Czech Technical University in Prague
Faculty of Civil Engineering
Department of mechanics

INVERSE RELIABILITY OPTIMIZATION

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Reliability-based Design Optimization

$$\min_{\mathbf{d} \in \mathrm{D}} C(\mathbf{x}, \mathbf{d})$$

Minimize costs (e.g. weight of the structure)

$$\max_{\mathbf{d} \in \mathrm{D}} \beta_j(\mathbf{x}, \mathbf{d}), j = 1, \dots, n_J$$

Maximize safety (Reliability)

$$\text{s.t. } h_i(\mathbf{d}) \leq 0, i = 1, \dots, n_I$$

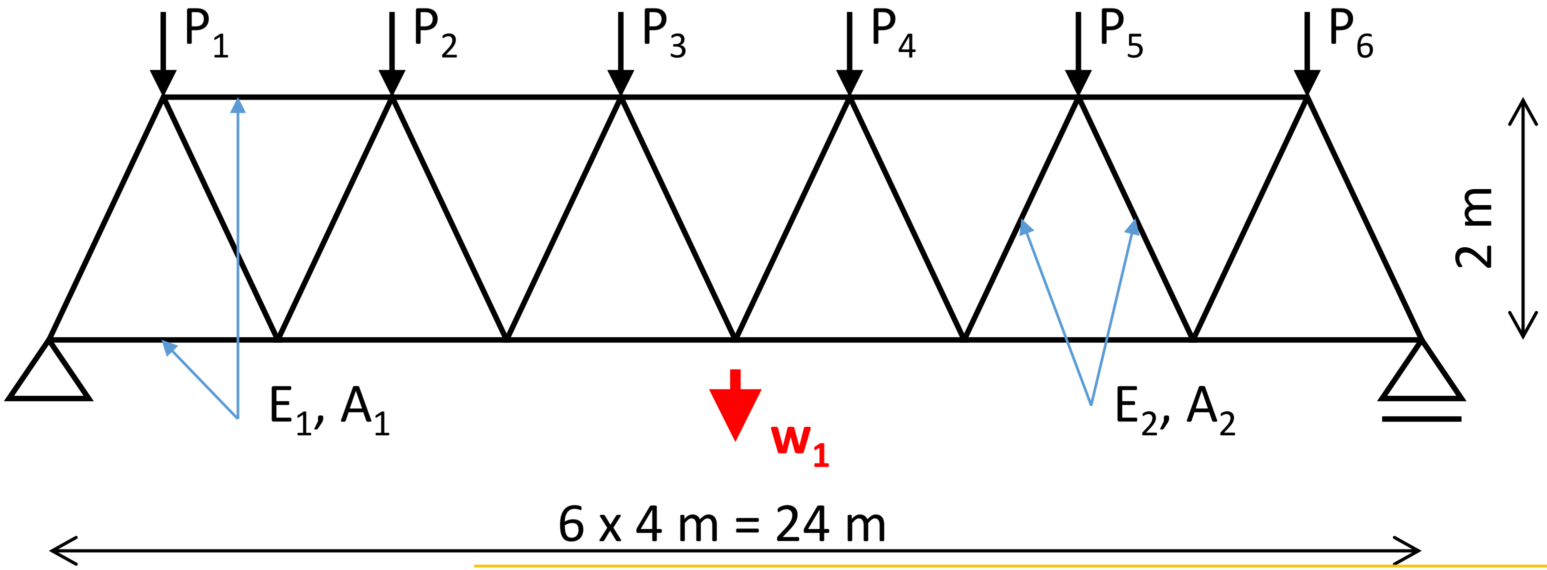
$$\beta_j(\mathbf{x}, \mathbf{d}) \geq \beta_j^{tol}, j = 1, \dots, n_J$$

Subject to constraints

Constraining Pareto-front from below

Multi-Objective Problem

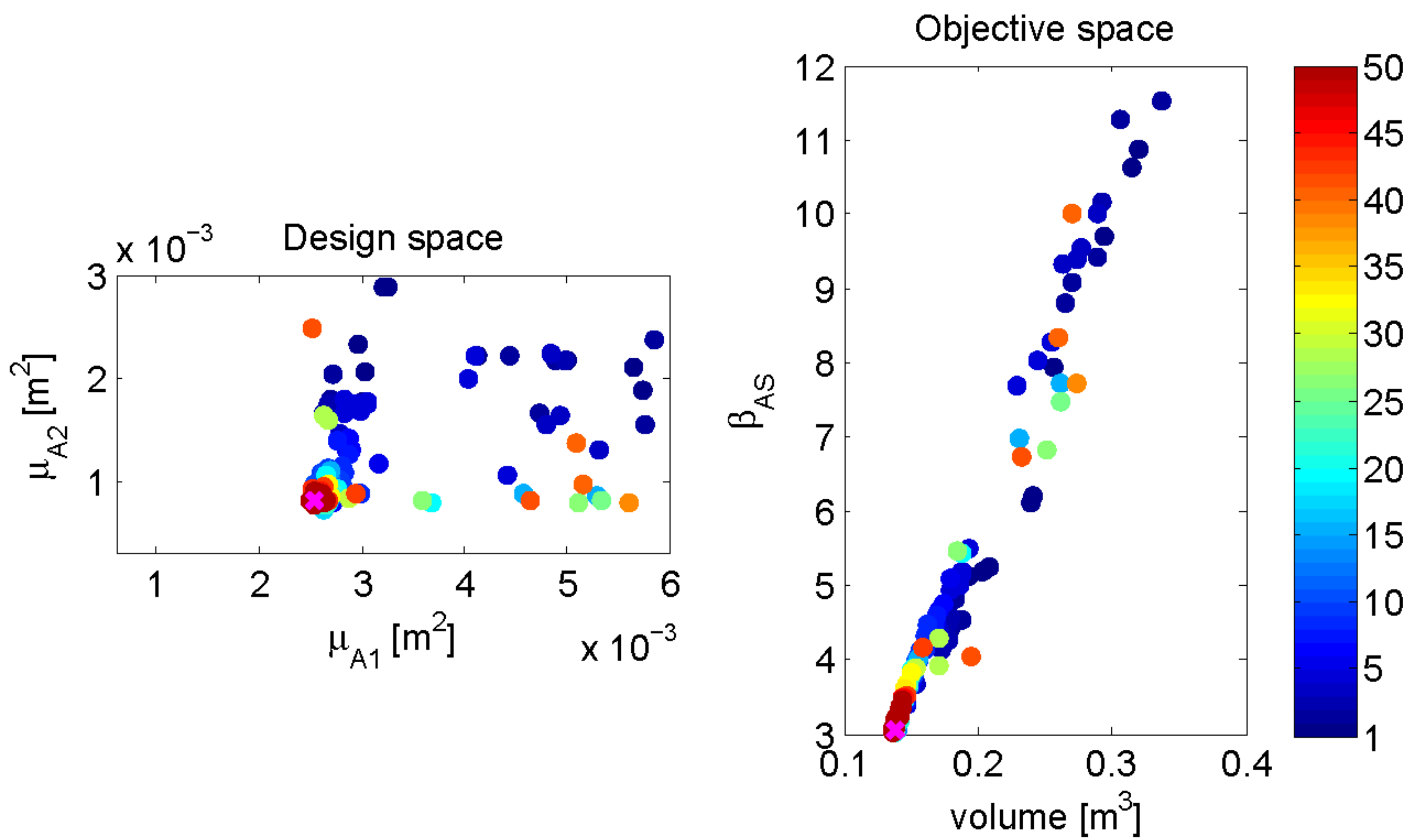
RELIABILITY-BASED DESIGN OPTIMIZATION



$g(x) = w_{max} - |w_1(x)|$
$w_{max} = 10$ cm

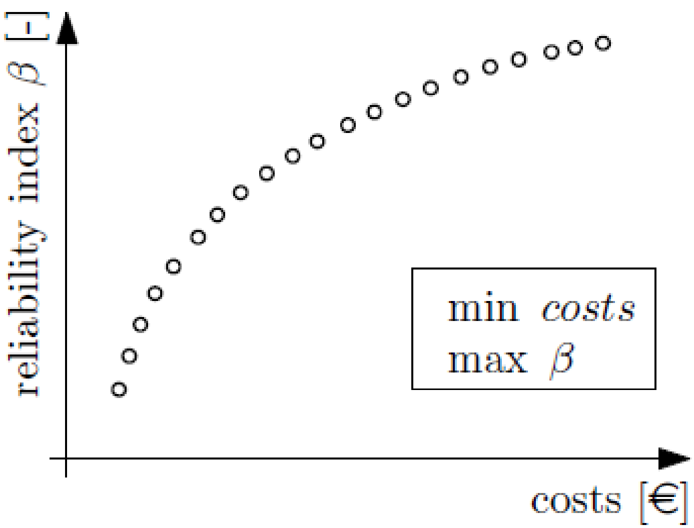
Variable		Distribution	Mean	Standard deviation
E_1, E_2	Pa	Lognormal	$2.1 \cdot 10^{11}$	$2.1 \cdot 10^{10}$
A_1	m^2	Lognormal	μ_{A1}	$2 \cdot 10^{-4}$
A_2	m^2	Lognormal	μ_{A2}	$1 \cdot 10^{-4}$
$P_1, \ldots, P_6$	N	Gumbel	$5 \cdot 10^4$	$7.5 \cdot 10^{-3}$

RELIABILITY-BASED DESIGN OPTIMIZATION

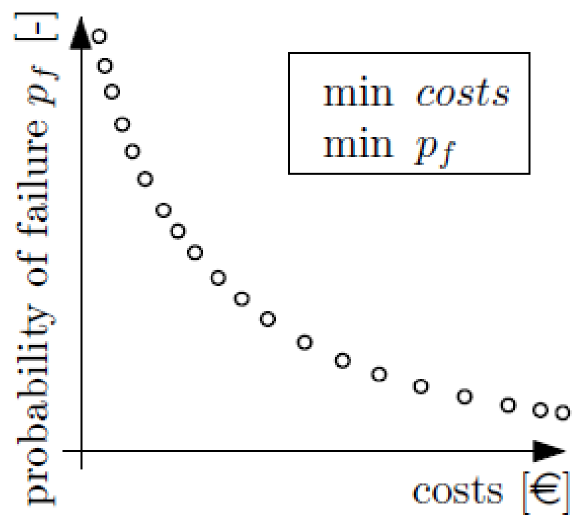


Reliability-based Design Optimization

Pareto-front with β-index



Pareto-front with p_f



Reliability-based Design Optimization

$$\min_{\mathbf{d}\in D} C(\mathbf{x}, \mathbf{d})$$

Minimize costs (e.g. weight of the structure)

$$\min_{\mathbf{d}\in D} p_f(\mathbf{x}, \mathbf{d}), j = 1, \ldots, n_J$$

Minimize probability of failure

$$\text{s.t.} \quad h_i(\mathbf{d}) \leq 0, i = 1, \ldots, n_I$$

$$p_f(\mathbf{x}, \mathbf{d}) \leq p_f^{tol}, j = 1, \ldots, n_J$$

Subject to constraints

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Multi-Objective Problem

Reliability-based Design Optimization

$$\min_{\mathbf{d} \in D} C(\mathbf{x}, \mathbf{d})$$

Minimize costs (e.g. weight of the structure)

$$\min_{\mathbf{d} \in D} p_f(\mathbf{x}, \mathbf{d}), j = 1, \dots, n_J$$

Minimize probability of failure

$$\text{s.t.} \;\; h_i(\mathbf{d}) \leq 0, i = 1, \dots, n_I$$

$$p_f(\mathbf{x}, \mathbf{d}) \leq p_f^{tol}, j = 1, \dots, n_J$$

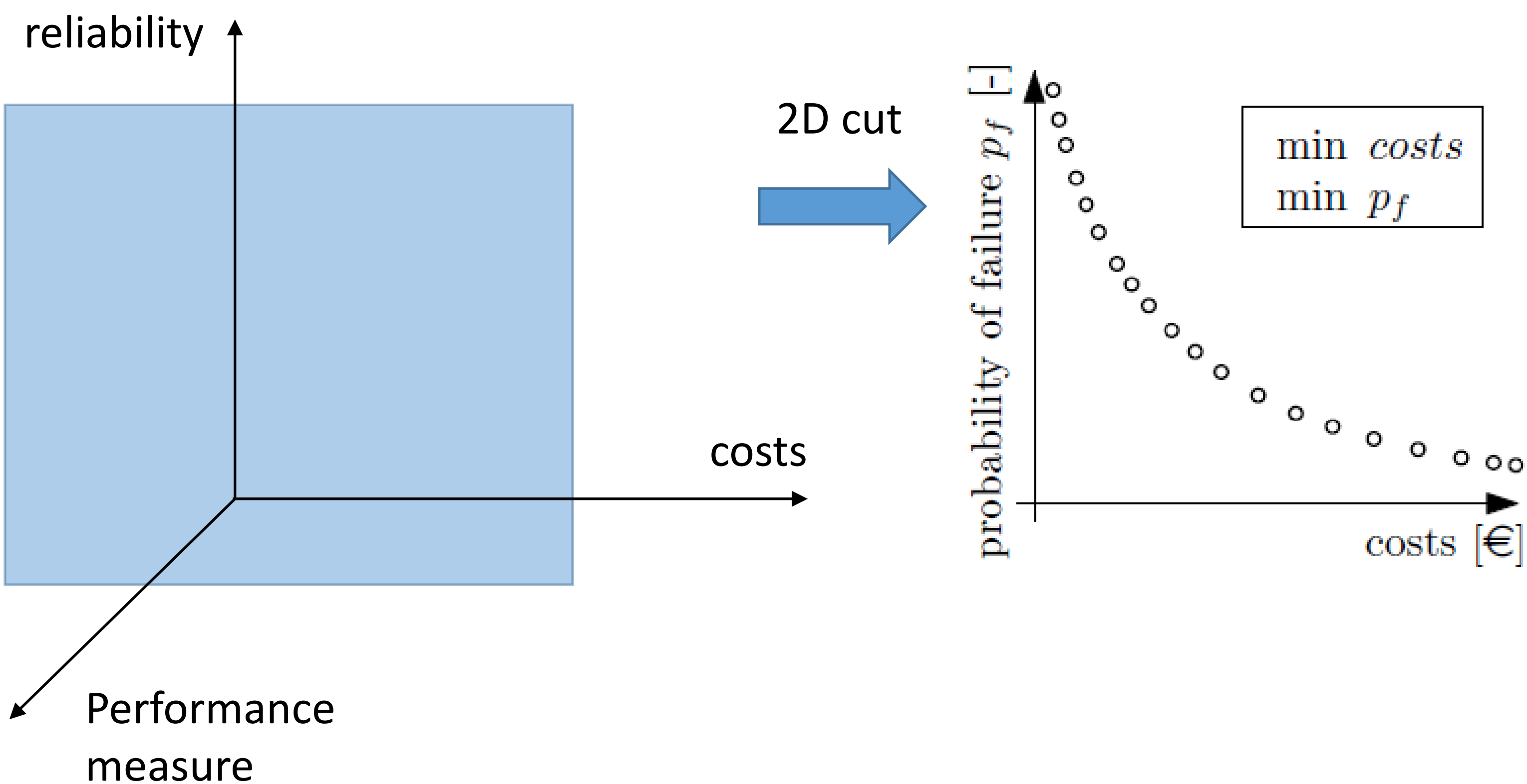
Subject to constraints

Performance measure

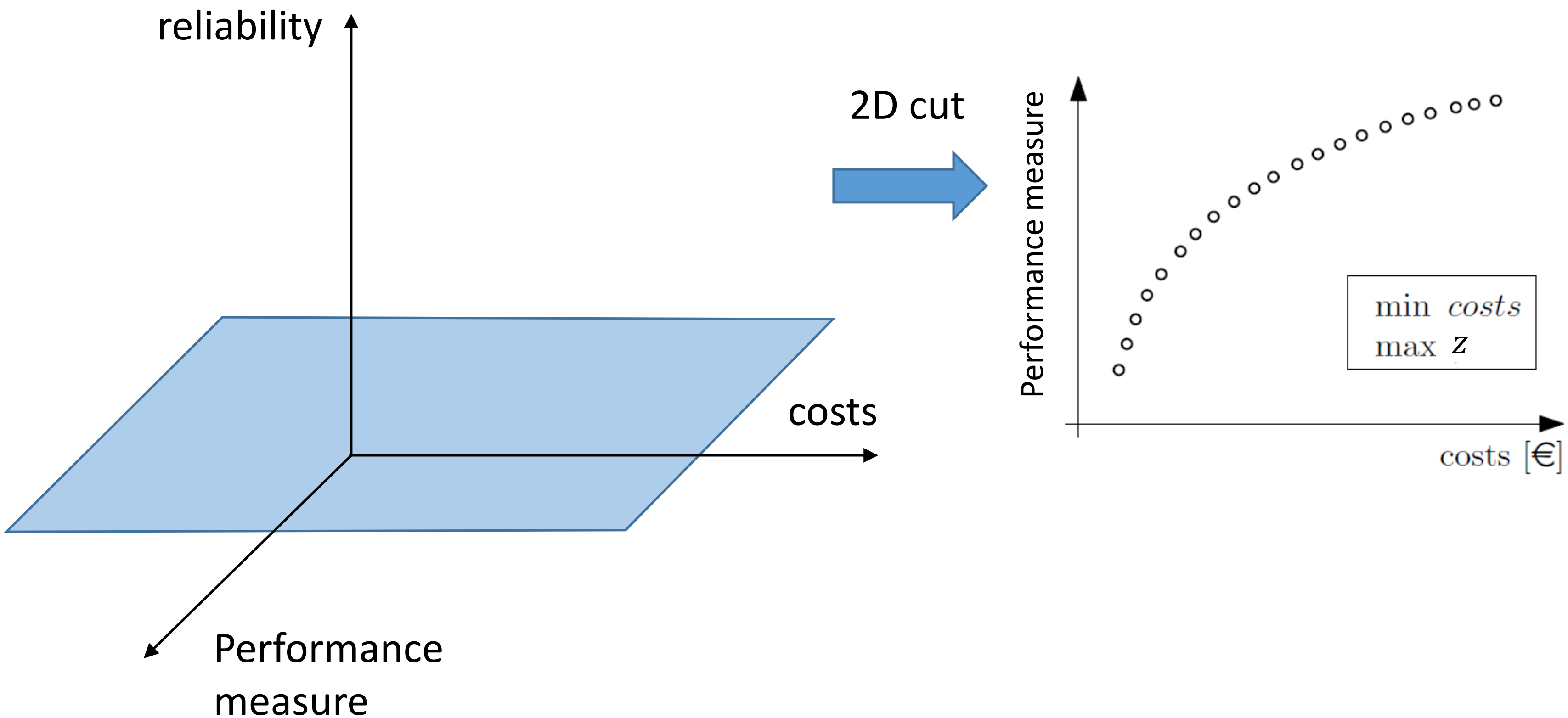
$$p_f(\mathbf{x}, \mathbf{d}) = \text{Prob}[g(\mathbf{X}) \leq \bar{z}] = \int_{g(\mathbf{X}) \leq \bar{z}} \dots \int f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = F_{\mathbf{X}}(\bar{z})$$

Multi-Objective Problem

Generalized Reliability Optimization



Generalized Reliability Optimization



INVERSE RBDO

$$\min_{\mathbf{d}\in D} C(\mathbf{x}, \mathbf{d})$$

Minimize costs (e.g. weight of the structure)

$$\max_{\mathbf{d}\in D} z(\mathbf{x}, \mathbf{d}), j = 1, \dots, n_J$$

Maximize performance measure

$$\text{s.t.} \quad h_i(\mathbf{d}) \le 0, i = 1, \dots, n_I$$

$$z(\mathbf{x}, \mathbf{d}) \ge z^{tol}, j = 1, \dots, n_J$$

Subject to constraints

Multi-Objective Problem

INVERSE RELIABILITY ANALYSIS

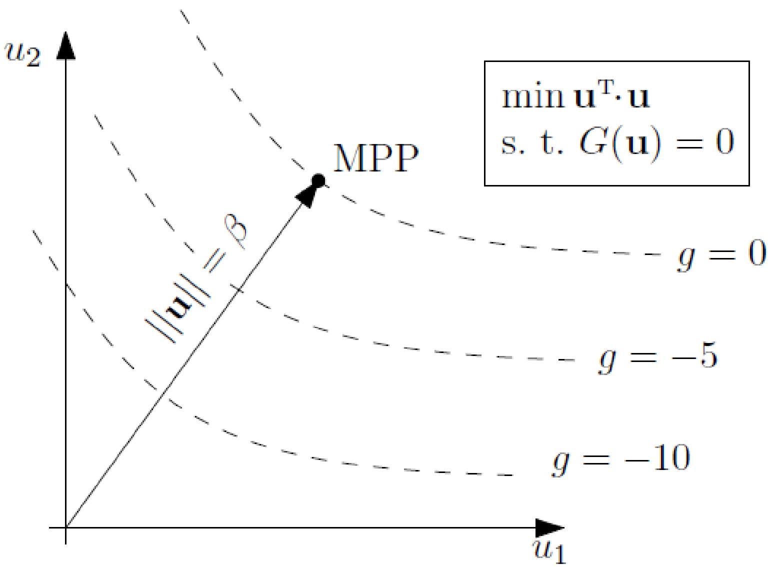


Figure 1: Forward reliability analysis [1]. MPP in standard normal space is the shortest distance between the origin and the limit state contour $g = \bar{z}$. The threshold value $\bar{z}$ is set to zero in this task.

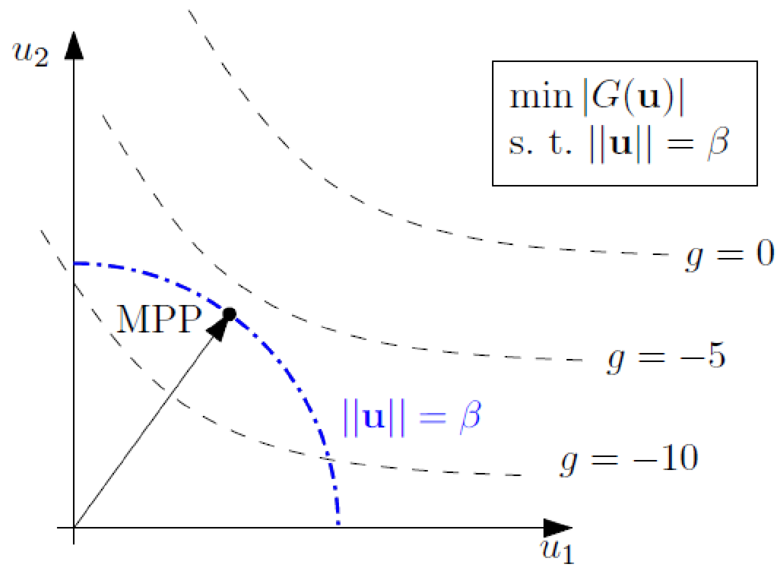


Figure 2: Inverse reliability analysis [1]. MPP in standard normal space lies on the circle with perimeter β and has the smallest absolute value of the performance function.

INVERSE MONTE CARLO

Monte Carlo: $$p_f = \frac{1}{m} \sum_{k=1}^{m} I_G(\mathbf{x}^{(k)}) = \frac{n_f}{m}$$

Inverse Monte Carlo: $$n_f = \bar{p}_f \cdot m$$