Title (2): Overview of Micro-Elasticity theories with emphasis on strain gradient elasticity: Part II – Solution of some boundary value problems with technological importance

Abstract (2)

Some fundamental plane problems of Fracture Mechanics, propagation of surface waves, and axial splitting, are revisited in the light of a strain gradient elasticity theory. Our study here reveals the necessity of enriching elasticity of materials with length parameters to model surface energy of free surfaces and predict non-classical dispersion phenomena and size effects. We present at some extent previously obtained solutions for the three fundamental crack deformation modes. These solutions reveal that the crack shape is no longer elliptical as is predicted by the classical theory but the crack lips take a cusp shape. This evidence may lead to the formulation of new fracture criterion instead of the classical Griffith type criterion of fracture that employs in an ad-hoc manner the concept of surface energy. After the revisit of the surface instability of an elastic solid in a Biot-type analysis, we embark on the study of the axial splitting of geological layers that is of high importance in Structural Geology. Subsequently to the study of the dependence of the ratio of spacing of cracks to the thickness of the layer in a first place, we study the dependence of the buckling load on the bed thickness, for fixed crack spacing to bed thickness ratio and a specified degree of anisotropy of the macrostructure. As is already known, the strain gradient theories are used for the prediction of size effects of elasticity and strength of solids, for dispersion effects in wave propagation problems, and for regularization of ill-posed problems like fracture localization in the post-peak loading regime of a structure. Here it is further shown that gradient elasticity can be also used for the improvement of performance of a three-dimensional indirect boundary element method developed by Crouch and Starfield (1990) that is based on classical elasticity. This is first demonstrated from the inverse gradient elastic solution of stresses produced from a crack of prescribed shape. It is shown that the intrinsic length scale parameter is an imaginary quantity that could be calibrated in such manner to give a more representative value of the stresses at the centroid of the element compared to the classical elasticity solution. Based on this result a new triangular displacement discontinuity element is created. In a verification stage the accuracy of the computational algorithm for the mixed-mode elliptic crack problem and for the rectangular crack is demonstrated. It is shown that the new element results in significantly improved estimation of the stress intensity factor at crack tips and crack displacements. An intermediate result of this work is the solution of the three fundamental dislocation problems in the three-dimensional space. Another conclusión that may be drawn from this overview, is that instead of using many additional gradient parameters in a generalized elasticity theory which presents great problems for practical applications, one may employ for each particular problem only the absolutely necessary (i.e. one to two) gradient parameters in addition to the two isotropic elasticity parameters in order to capture the desired phenomena.

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