Výpočet průhybu na pružnoplastickém nosníku

Semestrální práce z předmětu PRPE

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Vedoucí práce : Prof. Ing. Milan Jirásek DrSc.

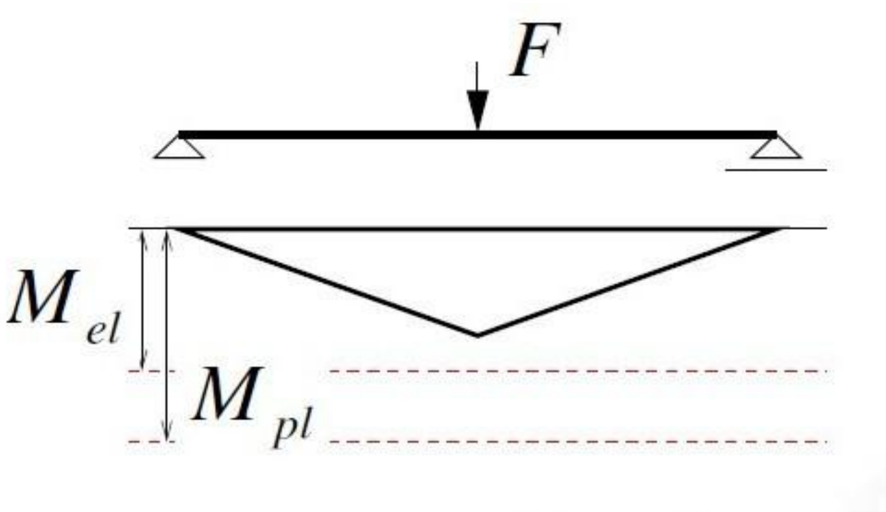
Zadání

Pružný stav

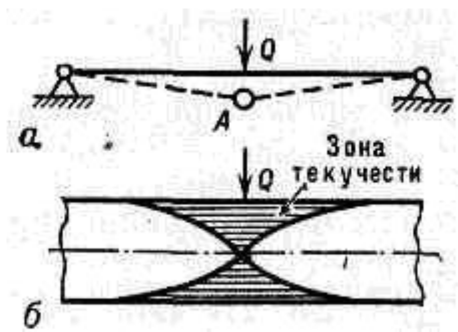
Pružnoplastický stav

Pružnoplastický s lineárním zpevněním

Zadání

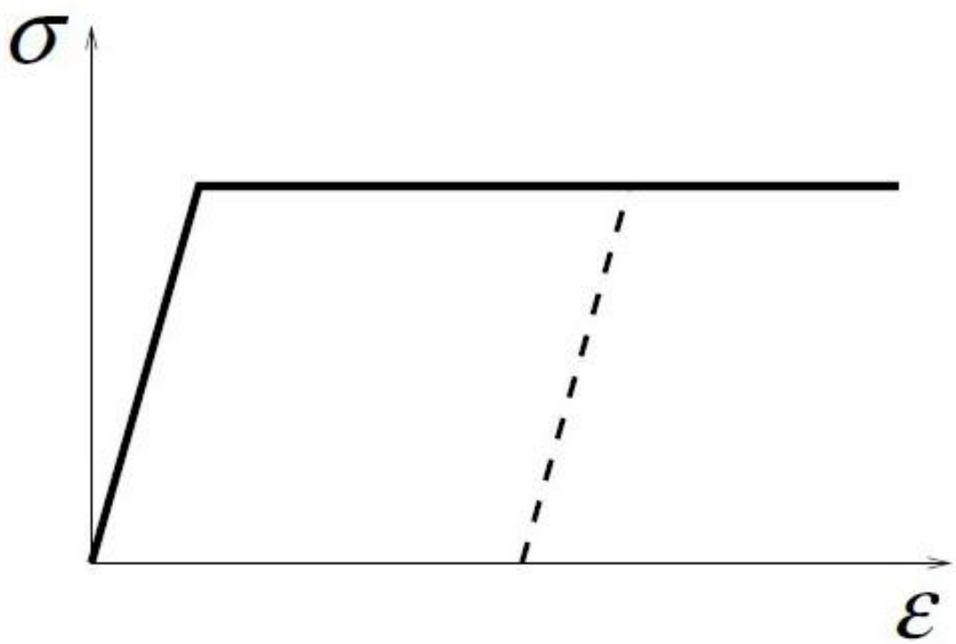


Plastický kloub

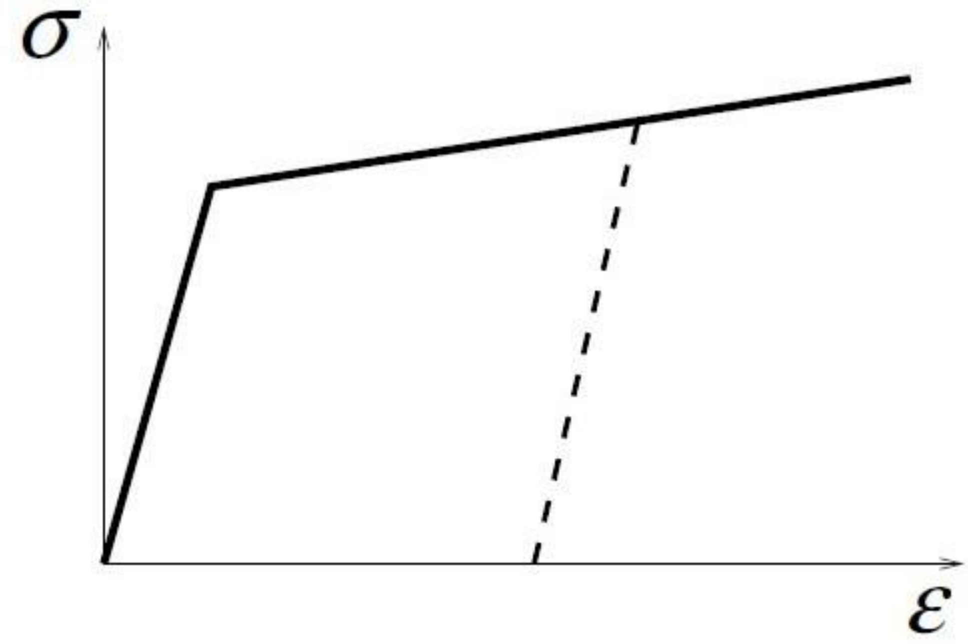


(Zdroj : 6. přednáška (30.10.2012), PRPE, Prof. Ing. Milan Jirásek DrSc.)

Zadání



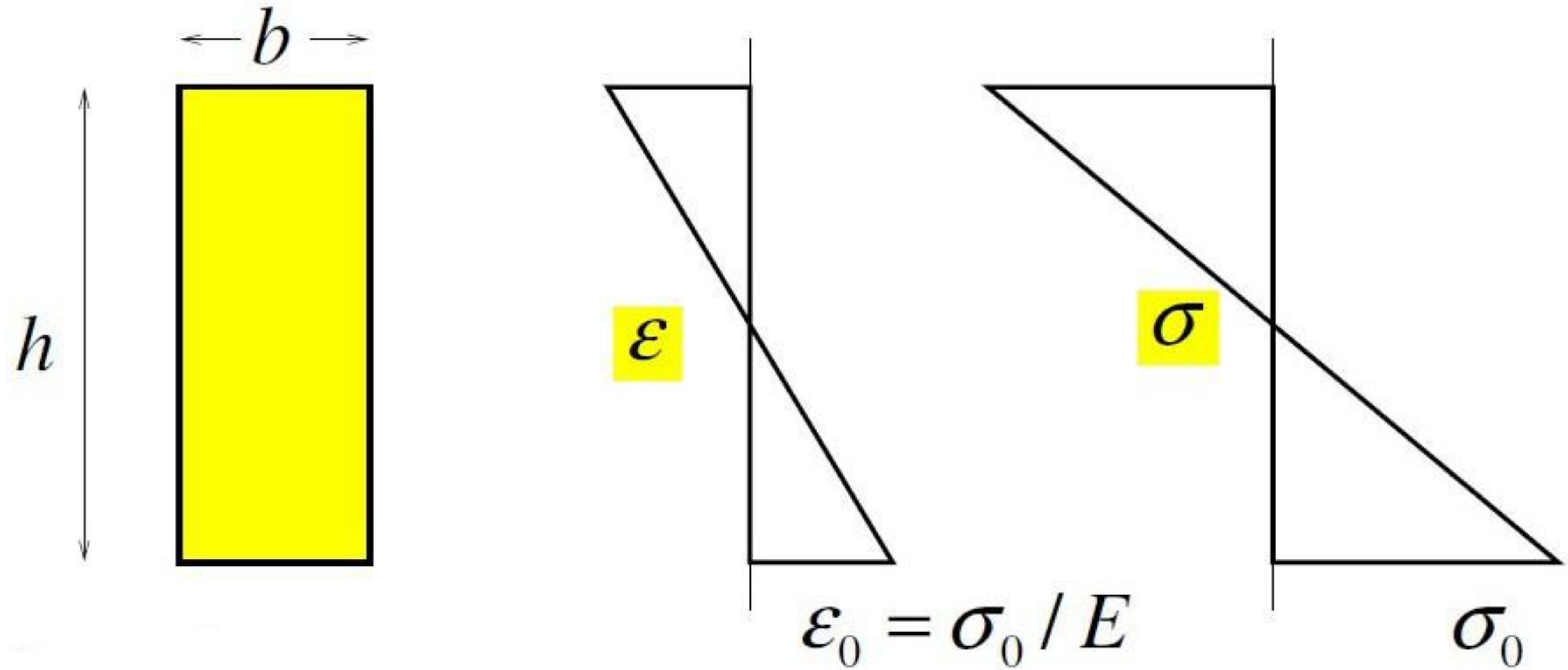
ideálně pružnoplastický model



pružnoplastický model se zpevněním

(Zdroj : 6. přednáška (30.10.2012), PRPE, Prof. Ing. Milan Jirásek DrSc.)

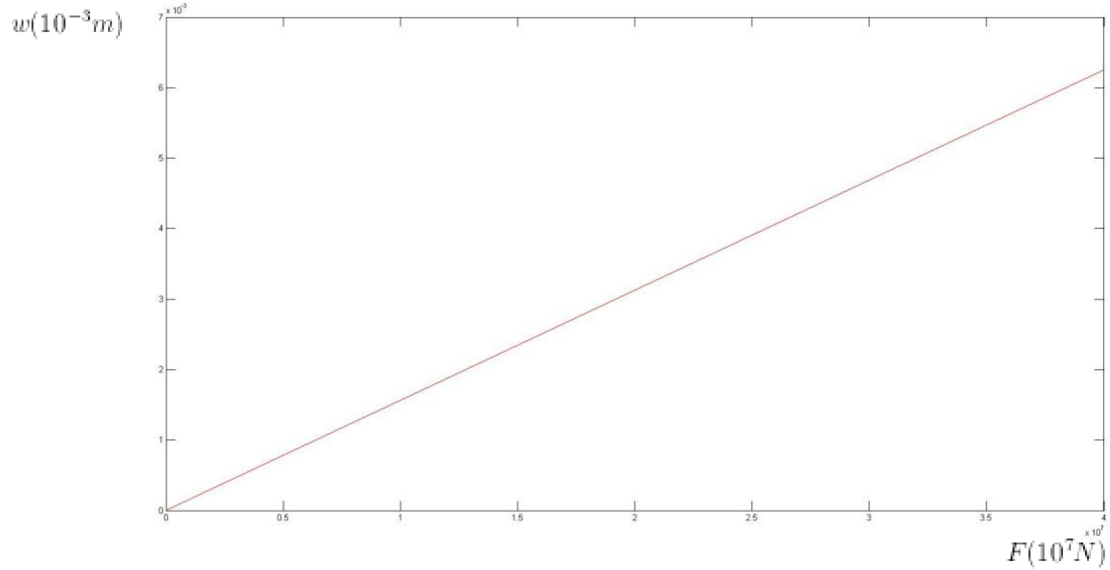
Pružný stav



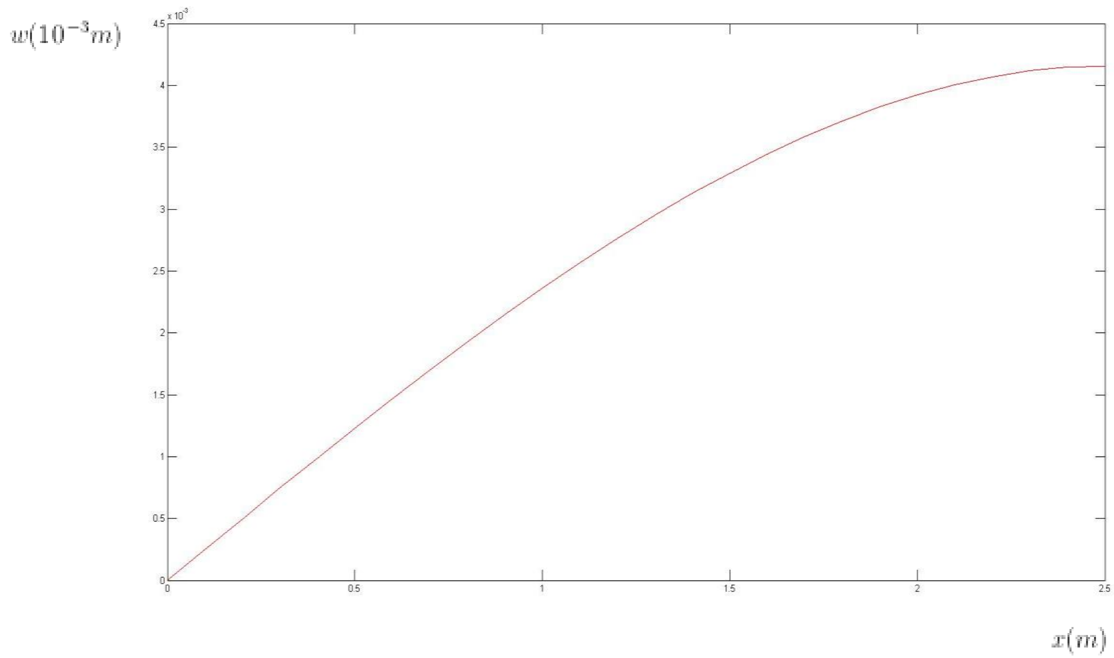
(Zdroj : 6. přednáška (30.10.2012), PRPE, Prof. Ing. Milan Jirásek DrSc.)

Pružný stav

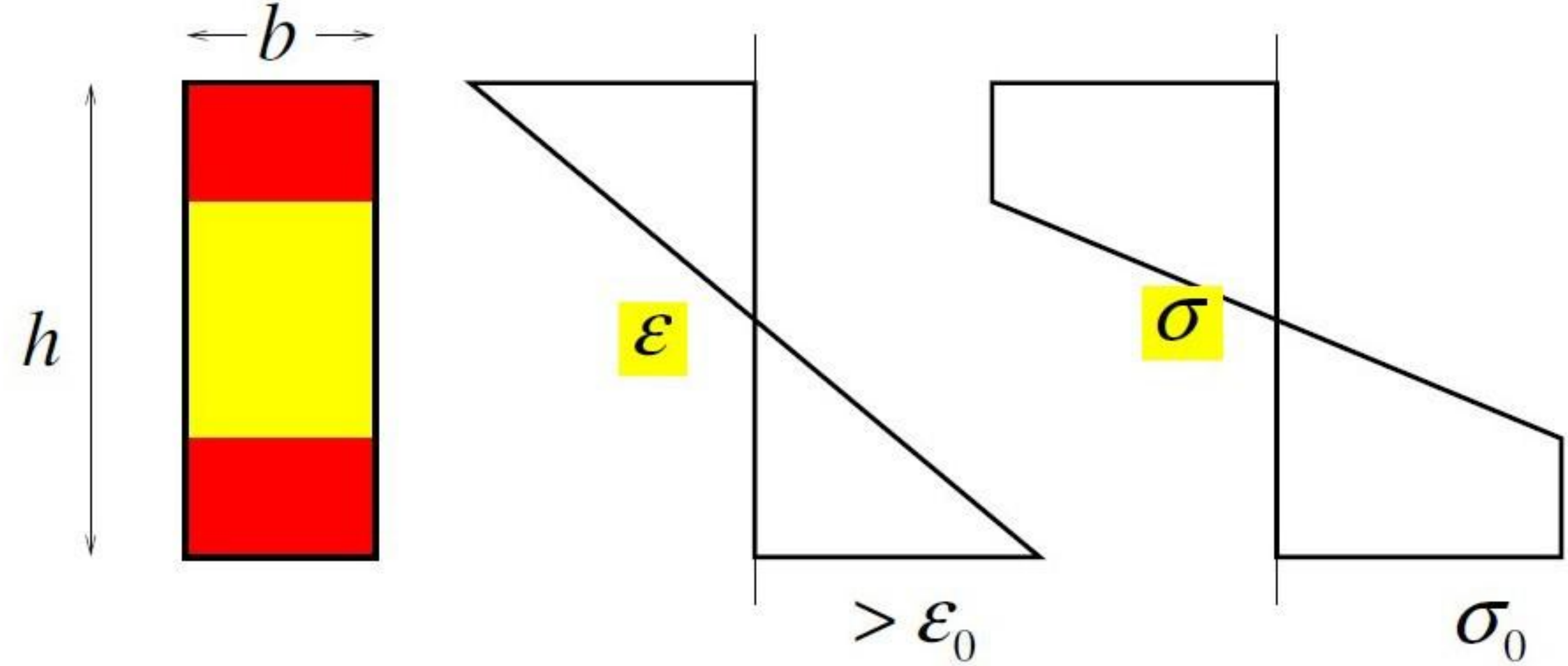
```
clear all;

E = 2.e11;
b = 1;
h = 1;
l = 5;
o = 2.e8;
step = 10000;
x = 0:step:4e7;


y =x/(E*b*h^3)*(-2.5^3+0.75*l^2*2.5);
plot(x,y,'r')
```




```
clear all;

E = 2.e11;
b = 1;
h = 1;
l = 5;
o = 2e8;
step = 0.1;
x = 0:step:2.5;
F = 2.66e7;

y =F/(E*b*h^3)*(-x.^3+0.75*l^2.*x)
plot(x,y,'r')
```



Pružnoplastický stav



(Zdroj : 6. přednáška (30.10.2012), PRPE, Prof. Ing. Milan Jirásek DrSc.)

Mezní síly

$$M_{el} = \frac{bh^2\sigma_0}{6}$$

$$M_{pl} = \frac{bh^2\sigma_0}{4}$$

$$F_{el} = \frac{4M_{el}}{L} = \frac{2bh^2\sigma_0}{3L}$$

$$F_{pl} = \frac{4M_{pl}}{L} = \frac{bh^2\sigma_0}{L}$$

```
E=2e11;
b=1;
h=1;
l=5;
o=2e8;

F=2/3*b*h^2*o/l
```

Fel = 2.6667e7

Fpl = 4e7

Plastická zóna

$$\sigma_0 = E\varepsilon \qquad \varepsilon = \frac{\kappa h_e}{2} \qquad M = \frac{Fx}{2}$$

$$\kappa = \frac{M}{EI} = \frac{Fx}{2EI} \qquad h_e = h$$

$$\sigma_0 = E\frac{Fxh}{4EI} = \frac{Fxh}{4I}$$

$$x = \frac{\sigma_0 bh^2}{3F}$$

Pružná část

$$\kappa(x) = \frac{M}{EI} = \frac{Fx}{2EI}$$

$$\varphi(x) = \frac{3Fx^2}{Ebh^3} + C_1$$

$$\omega(x) = \frac{-Fx^3}{Ebh^3} - C_1 x + C_2 \qquad C_2 = 0$$

Plastická část

$$M = \frac{b\sigma_0(3h^2 - h_e)}{12} \quad h_e = \sqrt{3h^2 - \frac{6Fx}{b\sigma_0}}$$

$$\sigma_0 = E\kappa(x)\frac{h_e}{2} \qquad \kappa(x) = \frac{2\sigma_0}{E\sqrt{3h^2 - \frac{6Fx}{b\sigma_0}}}$$

$$\varphi(x) = -\frac{2}{3}\frac{\sigma_0^2 b}{EF}\sqrt{3h^2 - \frac{6Fx}{b\sigma_0}} + C_1$$

Plastická část

$$\varphi(x) = -\frac{2}{3}\frac{\sigma_0^2 b}{EF}\sqrt{3h^2 - \frac{6Fx}{b\sigma_0}} + C_1$$

$$\varphi\left(\frac{L}{2}\right) = 0$$

$$\omega\left(x\right) = -\frac{2}{27}\frac{\sigma_0^3 b^2}{EF^2}\left(3h^2 - \frac{6Fx}{b\sigma_0}\right)^{\frac{3}{2}} - C_1 x + C_2$$

Výpočet konstant

Vzdálenost plastické zóny x

```
C1:
clear all
format long
F = 3e7;
o = 2e8;
E = 2e11;
b = 1;
h = 1;
l = 5;
x = o*b*h^3/(3*F);

C1 = 2/3*(o.^2*b)/(E*F)*(-sqrt(3*h.^2-(6*F*x/(b*o)))+sqrt(3*h.^2-(6*F*l/(2*b*o))))-
3*F*x.^2/(E*b*h.^3)
```

Výpočet konstant

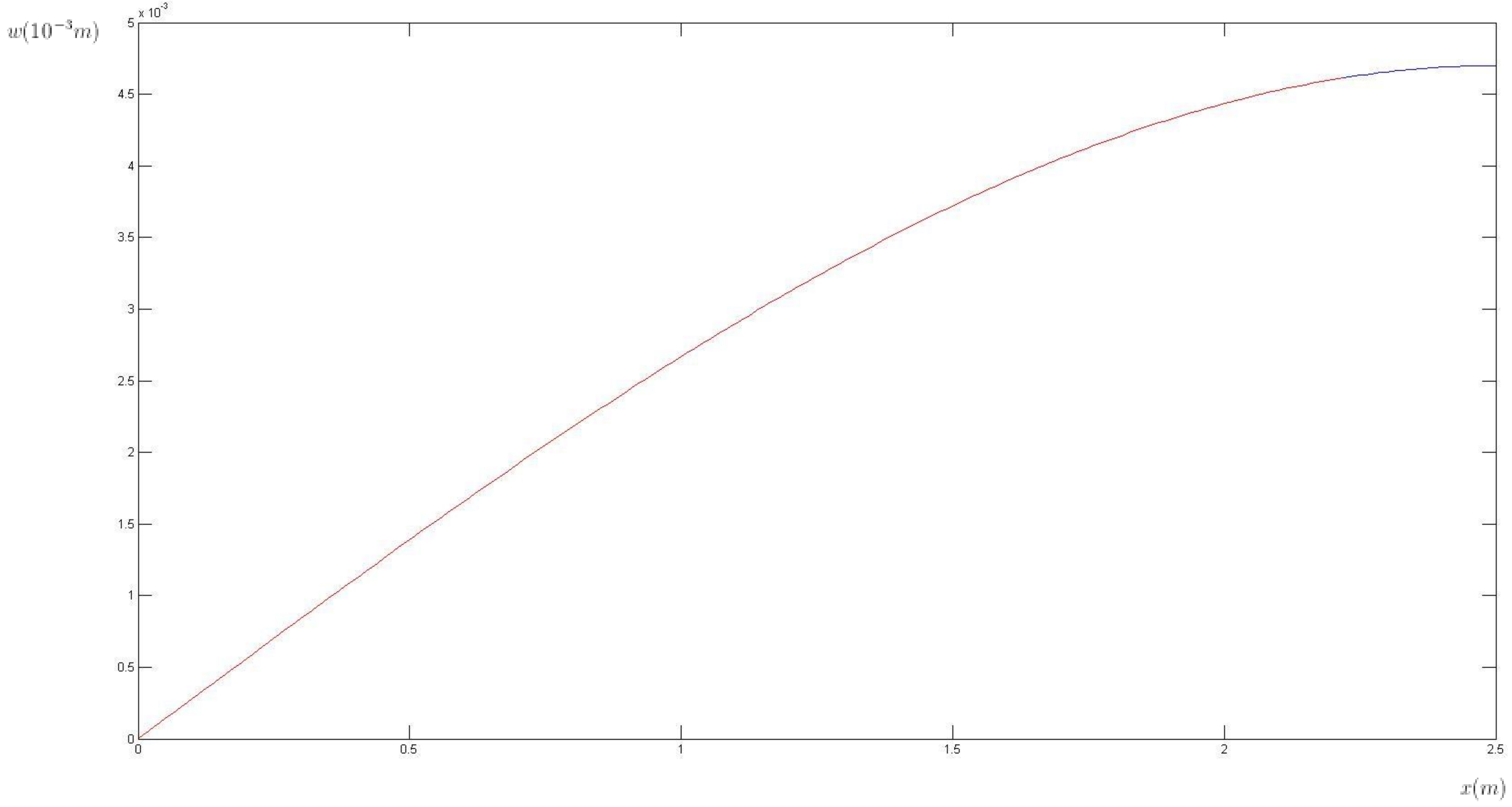
C2:

```
clear all
format long
F = 3e7;
o = 2e8;
E = 2e11;
b = 1;
h = 1;
l = 5;
x = o*b*h^3/(3*F);

C2= 2/27*(o.^3*b.^2)/(E*F.^2)*(3*h.^2-(6*F*x)/(b*o)).^1.5+2/3*o.^2*b/(E*F)*sqrt(3*h.^2-3*F*l/b/o)*x-F/(E*b*h.^3)*x.^3+C1*x
```

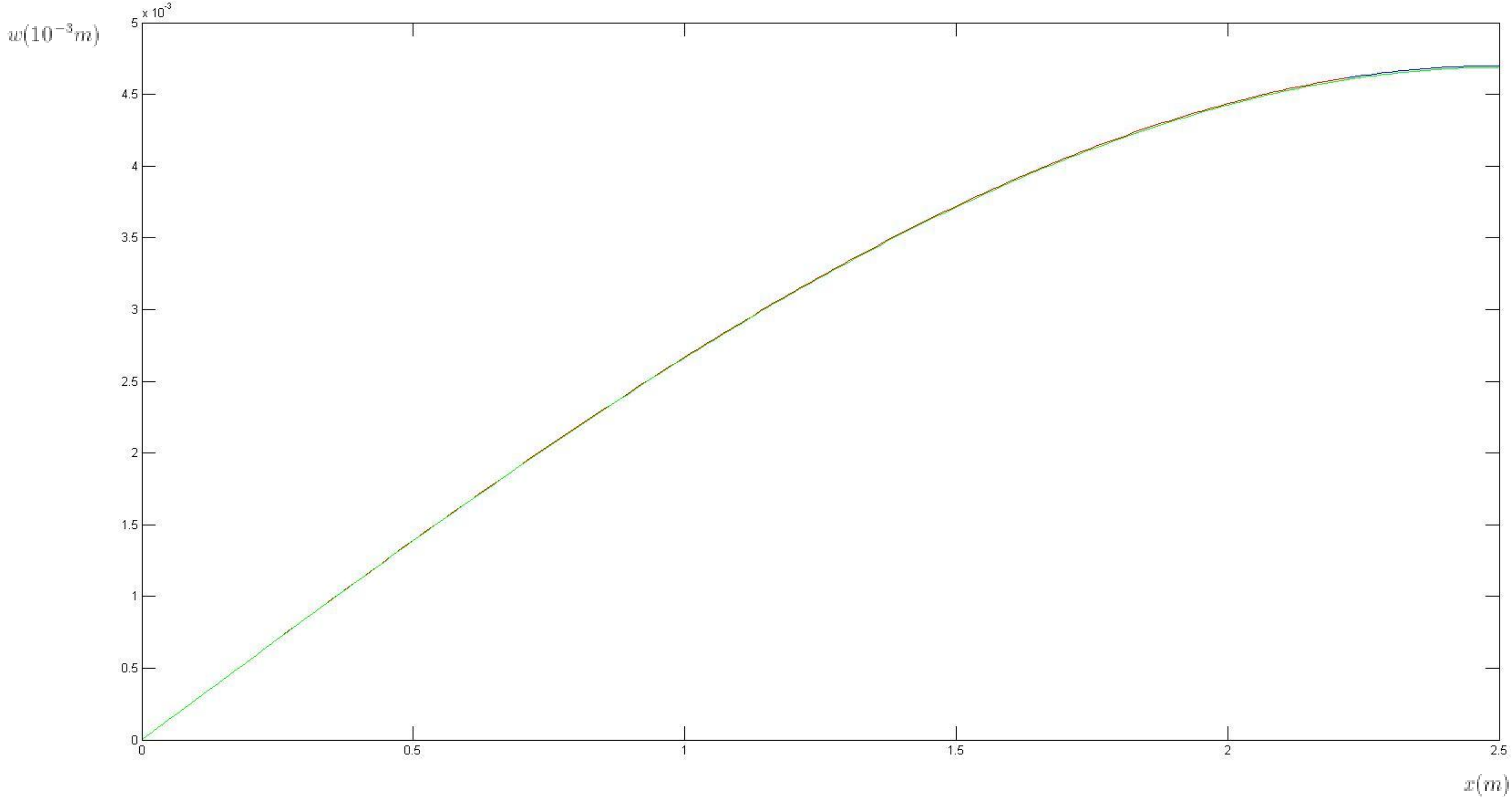
Skript

```
clear all;
E = 2e11;
F = 3e7;
b = 1;
h = 1;
l = 5;
o = 2e8;

x1 = 0:0.01:2.22;
x2 = 2.22:0.01:2.5;

y = -F/(E*b*h.^3)*x1.^3+0.00281766*x1;
u = -2/27*(o.^3*b.^2)/(E*F.^2)*(3*h.^2-(6*F*x2)/(b*o)).^1.5-2/3*o.^2*b/(E*F)*sqrt(3*h.^2-
      3*F*l/b/o)*x2+0.0164609;
plot(x1,y,'r',x2,u);
```

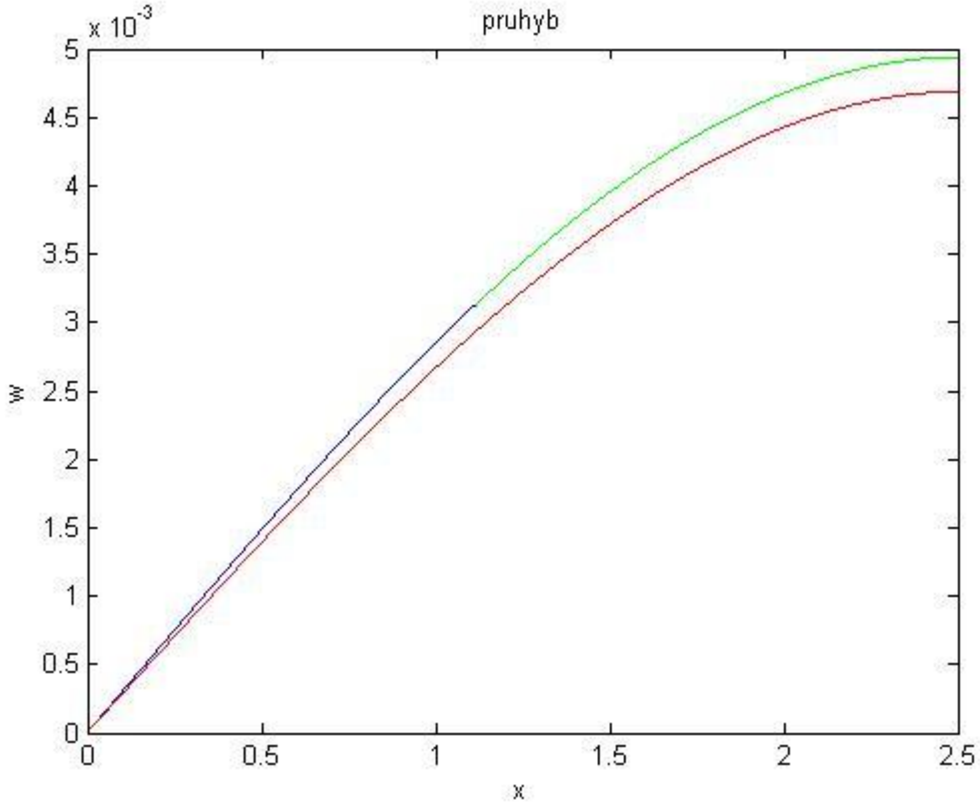
Vykreslení



Porovnání s pružným stavem



Porovnání s pružným stavem



Numerické řešení

$$w''(x_n) = \frac{w_{n+1} - 2w_n + w_{n-1}}{h^2}$$

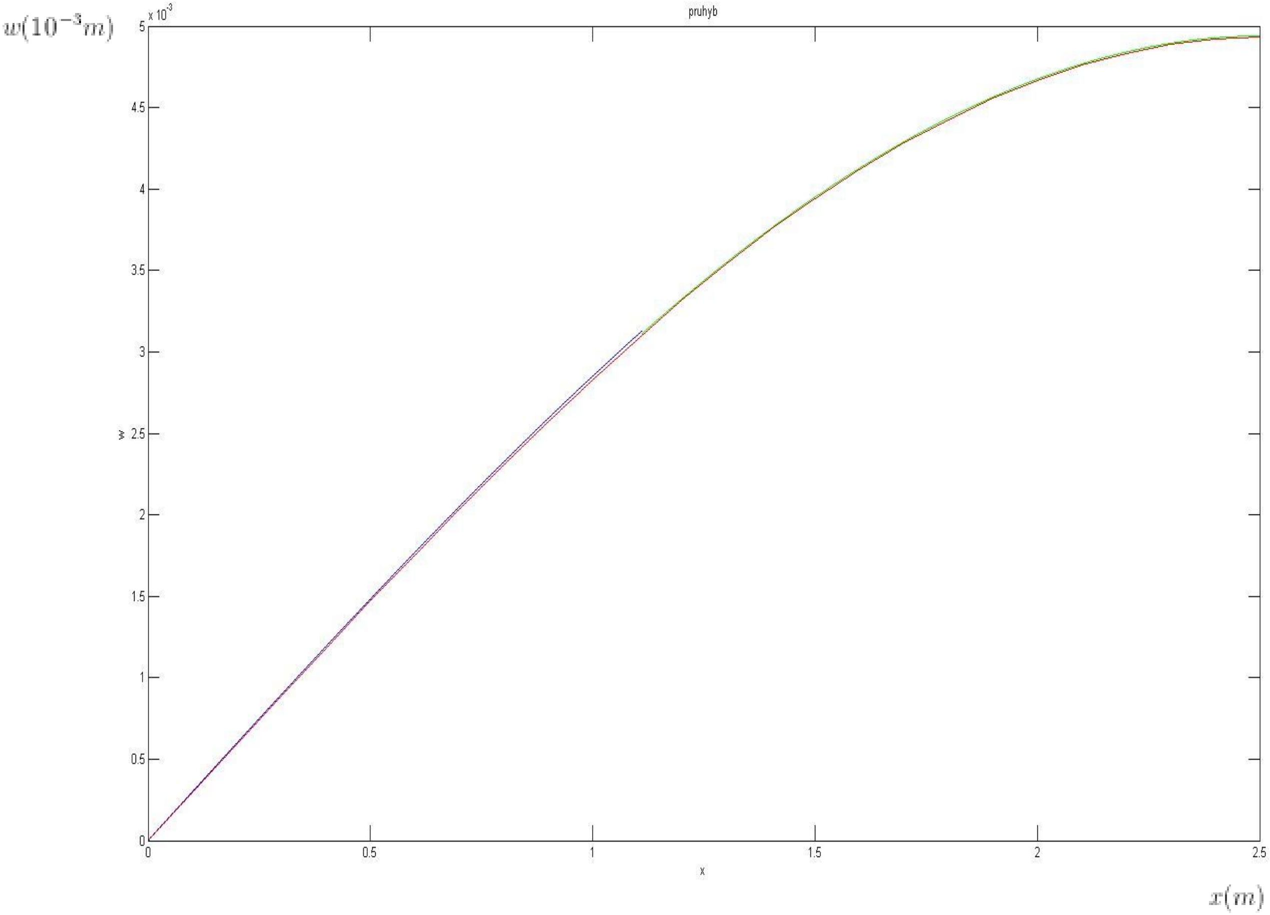
$$w(0) = 0$$

$$\omega\left(\frac{L}{2} - h\right) = \omega\left(\frac{L}{2} + h\right)$$

Numerické řešení

```
load 'a.mat'
E = 2e11;
b = 1;
h = 1;
l = 5;
o = 2e8;
F = 3e7;

threshold = o*b*h.^2/3/F;

x1 = 0:0.1:threshold;
x2 = threshold:0.1:2.5;

C1 = (-6*F*x1/(E*b*h.^3).*(0.1.^2))';
C2 = (-2.*o./E.*(0.1.^2)./sqrt(3*h.^2-6*F*x2./b./o))';

C3 = [C1;C2];
Y = A\C3;


plot (0:0.1:2.5,Y,'r')
```

Porovnání



Síla, průhyb

$$\varphi(x) = \frac{3Fx^2}{Ebh^3} + C_1$$

$$\omega(x) = \frac{-Fx^3}{Ebh^3} - C_1 x + C_2$$

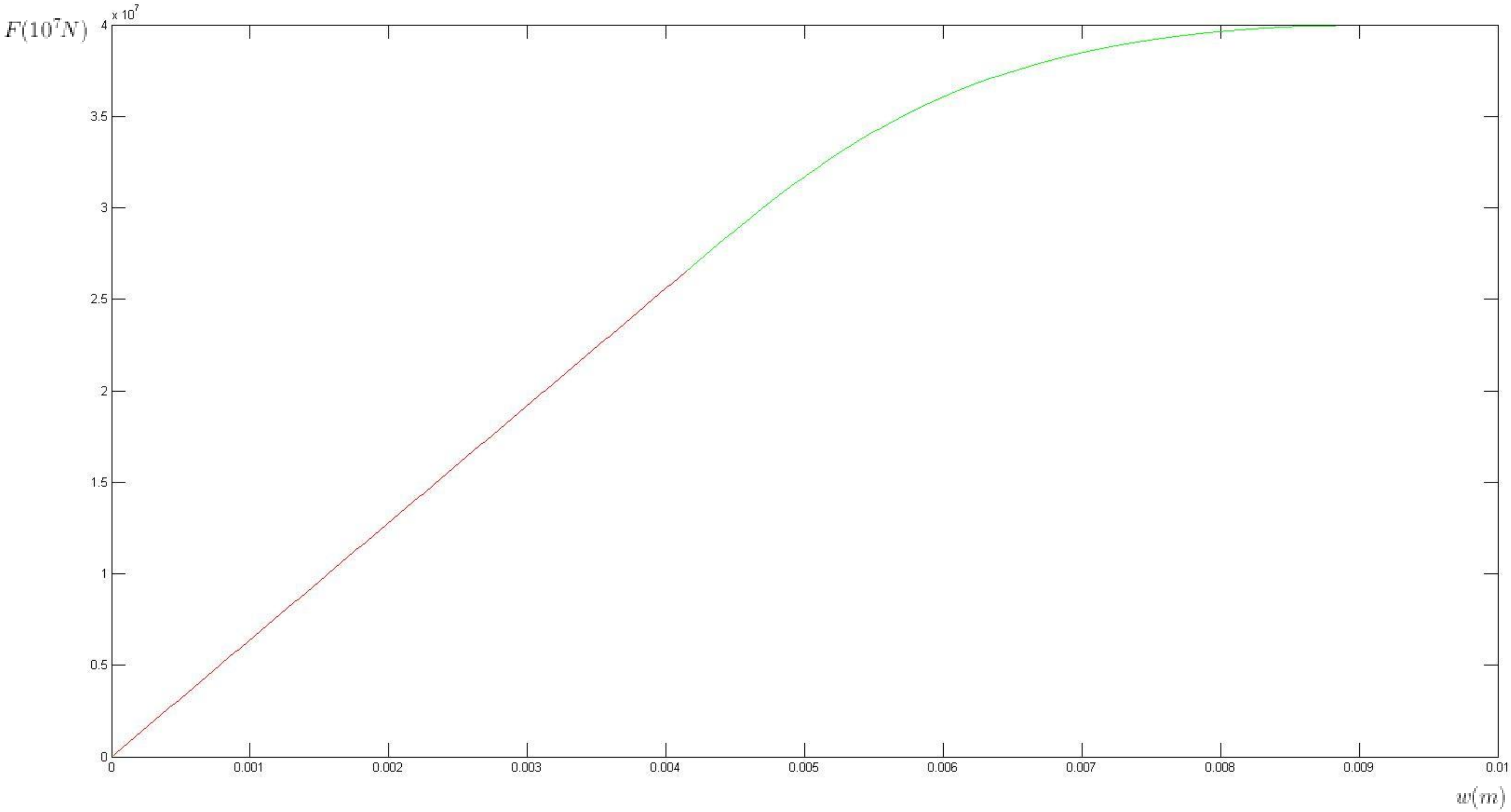
$$\varphi(x) = -\frac{2}{3}\frac{\sigma_0^2 b}{EF}\sqrt{3h^2 - \frac{6Fx}{b\sigma_0}} + C_1$$

$$\omega\left(x\right) = -\frac{2}{27}\frac{\sigma_0^3 b^2}{EF^2}\left(3h^2 - \frac{6Fx}{b\sigma_0}\right)^{\frac{3}{2}} - C_1 x + C_2$$

Síla, průhyb

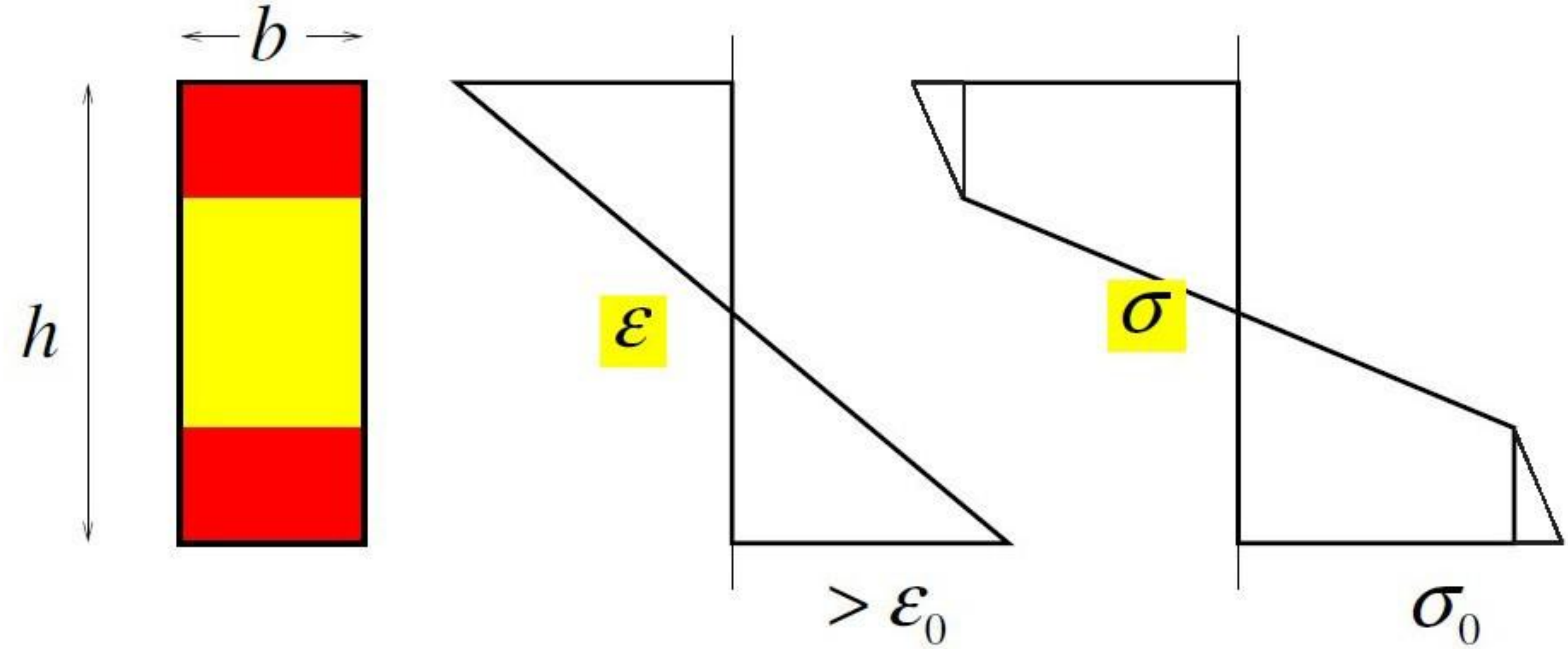
```
clear all

E = 2e11;
b = 1;
h = 1;
l = 5;
o = 2e8;
s = 2.5;
threshold = 2.66e7;

x1 = 0:1e5:threshold;
x2 = threshold:1e5:4e7;

r = -x1./(E.*b.*h.^3).*s.^3-(2./3.*(o.^2.*b)./(E.*x1).*(-sqrt(3.*h.^2-(6.*x1.*s./(b.*o)))+sqrt(3.*h.^2-(3.*x1.*l./(b.*o))))-
      3.*x1.*s.^2./(E.*b.*h.^3)).*s;
d = -2./27.*(o.^3.*b.^2)./(E.*x2.^2).*(3.*h.^2.-(6.*x2.*s)./(b.*o)).^1.5 ...
   -2./3.*o.^2.*b./x2./E.*sqrt(3.*h.^2-3.*x2.*l./b./o).*s ...
   +2./27.*(o.^3.*b.^2)./(E.*x2.^2).*(3.*h.^2-(6.*x2.*(o.*b.*h.^2./3./x2))./(b.*o)).^1.5 ...
   +2./3.*o.^2.*b./(E.*x2).*sqrt(3.*h.^2-3.*x2.*l./b./o).*(o.*b.*h.^2./3./x2)-x2./(E.*b.*h.^3).*(o.*b.*h.^2./3./x2).^3 ...
   -(2./3.*(o.^2.*b)./(E.*x2).*(-sqrt(3.*h.^2-(6.*x2.*(o.*b.*h.^2./3./x2)./(b.*o)))+sqrt(3.*h.^2-(3.*x2.*l./(b.*o))))-
      3.*x2.*(o.*b.*h.^2./3./x2).^2./(E.*b.*h.^3)).*(o.*b.*h.^2./3./x2);

plot(x1,r,'r',x2,d,'g')
view(90,270)
```

Síla, průhyb



$F(10^7N)$
x 10^7
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01
$w(m)$

Lineární zpevnění



(Zdroj : 6. přednáška (30.10.2012), PRPE, Prof. Ing. Milan Jirásek DrSc.)

Lineární zpevnění

E_{ep} – pružnoplastický modul

$$M = \frac{b\sigma_0(3h^2 - h_e^2)}{12}$$

$$N_+ = \frac{h - h_e}{4} b\sigma_0 \qquad r_+ = \frac{2h + h_e}{6}$$

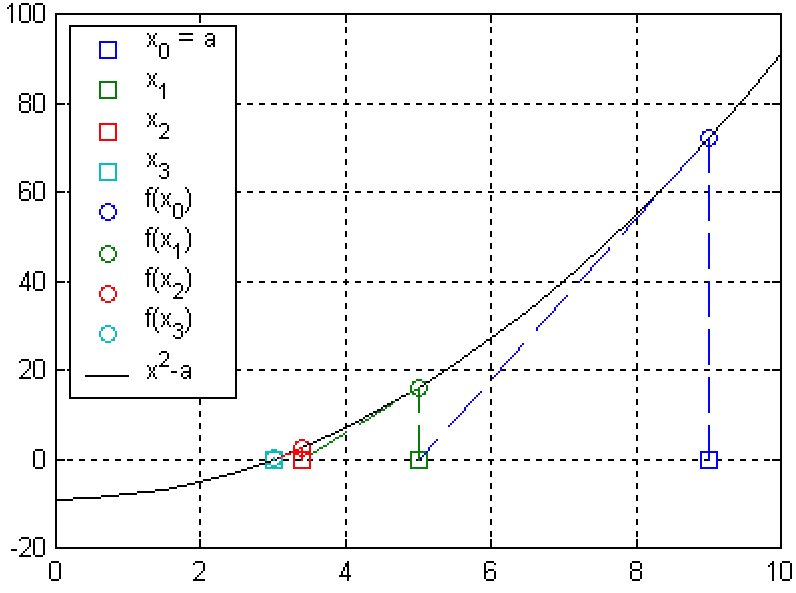
$$M = \frac{b\sigma_0(3h^2 - h_e^2)}{12} + bE_{ep}\kappa\frac{(h - h_e)^2(2h + h_e)}{24}$$

Lineární zpevnění

$$\sigma_0 = E\kappa \frac{h_e}{2} \qquad h_e = \frac{2\sigma_0}{E\kappa}$$

$$M(\kappa) = \frac{b\sigma_0(3h^2 - \frac{4\sigma_0^2}{E^2\kappa^2})}{12} + \kappa \frac{bE_{ep}}{24}\left(h - \frac{2\sigma_0}{E\kappa}\right)^2 \left(2h + \frac{2\sigma}{E\kappa}\right)$$

Metoda tečen



$$\kappa_1 = \frac{M}{EI}$$

(Zdroj :
wikipedie.cz)

Metoda tečen

$$\kappa_1 = \frac{M}{EI}$$

$$|M - M(\kappa_1)| < \varepsilon$$

$$\kappa_2 = \kappa_1 + \Delta\kappa$$

$$\Delta\kappa = \frac{M - M(\kappa_1)}{M'\kappa_1}$$

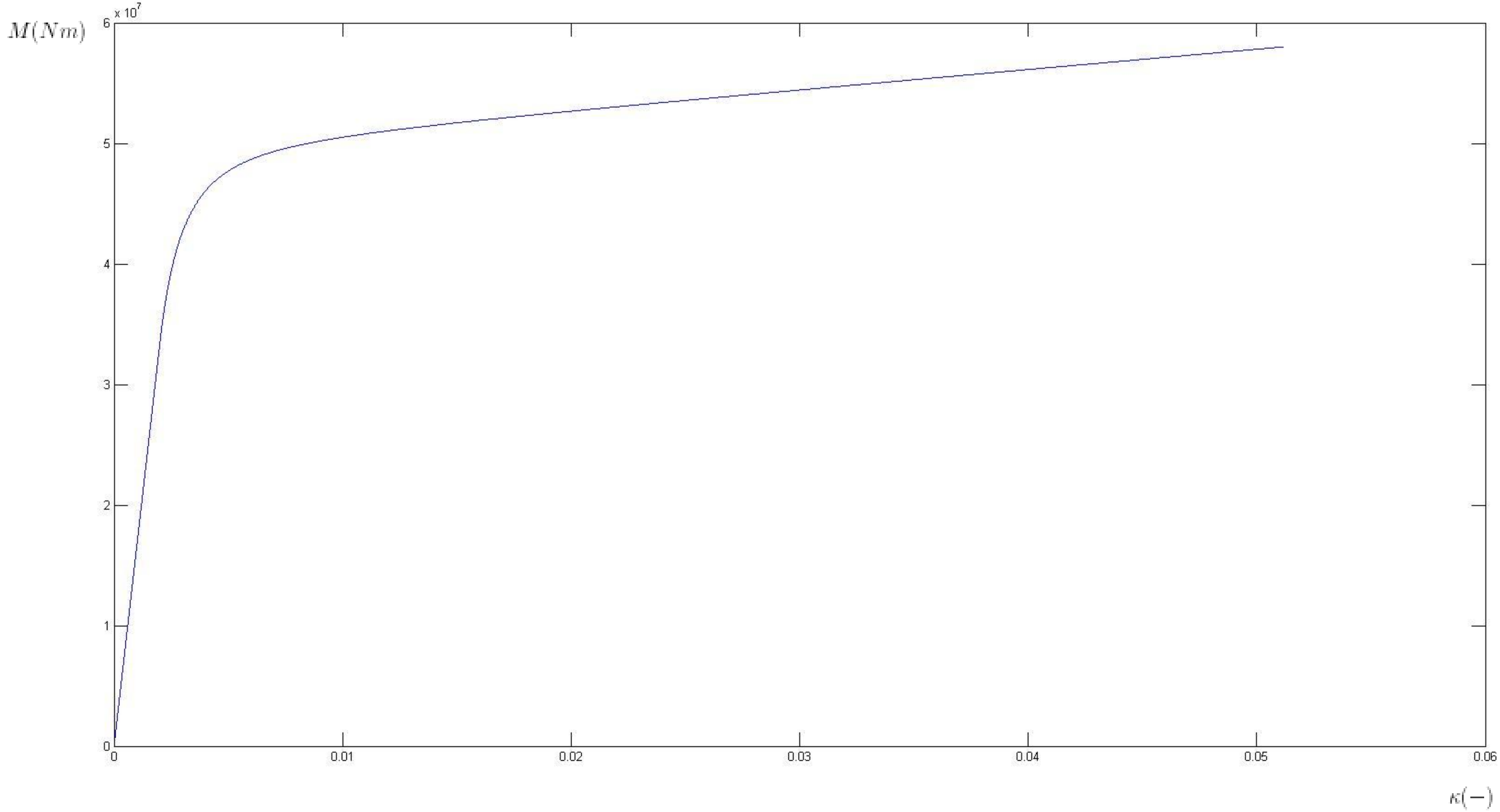
Křivost

```
clear all
E = 2e11;
Ee = E/100;
b = 1;
h = 1;
l = 5;
o = 2e8;
Mel = b*h^2*o/6;
eps = 0.1;

Mstep = 1.e4;
Mmax = 5.8e7;
M = Mstep:Mstep:Mmax;
   nsteps = size(M,2);
for i=1:nsteps;
   ds = 0.;
   s = 12.*M(i)/(E*b*h^3);
   if (M(i)>Mel)
      mkappa = b*o/12.*(3.*h^2-4.*o^2/(E^2*s^2))+s*b*Ee/24.*(h-2.*o/E/s)^2*(2.*h+2.*o/E/s);
   while abs(M(i)-mkappa) > eps
         mscaroukappa = 2./3.*b*o^3/E^2/s^3+1./12.*Ee*b*(h^3-8.*o^3./E.^3/s^3);
               ds = (M(i)-mkappa)/mscaroukappa;
               s = s + ds;
               mkappa = b*o/12.*(3.*h^2-4.*o^2/(E^2*s^2))+s*b*Ee/24.*(h-2.*o/E/s)^2*(2.*h+2.*o/E/s);
    end
   end
   Kappa(i) = s;
end
```



```
plot(Kappa,M);
```

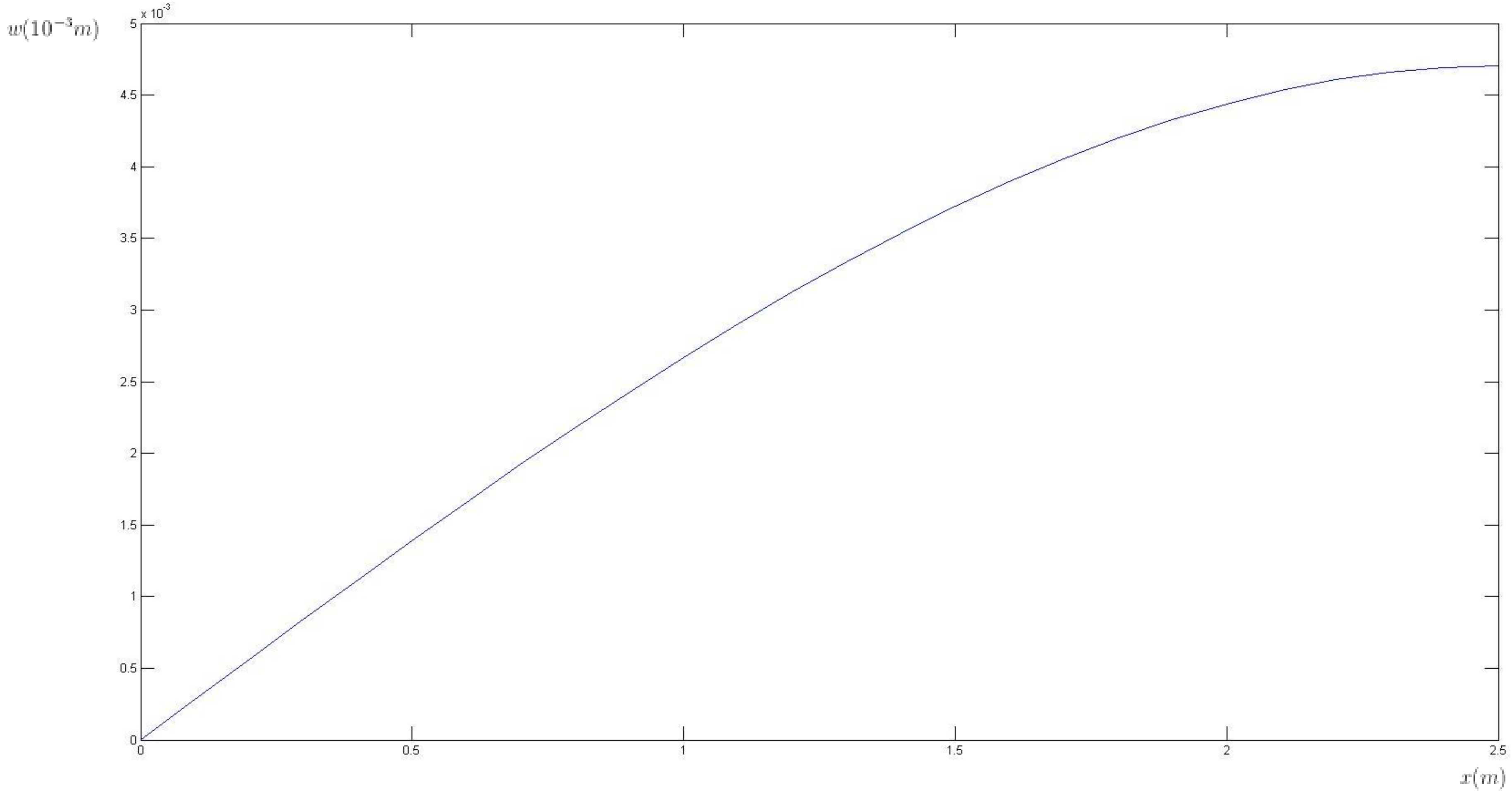
Vykreslení křivosti



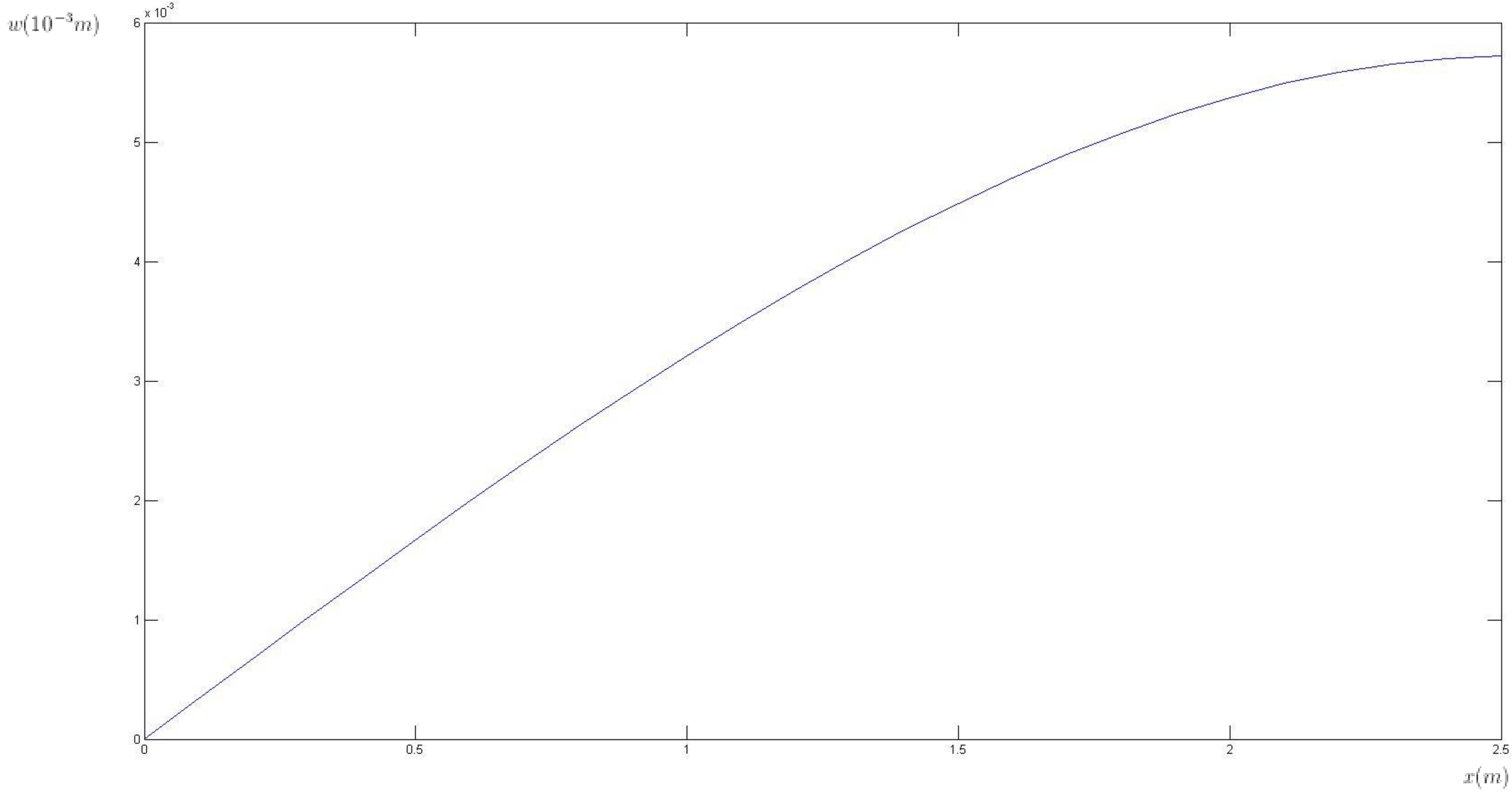
Skript

```
clear all
    load 'a.mat'
    E = 2e11;
    Ee = E/1000;
    b = 1;
    h = 1;
    l = 5;
    o = 2e8;
    F = 3.5e7;
    Mel = b*h^2*o/6;
    eps = 0.1;
    x = 0:.1:2.5;
    C = zeros(length(x), 1);
    M = x.*F./2;
    nsteps = size(M,2);

        for i=1:nsteps;
            ds = 0.;
            s = 12.*M(i)/(E*b*h^3);

            if (M(i)>Mel)
                mkappa = b*o/12.*(3.*h^2-4.*o^2/(E^2*s^2))+s*b*Ee/24.*(h-2.*o/E/s)^2*(2.*h+2.*o/E/s);
                    while abs(M(i)-mkappa) > eps
                    mscaroukappa = 2./3.*b*o^3/E^2/s^3+1./12.*Ee*b*(h^3-8.*o^3./E.^3/s^3);
                        ds = (M(i)-mkappa)/mscaroukappa;
                        s = s + ds;
                        mkappa = b*o/12.*(3.*h^2-4.*o^2/(E^2*s^2))+s*b*Ee/24.*(h-2.*o/E/s)^2*(2.*h+2.*o/E/s);
                    end
            end
            mkappa(i) = s;
            C(i) = -s.*0.1.^2;
        end
        Y = A\C;
plot(x, Y)
```

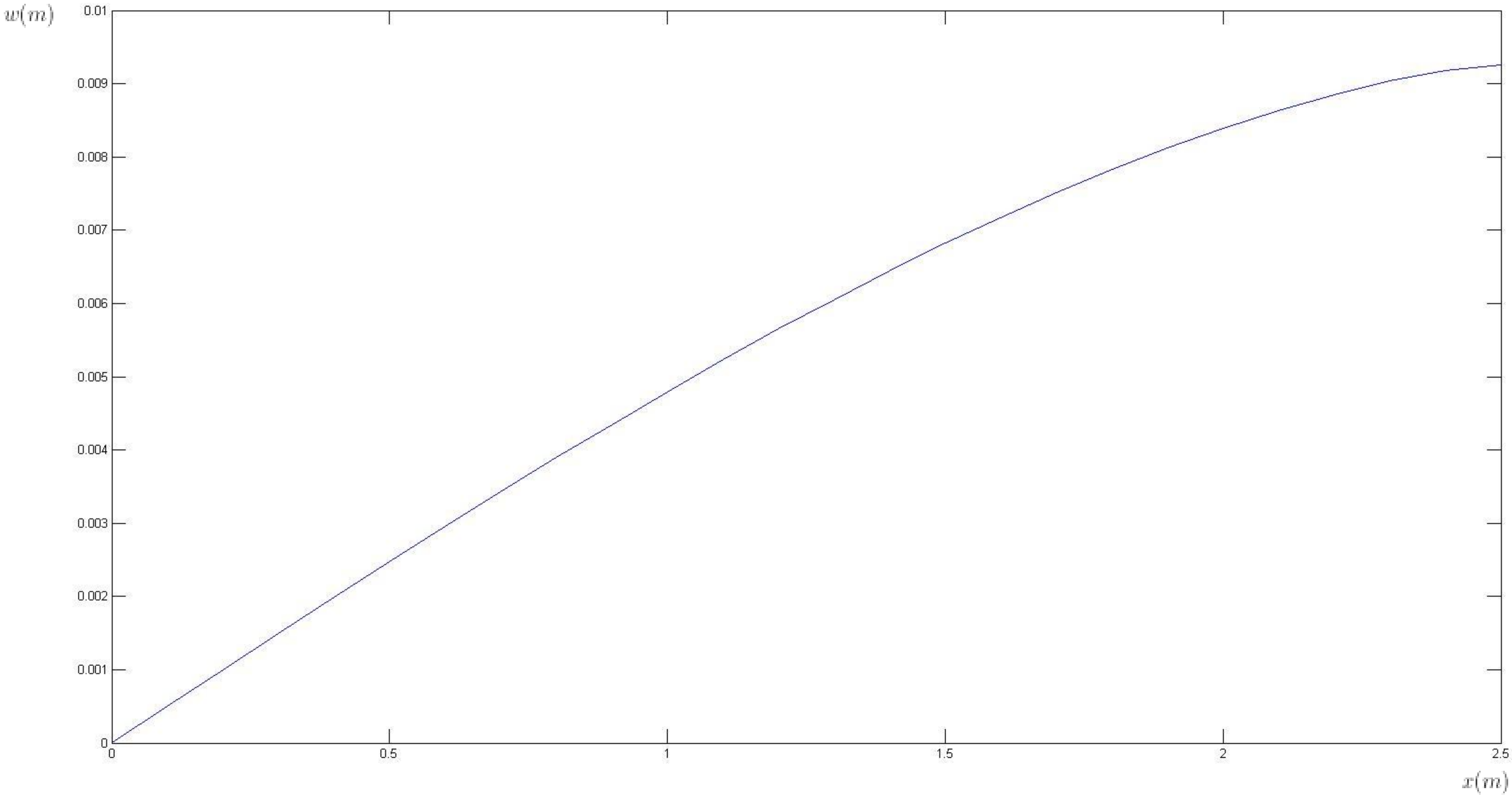
Vykreslení



Vykreslení

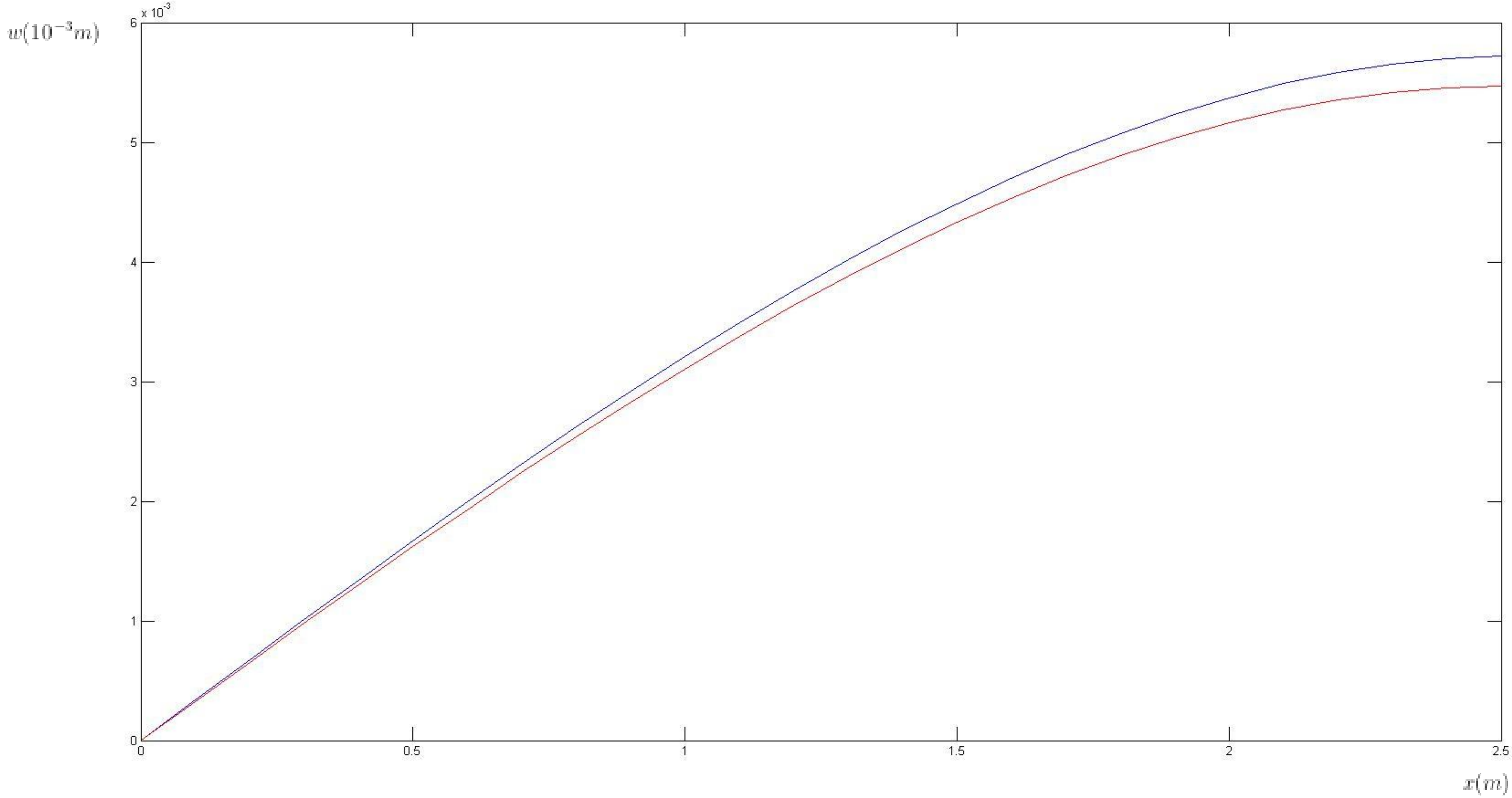


Vykreslení

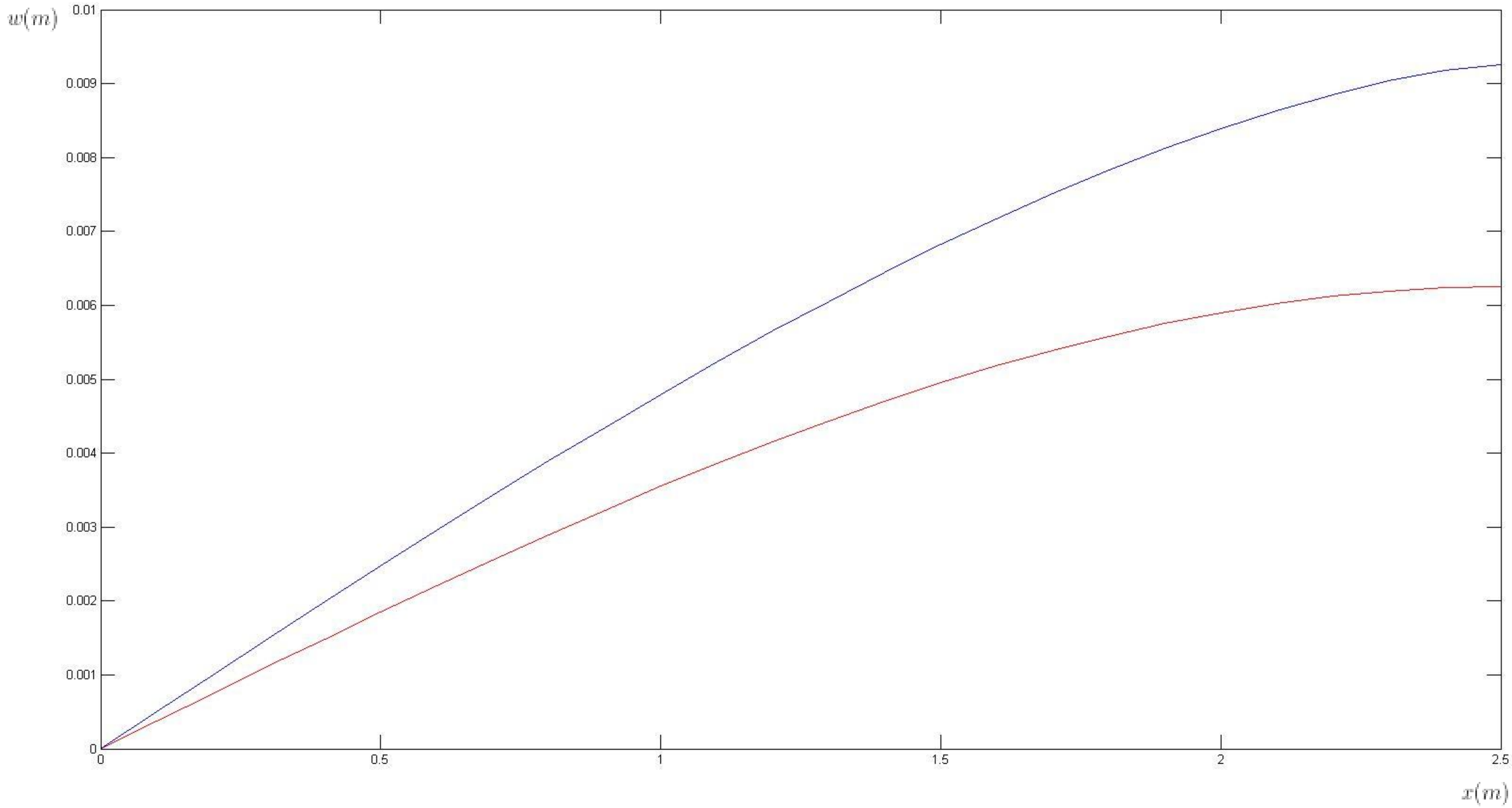


w(m)
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0
0.5
1
1.5
2
2.5
x(m)

Porovnání s pružným stavem



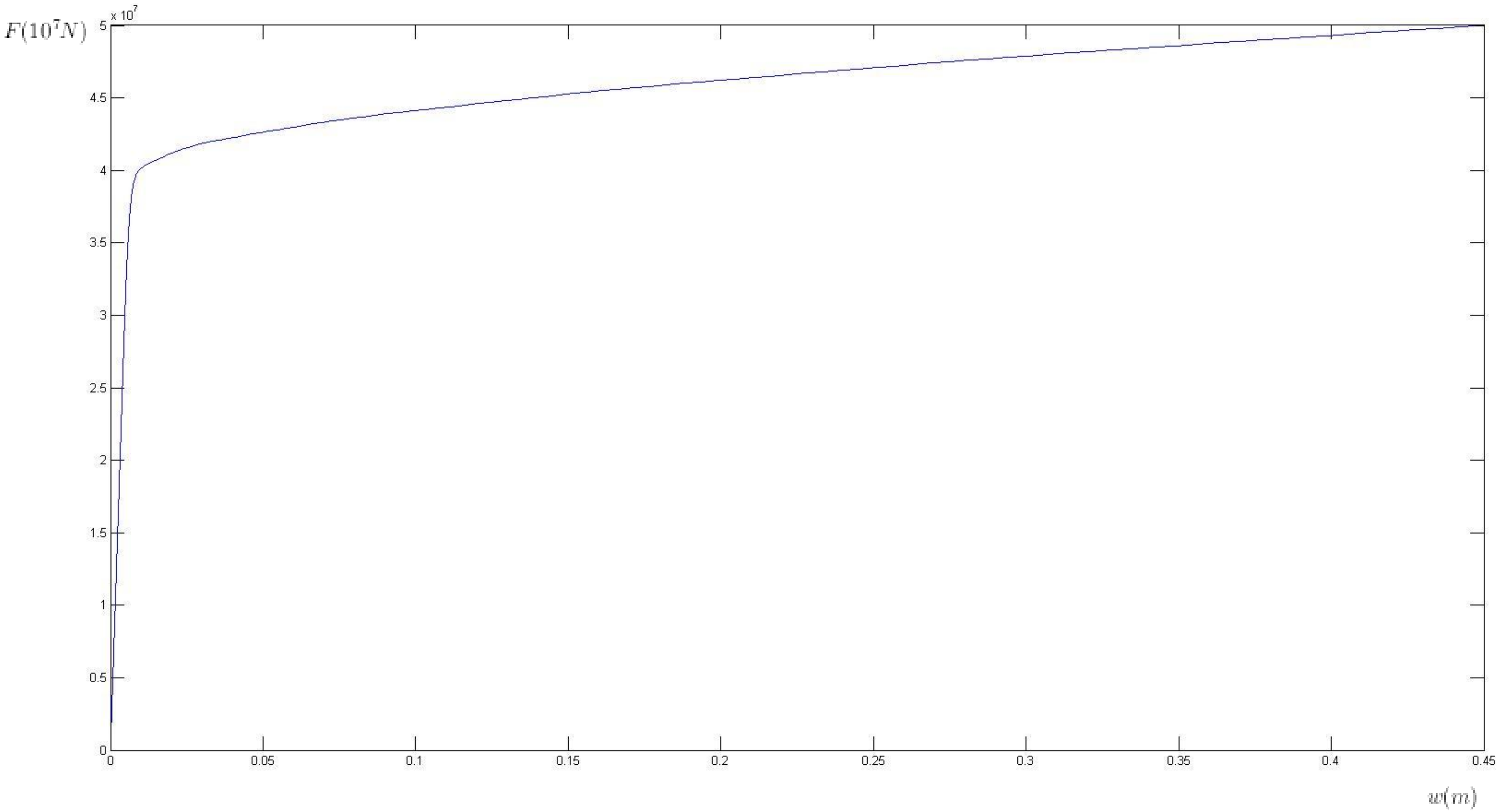
Porovnání s pružným stavem



Průhyb, síla

```
clear all
    load 'a.mat'
    E = 2e11;
    Ee = E/1000;
    b = 1;
    h = 1;
    l = 5;
    o = 2e8;
    F = 0:1e5:5e7;
    Mel = b*h^2*o/6;
    eps = 0.1;
    x = 0:.1:2.5;
    C2 = zeros(length(F), 1);
    for j = 1:length(F)
        C = zeros(length(x), 1);
        M = x.*F(j)./2;
        Kappa = M;
        nsteps = size(M,2);
        for i=1:nsteps;
            ds = 0.;
            s = 12.*M(i)/(E*b*h^3);
            if (M(i)>Mel)
                mkappa = b*o/12.*(3.*h^2-4.*o^2/(E^2*s^2))+s*b*Ee/24.*(h-2.*o/E/s)^2*(2.*h+2.*o/E/s);
                    while abs(M(i)-mkappa) > eps
                    mscaroukappa = 2./3.*b*o^3/E^2/s^3+1./12.*Ee*b*(h^3-8.*o^3./E.^3/s^3);
                        ds = (M(i)-mkappa)/mscaroukappa;
                        s = s + ds;
                        mkappa = b*o/12.*(3.*h^2-4.*o^2/(E^2*s^2))+s*b*Ee/24.*(h-2.*o/E/s)^2*(2.*h+2.*o/E/s);
                    end
            end
            Kappa(i) = s;
            C(i) = -s.*0.1.^2;
        end
        Y = A\C;
        C2(j) = Y(end);
    end
plot(F, C2)    view(90,270)
```

Vykreslení



$F(10^7 N)$
x 10^7
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
$w(m)$

Závěr