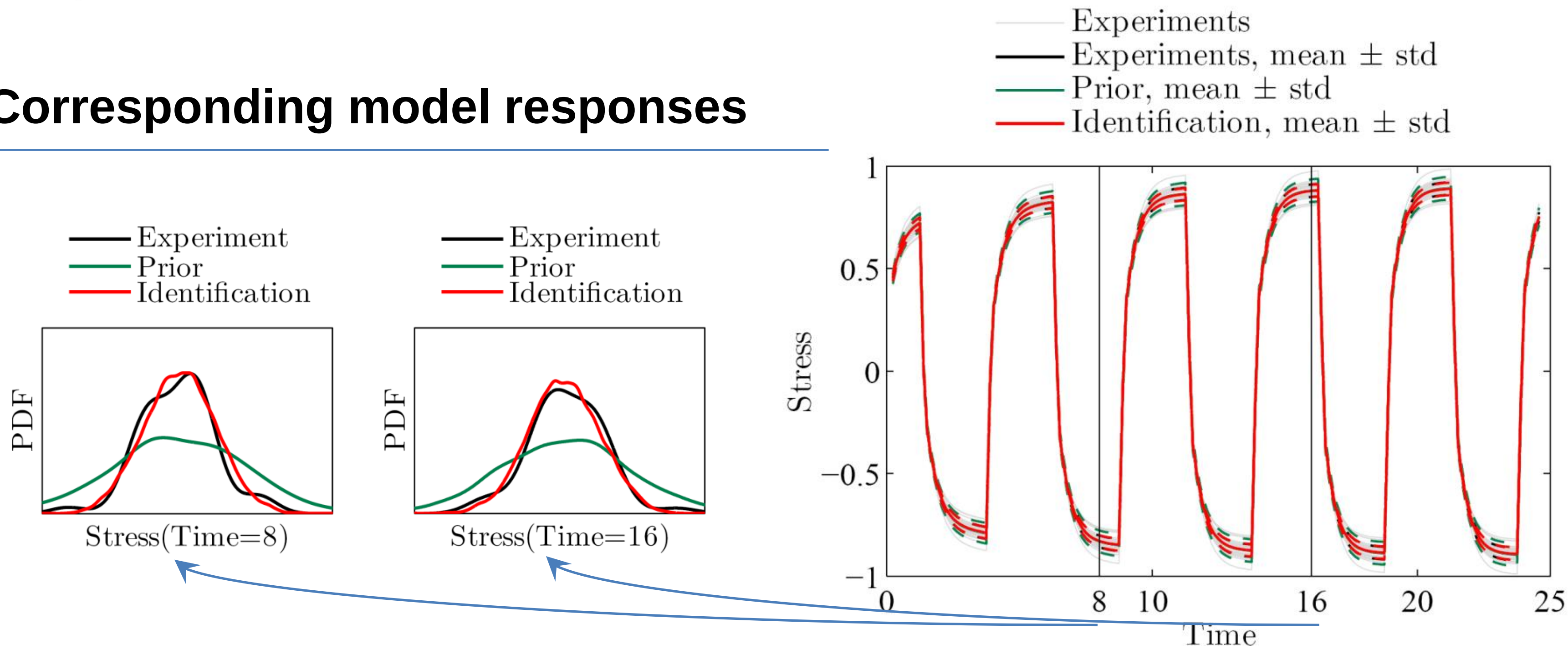


Results

Identified parameters' pdfs

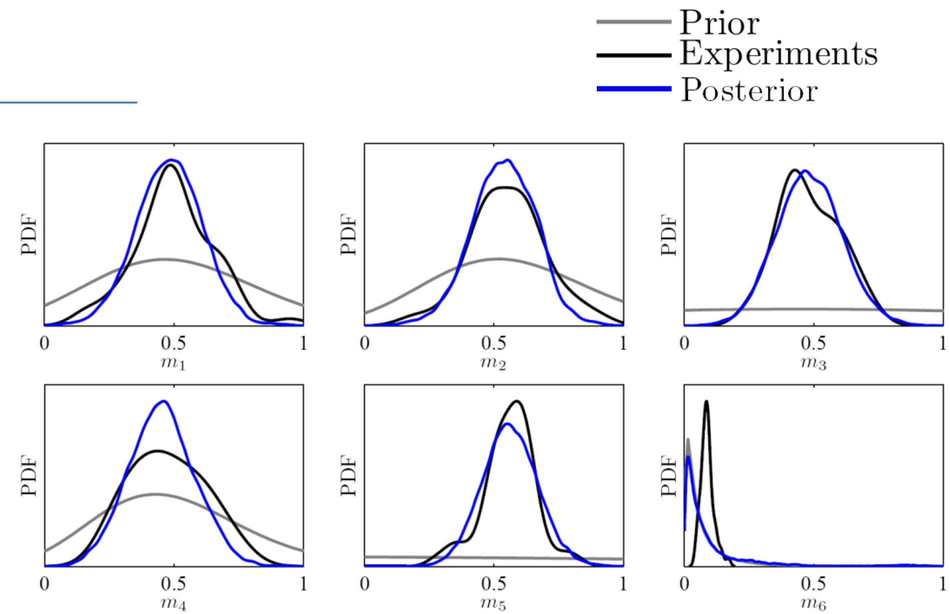


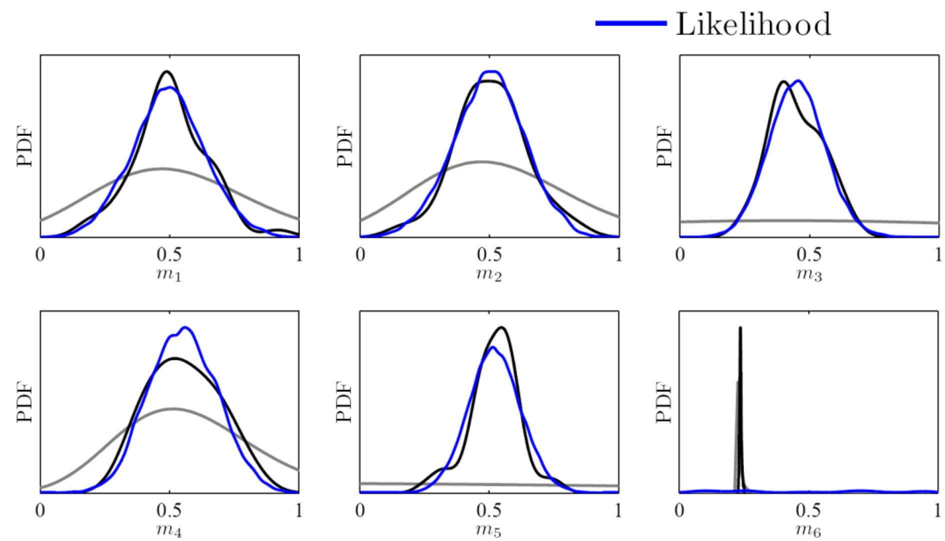
Corresponding model responses



Posterior vs. Likelihood

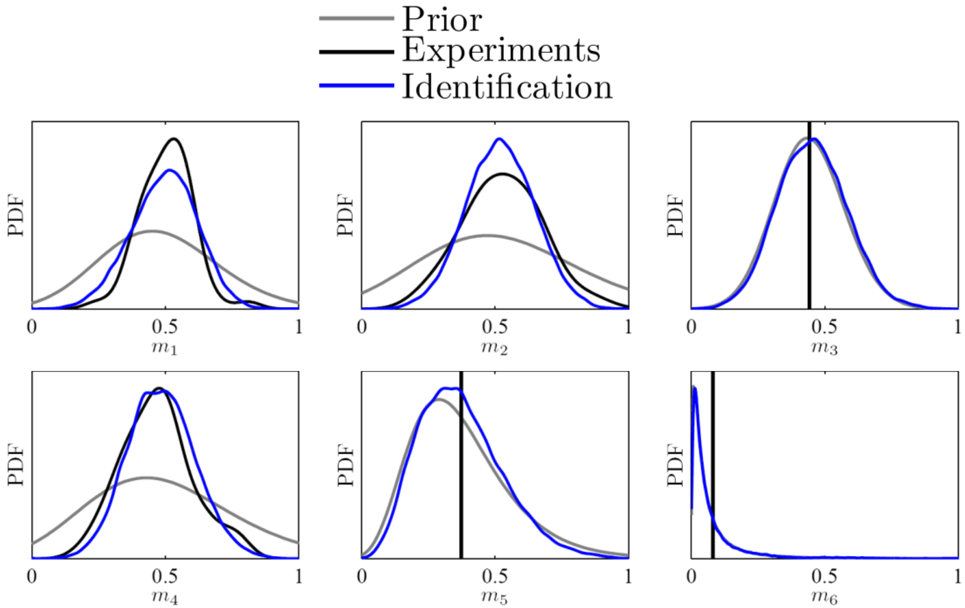
	m_1	m_2	m_3	m_4	m_5	m_6
			Prior			
MEAN	0.511	0.522	0.473	0.580	0.508	0.236
VAR	0.093	0.092	1.041	0.076	3.086	$5 \cdot 10^{-4}$
			Experiments			
MEAN	0.503	0.506	0.448	0.557	0.520	0.237
VAR	0.020	0.019	0.010	0.017	0.009	$1 \cdot 10^{-5}$
			Posterior			
MEAN	0.485	0.490	0.448	0.536	0.522	0.239
VAR	0.014	0.014	0.012	0.014	0.011	0.001
			Likelihood			
MEAN	0.496	0.498	0.452	0.552	0.522	0.475
VAR	0.020	0.019	0.012	0.018	0.011	0.095





Cyclic loading test – samples with fixed parameters

– Identification of the fixed samples from optimisation of particular curves



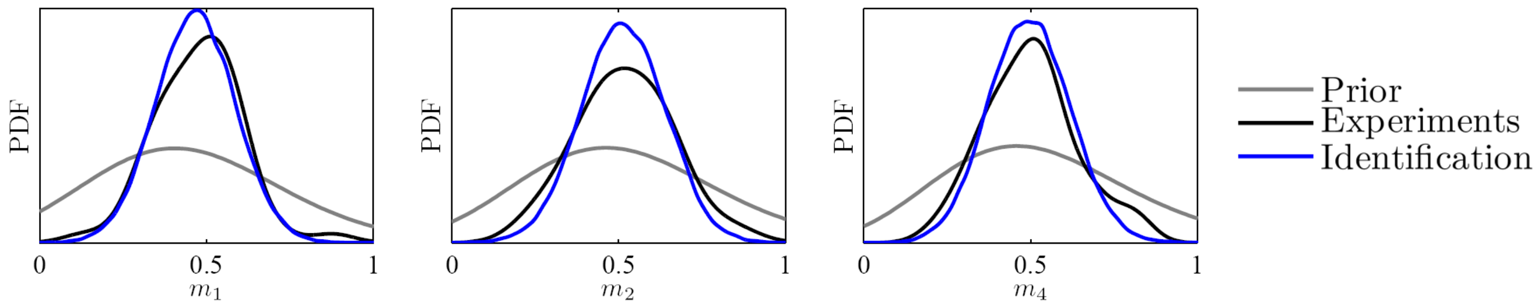
Experiment	m_1	m_2	m_3	m_4	m_5	m_6
1	0.469	0.226	0.451	0.365	0.373	0.100
2	0.529	0.529	0.481	0.345	0.375	−0.177
3	0.411	0.555	0.423	0.360	0.379	0.161
4	0.590	0.424	0.448	0.767	0.373	0.608
5	0.342	0.560	0.473	0.306	0.374	0.196
VAR	0.009	0.020	0.001	0.036	10^{-5}	0.079

	m_1	m_2	m_3	m_4	m_5	m_6
			Optimisation			
	0.505	0.530	0.441	0.469	0.373	0.269
	0.501	0.531	0.438	0.466	0.383	−0.059
	0.486	0.542	0.477	0.470	0.384	0.165
	0.499	0.528	0.421	0.480	0.377	0.440
	0.498	0.528	0.430	0.471	0.388	−0.145
	0.507	0.523	0.408	0.459	0.398	0.242
			Prior			
MEAN	0.482	0.523	0.443	0.498	0.373	0.080
VAR	0.050	0.092	0.017	0.080	0.036	0.018
			Posterior			
MEAN	0.496	0.520	0.453	0.486	0.370	0.079
VAR	0.015	0.017	0.017	0.016	0.024	0.013

Cyclic loading test – samples with fixed parameters

– 3 fixed parameters on optimised values, other are identified by MCMC

m_3	m_5	m_6
0.441	0.373	0.269



	$\varepsilon_{\mathrm{MEAN}}$	$\varepsilon_{\mathrm{STD}}$
Bayesian update of all parameters	0.626	1.336
Fixed γ, β and ν + Bayesian update of other parameters	0.227	0.390

Mean squared error (MSE) in mean value and standard deviation.

Example 03

Cyclic loading test – correlated samples

- Identification of the correlation from optimisation of particular curves

Experiment	m_1	m_2	m_3	m_4	m_5	m_6
1	0.812	0.812	0.458	0.495	0.356	0.235
2	0.549	0.525	0.431	0.455	0.360	0.155
3	0.666	0.672	0.444	0.691	0.374	0.074
4	0.480	0.487	0.480	0.451	0.375	0.144
5	0.484	0.496	0.452	0.479	0.382	−0.113
VAR	0.020	0.020	$3 \cdot 10^{-4}$	0.010	10^{-4}	0.017

$$\mathbf{R} = \begin{pmatrix} 1.000 & 0.995 & -0.153 & 0.396 & -0.661 & 0.586 \\ 0.995 & 1.000 & -0.092 & 0.426 & -0.590 & 0.538 \\ -0.153 & -0.092 & 1.000 & -0.294 & 0.306 & 0.116 \\ 0.396 & 0.426 & -0.294 & 1.000 & 0.199 & -0.104 \\ -0.661 & -0.590 & 0.306 & 0.199 & 1.000 & -0.841 \\ 0.586 & 0.538 & 0.116 & -0.104 & -0.841 & 1.000 \end{pmatrix}$$

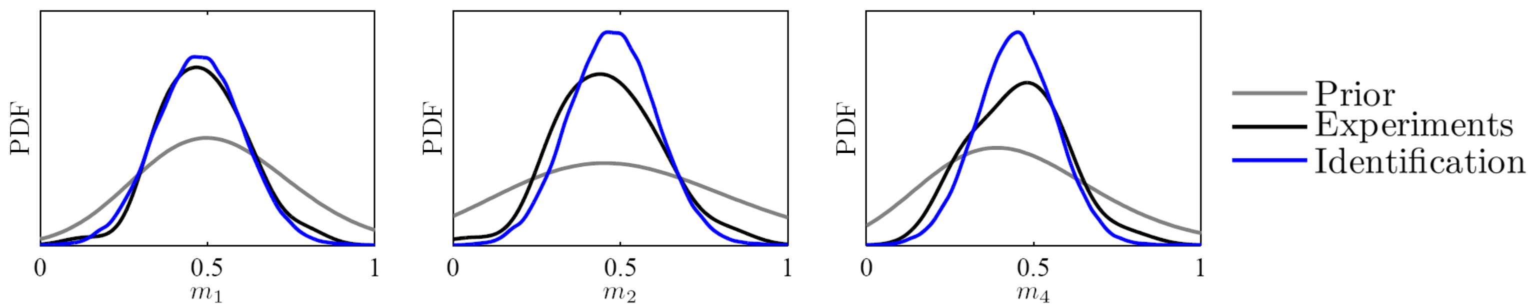
Example 03

Cyclic loading test – correlated samples

– 3 fixed parameters on optimised values, m_1 is calculated from m_2, other are identified by MCMC

m_3	m_5	m_6
0.441	0.373	0.106

$$m_1 = a m_2 + b$$

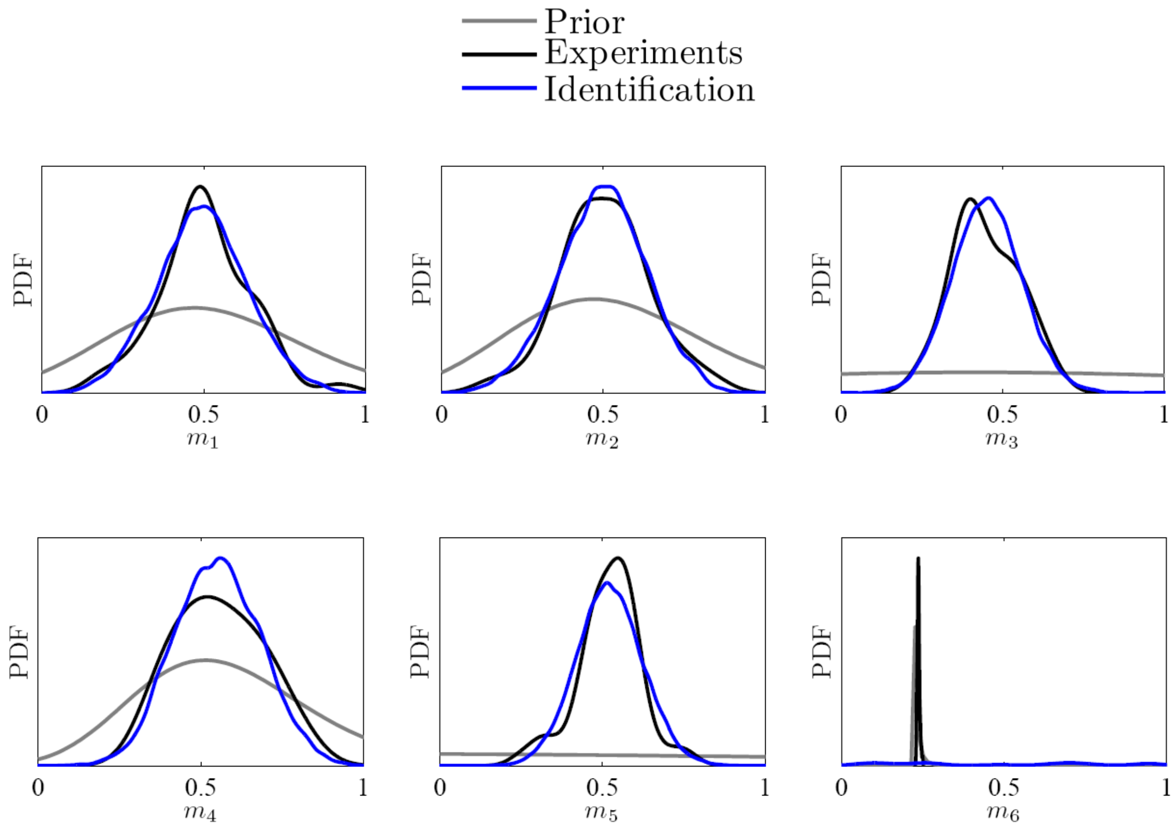


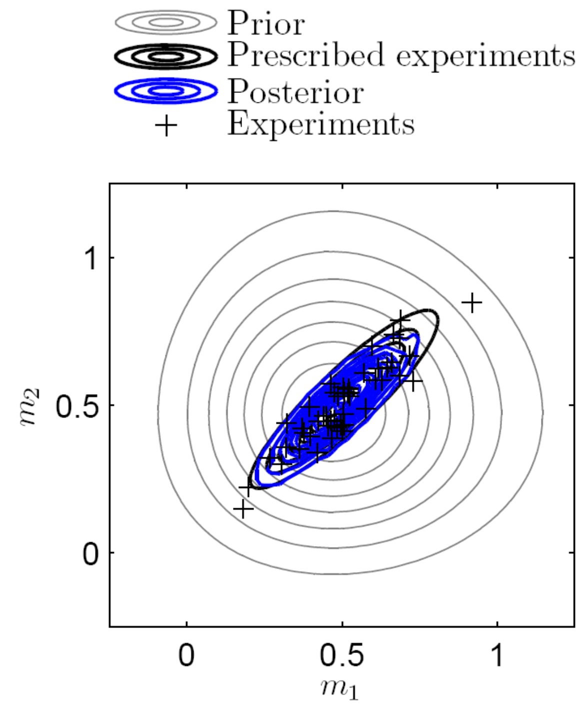
	$\varepsilon_{\mathrm{MEAN}}$	$\varepsilon_{\mathrm{STD}}$
Bayesian update of all parameters	0.203	2.504
Bayesian update of 5 parameters + calculation of σ_y	0.186	0.349
Fixed γ, β and ν + Bayesian update of X_∞ and R_∞ + calculation of σ_y	0.113	0.216

MSE in mean value and standard deviation.

Cyclic loading test – correlated samples

– Prescribed correlation 0.9 between m_1 and m_2, likelihood: 5 PC

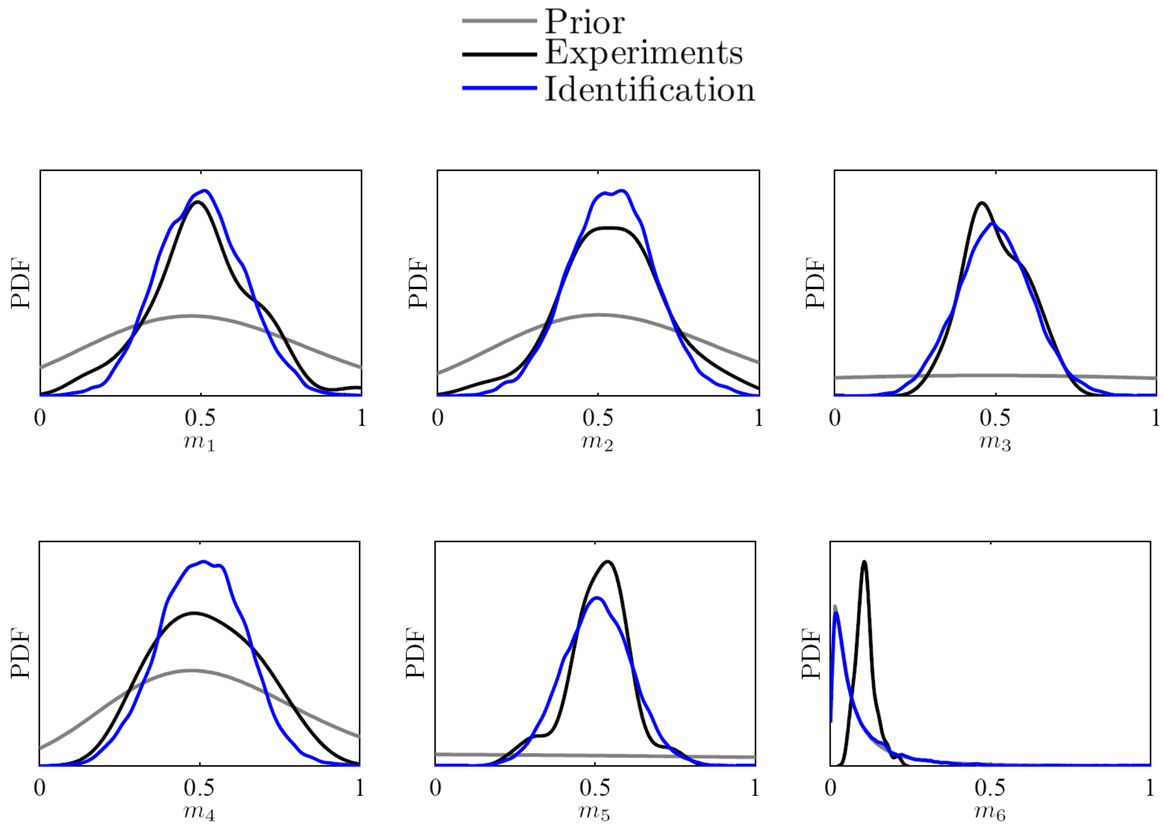


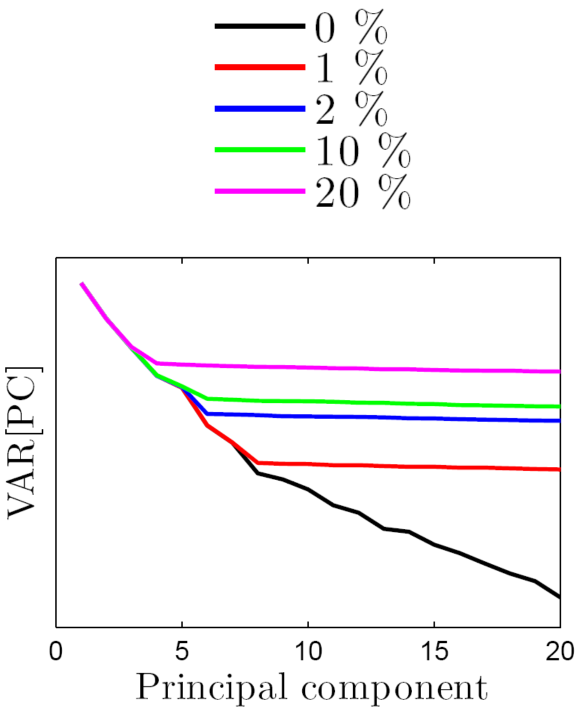


Example 04

Cyclic loading test – samples with experimental error

– Normal distributed error with STD equal to 2 % of corresponding output STD





Conclusion

Parameter identification based on a set of experimental curves

- Sensitivity analysis
  - Determination of relevant and irrelevant parameters
- Formulation of likelihood
  - Principal component analysis
  - Efficient number of principal components
- Optimisation
  - Fully correlated or fixed experimental samples
- MCMC sampling
  - Uncorrelated and not fully correlated experimental samples
  - Experimental errors up to critical size defined by PC variances

THANK YOU FOR YOUR ATTENTION.