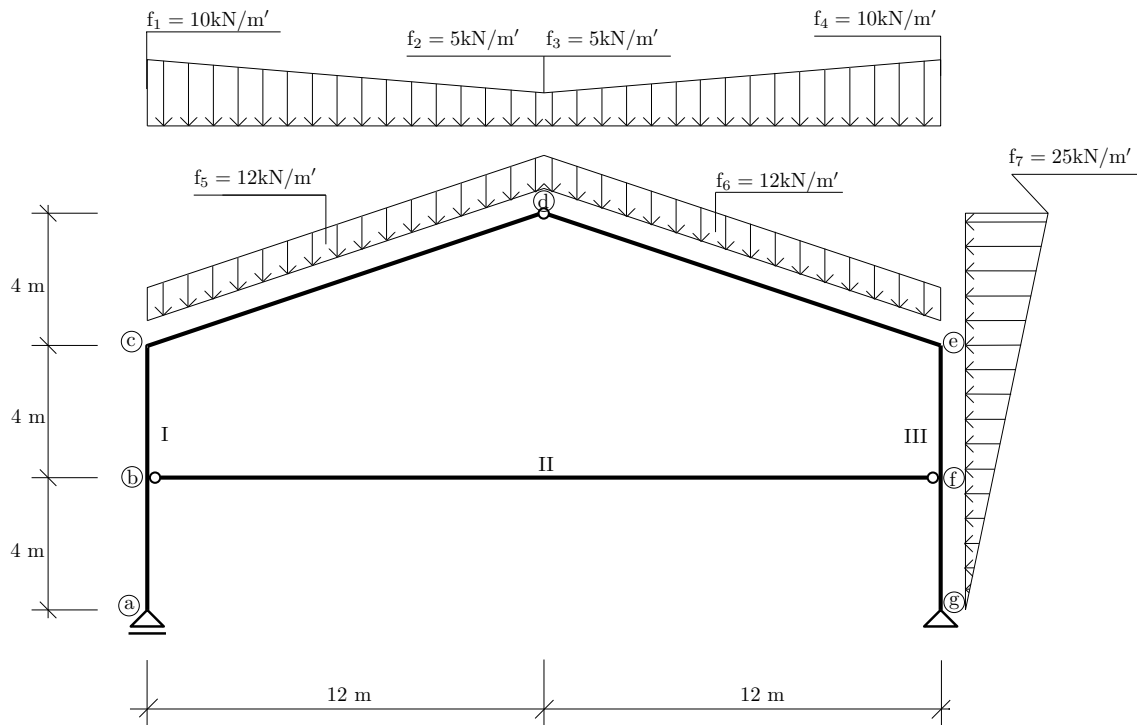


Výpočet vnitřních sil na složené soustavě

25. července 2010



Obrázek 1: Schéma průřezu.

Úkol: Vypočítejte reakce a nakreslete průběh normálové síly N , posouvající síly V a ohybového momentu M na celé konstrukci.

Řešení:

1) Výpočet reakcí:

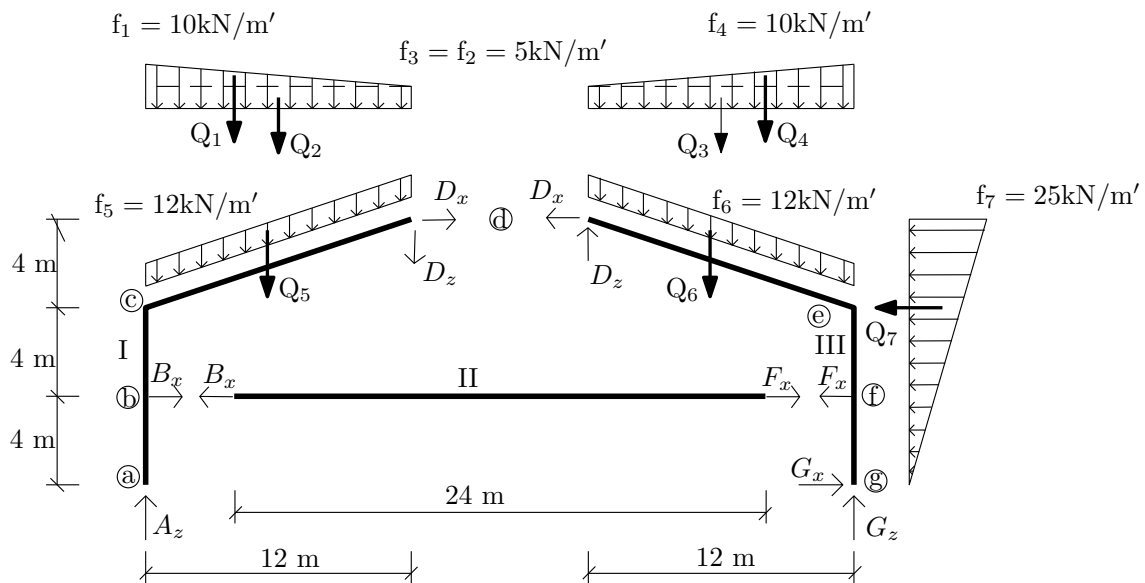
$$\rightarrow : G_x - Q_7 = 0 \Rightarrow \underline{G_x = 150 \text{ kN}} \quad (1)$$

$$\circlearrowleft g : Q_1 \cdot 20 + Q_2 \cdot 18 + Q_3 \cdot 6 + Q_4 \cdot 4 + Q_5 \cdot 18 + Q_6 \cdot 6 + Q_7 \cdot 8 - A_z \cdot 24 = 0 \Rightarrow \underline{A_z = 291,8 \text{ kN}} \quad (2)$$

$$\downarrow : Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 - A_z - G_z = 0 \Rightarrow \underline{G_z = 191,8 \text{ kN}} \quad (3)$$

Kontrola reakcí :

$$\circlearrowleft a : Q_1 \cdot 4 + Q_2 \cdot 6 + Q_3 \cdot 20 + Q_4 \cdot 18 + Q_5 \cdot 6 + Q_6 \cdot 18 + Q_7 \cdot 8 - G_z \cdot 24 = 0 \Rightarrow \underline{0 = 0} \quad (4)$$



Obrázek 2: Rozdělení na jednotlivé desky.

2) Výpočet vnitřních sil:

deska č.I

$$\circlearrowleft d : Q_1 \cdot 8 + Q_2 \cdot 6 + Q_5 \cdot 6 - A_z \cdot 12 + B_x \cdot 8 = 0 \Rightarrow \underline{B_x = 248,85 \text{ kN}} \quad (5)$$

deska č.II

$$\rightarrow : -B_x + F_x = 0 \Rightarrow \underline{F_x = 248,85 \text{ kN}} \quad (6)$$

deska č.III kontrola:

$$\circlearrowleft d : Q_3 \cdot 8 + Q_4 \cdot 6 + Q_6 \cdot 6 - B_x \cdot 12 - B_z \cdot 12 + F_x \cdot 8 = 0 \Rightarrow \underline{F_x = 248,85 \text{ kN}} \quad (7)$$

3) Analytické vyjádření průběhu vnitřních sil:

interval (a, b):

$$N^{ab} = -A_z \Rightarrow \underline{N^{ab} = -291,8 \text{ kN}} \quad (8)$$

$$N^{ba} = N^{ab} \Rightarrow \underline{N^{ba} = -291,8 \text{ kN}} \quad (9)$$

$$V^{ab} = \underline{0 \text{ kN}} \quad (10)$$

$$V^{ba} = \underline{0 \text{ kN}} \quad (11)$$

$$M^a = \underline{0 \text{ kNm}} \quad (12)$$

interval (b, c):

$$N^{bc} = \underline{-291,8 \text{ kN}} \quad (13)$$

$$N^{cb} = \underline{-291,8 \text{ kN}} \quad (14)$$

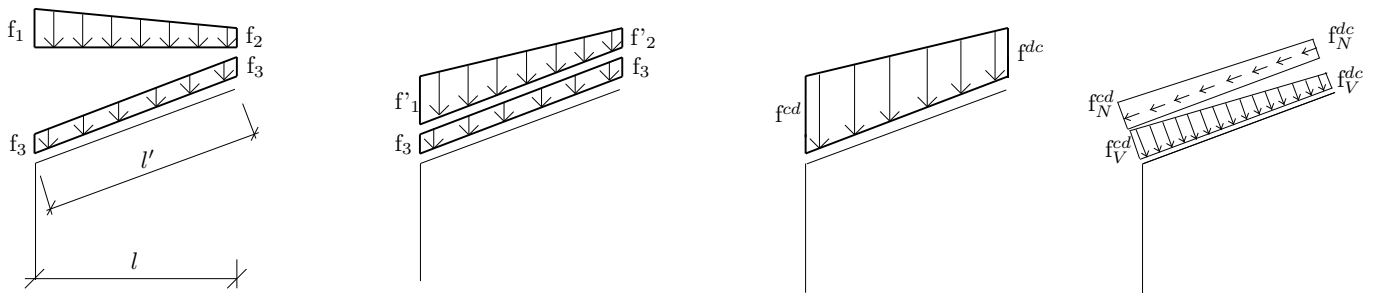
$$V^{bc} = -B_x \Rightarrow \underline{-248,85 \text{ kN}} \quad (15)$$

$$V^{cb} = V^{bc} \Rightarrow \underline{-248,85 \text{ kN}} \quad (16)$$

$$M^b = -B_x \cdot 4 \Rightarrow \underline{-995 \text{ kNm}} \quad (17)$$

interval (c, d):

Rozklad zatížení :



Obrázek 3: Rozklad zatížení.

$$f'_1 = \frac{f_1 \cdot l}{l'} = \frac{10 \cdot 12}{12,65} = 9,49 \Rightarrow f^{cd} = f'_1 + f_3 = 9,48 + 12 = \underline{21,49}$$

$$f'_2 = \frac{f_2 \cdot l}{l'} = \frac{5 \cdot 12}{12,65} = 4,74 \Rightarrow f^{dc} = f'_2 + f_3 = 4,74 + 12 = \underline{16,74}$$

$$\Rightarrow f_V^{cd} = f^{cd} \cdot \cos \alpha = 21,49 \cdot \cos 18,4336^\circ = \underline{20,3874}$$

$$\Rightarrow f_V^{dc} = f^{dc} \cdot \cos \alpha = 16,74 \cdot \cos 18,4336^\circ = \underline{15,881}$$

$$\Rightarrow f_N^{cd} = f^{cd} \cdot \sin \alpha = 21,49 \cdot \sin 18,4336^\circ = \underline{6,795}$$

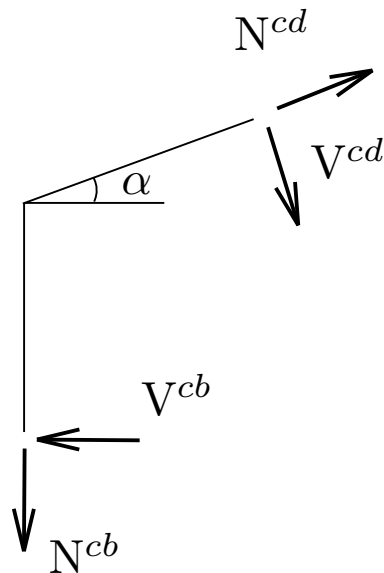
$$\Rightarrow f_N^{dc} = f^{dc} \cdot \sin \alpha = 16,74 \cdot \sin 18,4336^\circ = \underline{5,29}$$

Přepočet sil v bodě c :

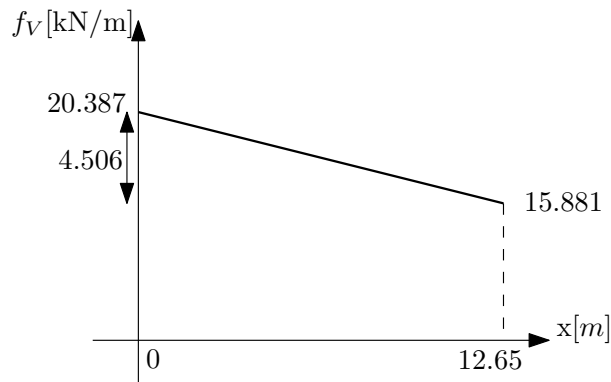
$$V^{cd} = N^{cb} \cdot \cos(\alpha) - V^{cb} \cdot \sin(\alpha) = \underline{198,14023 \text{ kN}}$$

$$N^{cd} = -N^{cb} \cdot \sin(\alpha) - V^{cb} \cdot \cos(\alpha) = \underline{-328,349 \text{ kN}}$$

$$N^{dc} = N^{cd} + 5,29 \cdot 12,65 + (6,795 - 5,29) \cdot \frac{12,65}{2} \Rightarrow \underline{-251,919 \text{ kN}}$$



Obrázek 4: Přepočítání sil.



Obrázek 5: Průběh zatížení.

$$f_V(x) = -\frac{4,506}{12,65} \cdot x + 20,387$$

$$V(x) = -\int f_V(x)dx = \frac{0,3562}{2} \cdot x^2 - 20,387 \cdot x + 198,14023$$

$$V(12,65) = \frac{0,3562}{2} \cdot 12,65^2 - 20,387 \cdot 12,65 + 198,14023 = \underline{-31,2553 \text{ kN}}$$

$$V(x) = 0 \Rightarrow \frac{0,3562}{2} \cdot x^2 - 20,387 \cdot x + 198,14023 = 0 \Rightarrow x_{\max} = \underline{10,724}$$

$$M(x) = \int V(x)dx = \frac{0,1781}{3} \cdot x^3 - \frac{20,387}{2} \cdot x^2 + 198,14023 \cdot x - 995,4$$

$$M(12,65) = \frac{0,1781}{3} \cdot 12,65^3 - \frac{20,387}{2} \cdot 12,65^2 + 198,14023 \cdot 12,65 - 995,4 = \underline{0 \text{ kNm}}$$

$$M_{\max} \Rightarrow M(10,724) = \frac{0,1781}{3} \cdot 10,724^3 - \frac{20,387}{2} \cdot 10,724^2 + 198,14023 \cdot 10,724 - 995,4 = \underline{30,3779 \text{ kNm}}$$

interval (b, f):

$$N^{bf} = Bx \Rightarrow \underline{248,85 \text{ kN}} \quad (18)$$

$$N^{fb} = N^{bf} \Rightarrow \underline{248,85 \text{ kN}} \quad (19)$$

$$V^{bf} = 0 \text{ kN} \quad (20)$$

$$V^{fb} = 0 \text{ kN} \quad (21)$$

$$M^{bf} = 0 \text{ kNm} \quad (22)$$

$$M^{fb} = 0 \text{ kNm} \quad (23)$$

interval (f, g):

$$N^{fg} = -G_z \Rightarrow \underline{-191,8 \text{ kN}} \quad (24)$$

$$N^{gf} = N^{fg} \Rightarrow \underline{-191,8 \text{ kN}} \quad (25)$$

$$V^{gf} = -G_x \Rightarrow \underline{-150 \text{ kN}} \quad (26)$$

$$V^{fg} = -G_x + \frac{8,333 \cdot 4}{2} \Rightarrow \underline{-133,334 \text{ kN}} \quad (27)$$

$$M^g = 0 \text{ kNm} \quad (28)$$

$$M^f = G_x \cdot 4 - \frac{8,333 \cdot 4}{2} \cdot \frac{4}{3} \Rightarrow \underline{577,778 \text{ kNm}} \quad (29)$$

interval (e, f):

$$N^{fe} = N^{gf} \Rightarrow \underline{-191,8 \text{ kN}} \quad (30)$$

$$N^{ef} = N^{fe} \Rightarrow \underline{-191,8 \text{ kN}} \quad (31)$$

$$V^{fe} = V^{fg} + F_x \Rightarrow \underline{115,516 \text{ kN}} \quad (32)$$

$$V^{ef} = V^{fe} + 8,333 \cdot 4 + \frac{(16,666 - 8,333) \cdot 4}{2} \Rightarrow \underline{165,514 \text{ kN}} \quad (33)$$

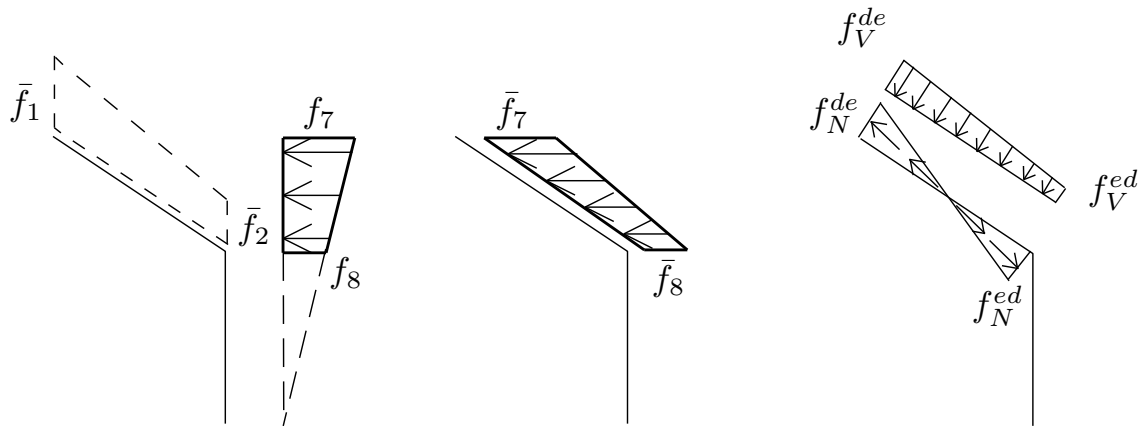
$$M^e = 150 \cdot 8 - 248,85 \cdot 4 - \frac{16,666 \cdot 8}{2} \cdot \frac{8}{3} \Rightarrow \underline{26,83 \text{ kNm}} \quad (34)$$

interval (d, e):

Rozklad zatížení :

$$\bar{f}_7 = \frac{f_7 \cdot l}{l'} = \frac{25 \cdot 4}{12,65} = \underline{7,91}$$

$$\bar{f}_8 = \frac{f_8 \cdot l}{l'} = \frac{16,666 \cdot 4}{12,65} = \underline{5,27}$$



Obrázek 6: Rozklad zatížení.

- Celkové zatížení:

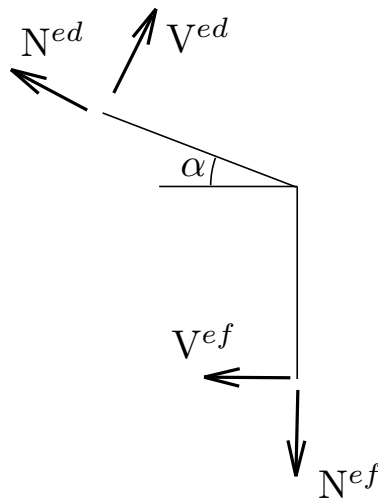
$$\Rightarrow f_V^{de} = f_V^{dc} + \bar{f}_7 \cdot \sin(\alpha) = \underline{18,381 \text{ kN}}$$

$$\Rightarrow f_N^{de} = f_N^{dc} - \bar{f}_7 \cdot \cos(\alpha) = \underline{-2,21 \text{ kN}}$$

$$\Rightarrow f_V^{ed} = f_V^{cd} + \bar{f}_8 \cdot \sin(\alpha) = \underline{22,0534 \text{ kN}}$$

$$\Rightarrow f_N^{ed} = f_N^{cd} - \bar{f}_8 \cdot \cos(\alpha) = \underline{1,795 \text{ kN}}$$

Přepočítání sil v bodě e :



Obrázek 7: Přepočítání sil.

$$V^{ed} = -N^{ef} \cdot \cos(\alpha) + V^{ef} \cdot \sin(\alpha) = \underline{-129,6236 \text{ kN}}$$

$$N^{ed} = -N^{ef} \cdot \sin(\alpha) - V^{ef} \cdot \cos(\alpha) = \underline{-217,67 \text{ kN}}$$

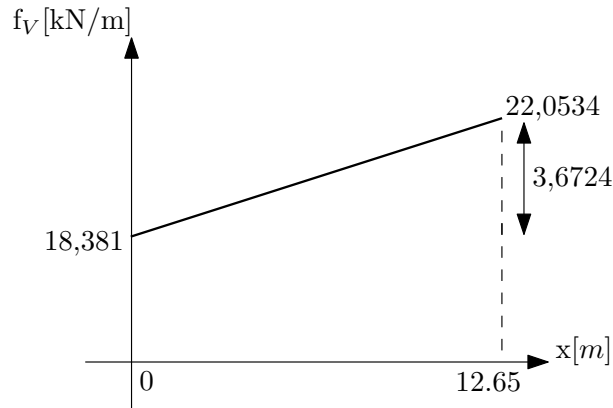
$$f_N(x) = -\frac{1,795 + 2,21}{12,65} \cdot x + 1,795$$

$$N(x) = \int f(x)dx = -\frac{0,317}{2} \cdot x^2 + 1,795 \cdot x - 217,67$$

$$N(12,65) = -\frac{0,317}{2} \cdot 12,65^2 + 1,795 \cdot 12,65 - 217,67 = \underline{-220,327 \text{ kN}}$$

$$f_N(x) = 0 \Rightarrow -\frac{1,795+2,21}{12,65} \cdot x + 1,795 = 0 \Rightarrow x_{\max} = 5,669$$

$$N(5,669) = -\frac{0,317}{2} \cdot 5,669^2 + 1,795 \cdot 5,669 - 217,67 = \underline{\underline{-212,588 \text{ kN}}}$$



Obrázek 8: Průběh zatížení.

$$f_V(x) = -\frac{3,6724}{12,65}x + 22,0534$$

$$V(x) = \int f(x)dx = \frac{0,29}{2} \cdot x^2 + 22,0534 \cdot x - 129,6236$$

$$V(12,65) = \frac{0,29}{2} \cdot 12,65^2 + 22,0534 \cdot 12,65 - 129,6236 = \underline{\underline{126,1486 \text{ kN}}}$$

$$V(x) = 0 \Rightarrow \frac{0,29}{2}x^2 + 22,0534 \cdot x - 129,6236 = 0 \Rightarrow x_{\max} = \underline{\underline{6,124}}$$

$$M(x) = -\int V(x)dx = \frac{0,145}{3} \cdot x^3 - \frac{22,0534}{2} \cdot x^2 + 129,6236 \cdot x + 26,83$$

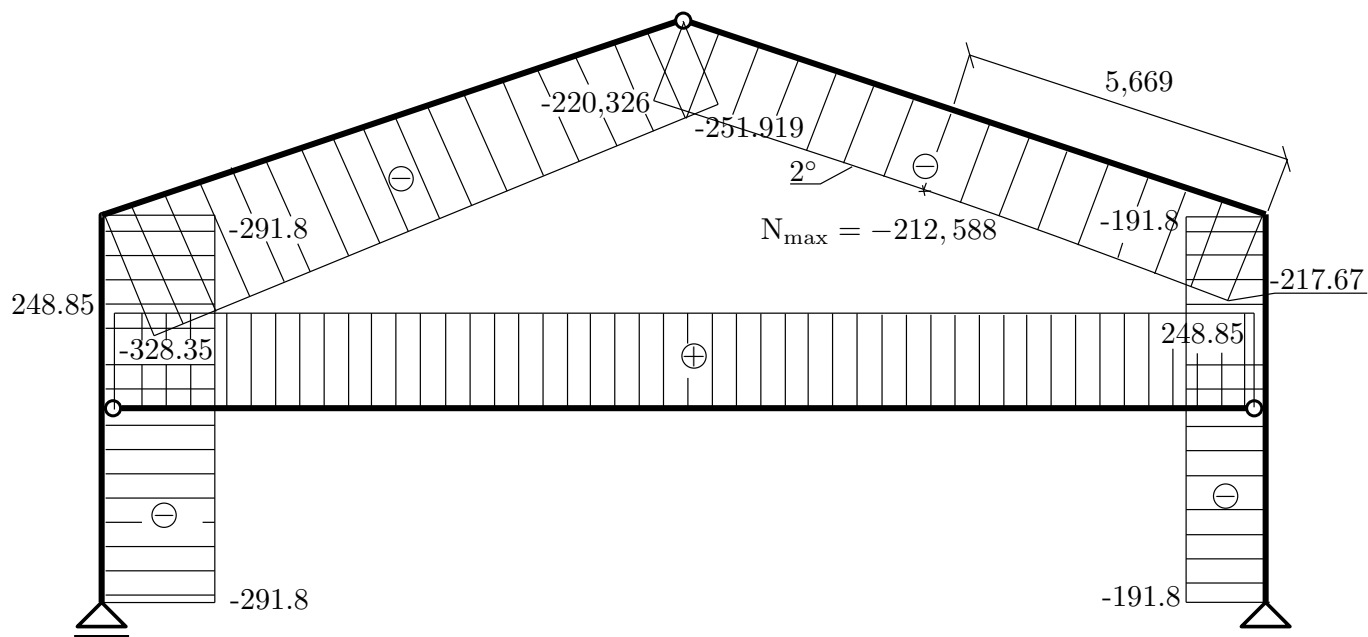
$$M_{(12,65)} = \frac{0,145}{3} \cdot 12,65^3 - \frac{22,0534}{2} \cdot 12,65^2 + 129,6236 \cdot 12,65 + 26,83 = \underline{\underline{0 \text{ kNm}}}$$

$$M_{\max} \Rightarrow M(6,124) = \frac{0,145}{3} \cdot 6,124^3 - \frac{22,0534}{2} \cdot 6,124^2 + 129,6236 \cdot 6,124 + 26,83 = \underline{\underline{418,207 \text{ kNm}}}$$

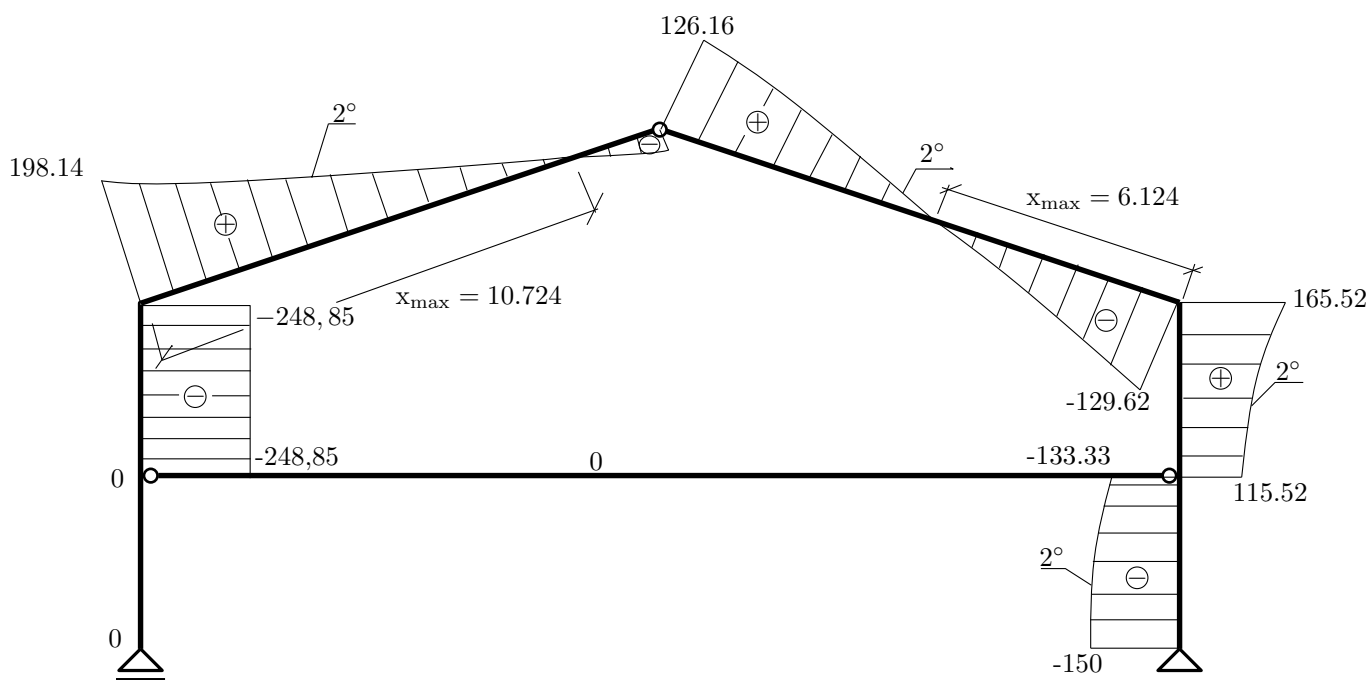
Styčnicková kontrola v bodě d:

$$\downarrow : N^{dc} \cdot \sin \alpha - V^{dc} \cdot \cos \alpha = N^{de} \cdot \sin \alpha - V^{de} \cdot \cos \alpha \Rightarrow \underline{\underline{0 = 0}} \quad (35)$$

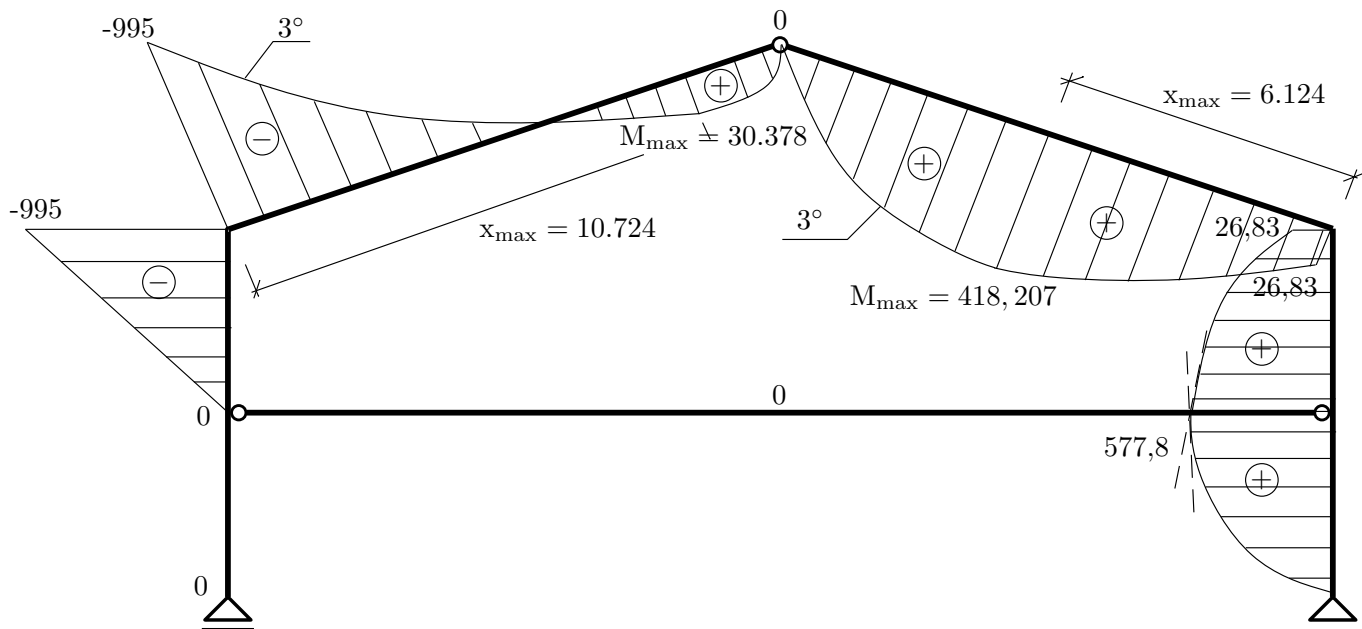
$$\rightarrow : N^{dc} \cdot \cos \alpha + V^{dc} \cdot \sin \alpha = N^{de} \cdot \cos \alpha + V^{de} \cdot \sin \alpha \Rightarrow \underline{\underline{0 = 0}} \quad (36)$$



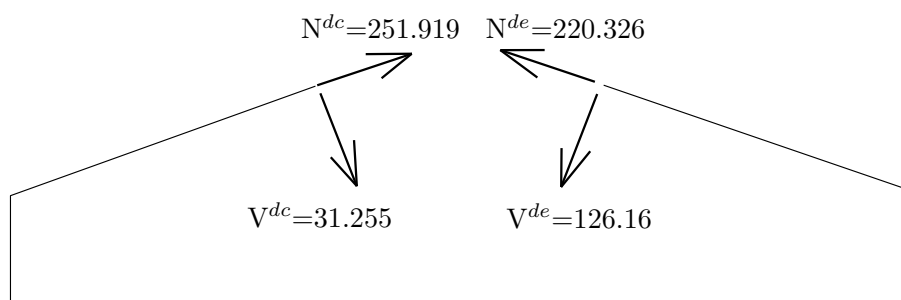
Obrázek 9: Normálové síly.



Obrázek 10: Posouvající síly.



Obrázek 11: Ohybové momenty.



Obrázek 12: Styčnicková kontrola