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Dedication

We dedicate this book to Prof. Jiří Šejnoha, who is a principal representative of research in the field of reliability analysis and uncertainty modelling in the Czech Republic. We would like to thank him for his patient guidance, understanding, stimulating suggestions and encouragement through whole our carrier as well as for his help with the organization of this symposium.

Editorial board

*“Not to be absolutely certain is,
I think, one of the essential things
in rationality.”*

Bertrand Russell

Contents

Preface

This volume comprises the extended abstracts of contributions presented at the 1st International Symposium on Uncertainty Modelling in Engineering (ISUME 2011) held in Prague, Czech Republic from 2 to 3 May 2011. Topic of the symposium is highly relevant today, as the complexity and demand on engineering systems increase while the society still deals with limited amount of resources. Engineers are asked to provide solutions with optimal level of performance and safety with respect to the costs. The optimum cannot be found without a systematic analysis of related uncertainties and risks. Sophisticated but also practically applicable models for such analyses are needed, recent developments in this field have been presented on the symposium. The symposium is intended to establish a tradition of regular meetings of researchers in the field of uncertainty modelling and risk and reliability analysis. The meeting aims to deepen their mutual cooperation and to facilitate the discussion for a better understanding and management of uncertainty and risk in all aspects of engineering.

The overall theme of the event is probabilistic modelling of complex systems and optimization of decisions under uncertainties. In particular, it focuses on the following topics:

- Uncertainty quantification in engineering problems
- Probabilistic modelling in material engineering
- Information updating with measurements and investigations
- Bayesian statistics
- Stochastic processes
- Stochastic finite elements
- Reliability analysis and structural safety
- Risk-based design and life-cycle management
- Natural hazards modelling

The abstracts are also available on the symposium webpage:
`http://klobouk.fsv.cvut.cz/~anicka/isume/isume.html`

Modeling Uncertainty with Emphasis on Non-Stochastic Approaches

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Two main areas of modeling uncertainty can be distinguished: stochastic methods and non-stochastic methods. The former are widely used and can deliver results that are strong and invaluable in applications. The latter do not seem so widely used even though, for example, fuzzy set theory has gained increased attention and has materialized into numerous applications.

However, the strong results of the stochastic methods are usually not for free. Often, rather detailed information on the probabilistic features of input data is required to guarantee sufficiently exact probabilistic conclusions about output data. In practice, the required input information can be hard to obtain, or its obtaining is too expensive, or the information is even unavailable.

Non-stochastic approaches to uncertainty are less demanding regarding the knowledge about input data features and relationships. To give an extremal example, let us take the worst (case) scenario method (WSM) [9] that, unlike stochastic methods or fuzzy set theory, does not attribute any weight to individual members of input data set. As a consequence, the outputs are also unweighted, which can be considered a drawback in applications.

However, a closer look reveals that the worst scenario approach is an integral part of methods using weighted input data. Even in the stochastic methods, the analyst should be aware of the "be-on-the-safe-side" rule that is essential in many engineering problems, and, consequently, should consider the least favorable probability distribution among the distributions that are relevant to the available input data set.

Let us take two representatives of non-stochastic approaches, namely fuzzy set theory [1, 6, 8, 11] and the Dempster-Shafer evidence theory [1, 6, 7, 10]. In the former approach, items of input data are weighted by some possibility value between 0 and 1, in the latter one, (sub)sets of input data are weighted by values between 0 and 1 that roughly resemble a sort of probability.

To fix ideas, let us assume that we search for $u(a) \in V$, the solution to a problem represented by an operator equation $A(a)u = f(a)$ where both the operator A and the right-hand side f depend on a that represents an input parameter whose value is uncertain but belonging to $\mathcal{U}_{\mathsf{ad}}$, a known set of admissible input values. The solution $u(a)$ is evaluated by $\Psi(a)$, a quantity of interest; take, for example, a local temperature, a local mechanical stress, or the energy norm. In the WSM, we search for

$$a^0 = \arg\max_{a \in \mathcal{U}_{\mathsf{ad}}} \Psi(a) \quad \text{or/and} \quad a_0 = \arg\min_{a \in \mathcal{U}_{\mathsf{ad}}} \Psi(a). \tag{1}$$

The range of $\Psi|_{\mathcal{U}_{\mathsf{ad}}}$ is determined by

$$I_\Psi = [\Psi(a_0), \Psi(a^0)]. \tag{2}$$

In the two non-stochastic approaches mentioned above, the ultimate goal is to infer the weight of $\Psi(a)$ if a belongs to a weighted set $\mathcal{U}_{\mathsf{ad}}$. It can be shown that in achieving this goal, (1)-like problems have to be solved on $\mathcal{U}^w_{\mathsf{ad}} \subset \mathcal{U}_{\mathsf{ad}}$ where $\mathcal{U}^w_{\mathsf{ad}}$ comprises inputs relevant to weight $w \in (0, 1]$. To construct the representation of the weight attributed to $\{\Psi(a) : \ a \in \mathcal{U}_{\mathsf{ad}}\}$, (2)-like ranges I^w_Ψ are used.

It is also worth noting that efforts have been made to transform stochastic problems to deterministic problems. Indeed, with the help of the Karhunen-Loève expansion, stochastic differential equations can be transformed into multi-dimensional problems [2, 3, 4, 5].

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Optimal Decision Making with Incomplete Information or What's the Point of Modeling Uncertainty in Engineering?

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Every good engineer is aware that his/her models cannot predict with certainty. However, few engineers employ a systematic (and quantitative) approach to dealing with these uncertainties. To date, probabilistic modeling and reliability analysis of engineering systems has found limited application in civil engineering practice, despite its success in the academic world. This talk starts out by reviewing how – in an ideal world – the engineer should consider uncertainty in the design and management of civil systems, utilizing the framework of Bayesian decision theory. This is illustrated by two applications of the theory in practice, on the assessment of avalanche risks and the optimization of inspections of deteriorating structures. A special focus will be put on how uncertainty arising from incomplete information is modeled and managed through Bayesian approaches. The talk will conclude with a discussion on what is needed (and what not) to enhance the relevance of risk and reliability tools in engineering practice.

Parametric Problems, Uncertainty Quantification, and Model Reduction

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Parameter dependent problems — one variant of which are stochastic problems with associated uncertainty quantification — posed in the form of partial differential equations lead upon discretisation to very high-dimensional problems. For many computations it would therefore be advantageous to reduce the model complexity.

All those problems share an inherent common structure based on decomposition of certain operators and subsequent possibility of representation.

The problems alluded to are naturally posed in tensor product spaces, and this property is used here in the model reduction process. Our aim is to determine the model reduction while we compute the solution. The ultimate goal is to actually reduce the model input, operate on the reduced model, and only compute a solution in reduced format.

Recent Developments of Fractional Calculus Based Models in Engineering

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The idea of real order differentiation was already present in the first speculations of the pioneers of the 17th century who contributed to the development of what nowaday is known as ordinary differential calculus. Although the latter undoubtedly represents the foundation of applied science, its limits became evident in dealing with physical phenomena exhibiting memory. Loosely speaking, this drawback is the consequence of the local character of ordinary derivatives which refer to limit properties in the neighbourhood of a point.

Parallel to the development of the ordinary differential calculus many mathematicians contributed to set the theoretical framework for its generalization, elaborating on a question De L'Hospital addressed to Leibnitz on 30 September 1695: "What if n be $1/2$ (in (d^n/dx^n))?". The corpus of theory which consents to calculate generalized derivatives, with n being a real or even a complex number, born from that famous question, is known as "Fractional Calculus" [1].

In this talk, some recent results on new models and methods based on the fractional calculus will be presented. In contrast with the local nature of integer derivatives, the non-local features of fractional derivatives will be highlighted. Moreover, a generalization of Taylor expansion [2], relevant in many applications, will be sketched.

It will be shown that fractional derivatives are suited to describe: i) physical problems with memory, such as in the description of stochastic visco-elastic oscillators excited by white noises processes [3]; ii) long-range effects in non-local continuum mechanics [4]; iii) long-correlated univariate and multivariate stochastic processes [5] and their digital generation as output of fractional differential equations [6].

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Pragmatic Probabilistic Models for Quantification of Tunnel Excavation Risk

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This communication is focused on the prediction of risks due to hardly predictable geotechnical conditions affecting the construction phase of the tunnel. To begin with, a probabilistic concept and categorization of failures emerging during tunnel excavation are addressed. Then a simple model is proposed to capture both the time dependent response of a surface structure to the tunnel construction process and to a cave-in accident. To predict risks, a pragmatic solution approach is outlined and a lucid methodology is proposed. Two specific problems are discussed in the sequel. First, a stochastic approach based on the Poisson process is presented to entertain discrete geotechnical parameters typical of jointed rocks. Another problem, engineers have to handle, relates to continuously changing geotechnical parameters, such as, e.g. a variable depth of rock overburden. This problem is treated by utilizing the Gaussian process.

The following types of failure will be discussed [1]: (i) Extensive deformations of the tunnel tube, (ii) exceeding of acceptable progress of the subsidence trough, (iii) cave-in collapse, (iv) occurrence of a tunnel segment surrounded by a suddenly weakened rock, (v) occurrence of overburden with randomly diminishing thickness. Disturbance of water regime in the surroundings is a very dangerous source of failure which, however, will not be discussed in detail in this paper.

This paper is aimed at proposing a methodology allowing us to describe the probable effect of subsidence trough on the ground surface that gives rise a surface structure to fail. Its shape can be approximated by a stochastic function in the form (Fig. 1)

$$W(x,y) = \overline{W}(x,y) + w^*(x,y), \tag{1}$$

where $\overline{W}$ is a deterministic function covering the global shape of the subsidence trough and reflecting the effect of soil-structure interaction; w^* describes random surface fluctuations. Considering this assumption, the subsidence trough can be expressed as

$$W(x;y) = \overline{W}(0;0)[g_1(x) + g^*][g_2(y) + g^*] \cong \overline{W}(x;y) + w^*[g_1(x) + g_2(y)], \tag{2}$$

where $\overline{W}(x,y) = \overline{W}_0 g_1(x) g_2(y)$, $w^* = \overline{W}_0 g^*$.

Deterministic functions g_1 and g_2 are to be estimated either by computer simulation or with the help of subsidence measurement. Note that $g_1(0) = g_2(0) = 1$. These functions tend to diminish with the distance from the tunnel heading. In Eq. 2, g^* is a random variable.

The loading effect due to differential settlement with respect to three selected points $x - d_x$; x; $x + d_x$ can be expressed by the second order difference (see Fig. 1)

$$\Delta_{2x} W(x;0) \cong \Delta_{2x} \overline{W}(x;0) + \Delta_{2x} w^*(x). \tag{3}$$

Substituting Eq. 2 into 3, written for both directions x and y, yields

$$\Delta_{2x} W(x;y) \cong \frac{\overline{W}_0 d_x^2}{2} g_1^{''}(x) g_2(y),\ \Delta_{2x} w^* \cong \overline{W}_0 g^* \frac{d_x^2}{2} g_1^{''}(x) = w^* \frac{d_x^2}{2} g_1^{''}(x), \tag{4}$$

$$\Delta_{2y} W(x;y) \cong \frac{\overline{W}_0 d_y^2}{2} g_1(x) g_2^{\bullet\bullet}(y),\ \Delta_{2y} w^* = w^* \frac{d_y^2}{2} g_2^{\bullet\bullet}(y). \tag{5}$$

On this premise, the rate of fluctuations is stationary throughout the whole subsidence trough. The surface structure fails if any value of loading effects, $\Delta_{2x} W$ and/or $\Delta_{2y} W$, exceeds the structure resistance, R_x and/or R_y, , that can be treated as the limit surface curvature the structure is able to sustain. Considering this state to apply to the segment located at a distance x from the tunnel heading (Fig. 1) allows the probability of failure, conditioned by this position, to be obtained as

$$p_f(x) = p_{fx}(x) + p_{fy}(x) - P\left[R_x < \Delta_{2x} W(x;0) \cap R_y < \Delta_{2y} W(x;0)\right]. \tag{6}$$

The conditional probabilities of structure failure are defined by the well known formulae [2]:

$$p_f(x) = P\left[R_x < \Delta_{2x} W(x;0)\right] = \\ = \int\limits_{\text{range of } w^*} F_{Rx}\left[\left(\overline{W}_0 + w^*\right) g_1^{''}(x) \frac{d_x^2}{2}\right] f_{W^*}(w^*) \mathrm{d}w^*, \tag{7}$$

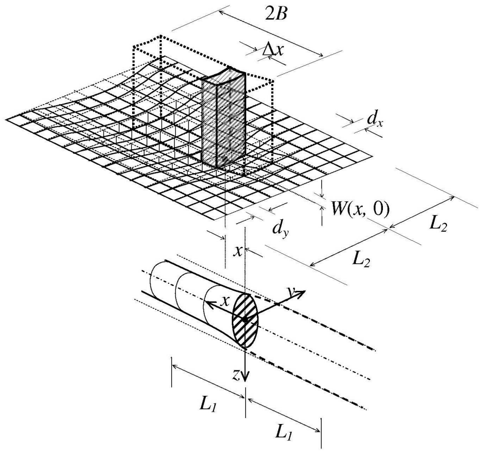

Figure 1: Development of subsidence trough giving rise to surface structure deformation and damage (L_1, L_2 indicate the extent of influence zone)

$$p_f(y) = P\left[R_y < \Delta_{2y} W(x;0)\right] = \\ = \int\limits_{\text{range of } w^*} F_{Ry} \left[\left(\overline{W}_0 g_1(x) + w^*\right) g_2^{\bullet\bullet}(0) \frac{d_y^2}{2} \right] f_{W^*}(w^*) \mathrm{d}w^*, \tag{8}$$

where F_{Rx} and F_{Ry} are the distribution functions of structure resistance in directions x and y evaluated for $\Delta_{2x}W(x;0)$ and $\Delta_{2y}W(x;0)$, respectively; f_{W^*} is the probability density function of a random fluctuation w^*.

Assuming x to be uniformly distributed on the interval $\langle -L_1 - B,\ L_1 + B \rangle$, the expression for unconditional probability reads

$$p_f = \frac{1}{2(L_1 + B)} \int\limits_{-L_1-B}^{L_1+B} p_f(x) \mathrm{d}x. \tag{9}$$

This paper suggests theoretical instruments making possible to analyze most serious problems tunnel engineering has to face. All the phenomena discussed within the scope of this paper have been recently met during the excavation of the Blanka tunnel in Prague [3].

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Statistical Modelling of the Thickness of Tunnel Primary Lining Using Random Functions and Principal Component Analysis

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Development of modern measurement techniques enables collecting of more and more data. From the statistical point of view, methods that are able to recognize valuable information amongst huge amount of data are needed.

A methodology and results of analysis of measurements of the tunnel primary lining thickness will be presented. Each profile of the tunnel was measured in 60 points from left to right clockwise. Later, the number of measured points was reduced to 53 points. The distance between measured profiles along the tunnel axis was 1m. Data were obtained in form of repeating measurements of 53-dimensional vectors. The statistical analysis aimed to describe changes of the primary support thickness along the tunnel axis. The dimensionality of the problem may be reduced by the method of principal components. The method is especially powerful in cases, when the high dimensional vector can be represented by a smooth function.

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Dynamic Bayesian Network Model for Assessment of Tunnel Excavation Risk

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Reliable estimates of construction cost and time of infrastructure projects are crucial information for decision makers during the planning and construction process. At present, these estimates are mostly obtained deterministically by means of expert judgement. This approach neglects the uncertainty associated with the estimates and leads to unexpected cost/time escalations. Real construction costs of transport infrastructure projects constructed over the past 70 years were on average 28% higher than the cost forecasted at the time of deciding to build [4]. New methods, which would enable a quantification of uncertainties associated with these estimates are therefore needed. They should not be based solely on expert judgement, but should ideally be based on systematic learning from past projects. In our experience, the expert estimates are reliable in case of assessing mean values of commonly observed variables, but for the determination of their variances and for the quantification of parameters describing rare events, a statistical analysis of historic data is necessary.

The uncertainty level connected with tunnel projects is high because of the limited knowledge of geotechnical conditions in which the tunnel is to be built. Existing probabilistic models quantifying the risk of tunnel construction utilize Monte Carlo simulation [6, 3, 9], Bayesian networks [10], artificial neural networks [1] or analytical solutions [5]. Most of these models describe in detail the uncertainties in the prediction of geotechnical conditions and common variations of performance rates or unit costs, but in general they fail to consider the impact of other factors. These include extraordinary events (e.g. cave-in collapse, fires, flooding) as well as human and organizational factors.

A new model utilizing dynamic Bayesian Networks (DBN) for probabilistic modelling of tunnel construction process that attempts to overcome the above mentioned gaps is developed. The excavation process, which is commonly modelled as a spatial Markov process [2], is discretized into a Markov chain for representation by means of DBN. Each slice of the DBN represents a section of the tunnel with length Δl. The model includes three types of uncertain factors: uncertain geotechnical conditions (GC), common variations in construction performance (CP) and extraordinary events (EE) such as cave-in collapses, flooding or fires. The modelling of EE is based on a Poisson model as described in [11]. Based on these variables, the full probability distributions of construction time (T_{cum}) and cost (C_{cum}) is calculated. The generic DBN model is depicted in Fig. 1(a).

The novel feature of the presented model is that it explicitly considers the quality of planning and construction as an important but hardly quantifiable factor introducing strong dependencies into the project. Graphical representation of the DBN model enables better communication of the model assumptions to tunnelling professionals. In addition, DBN facilitates updating of predictions during the construction phase, when observations about real geotechnical conditions, costs and advance rates are available. For exact inference of the DBN model, a

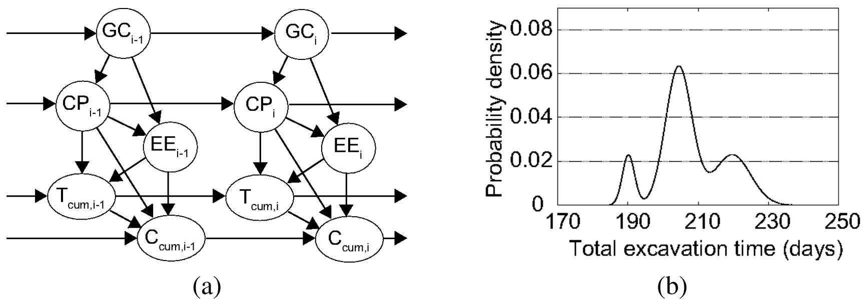

Figure 1: (a) Scheme of a generic DBN model for tunnel construction process; (b) PDF of total excavation time for a tunnel section with length of 610 m

modified frontier algorithm is applied. The frontier algorithm [8] requires discretization of all random variables in the DBN. Its modified version enables to efficiently deal with large number of states of the random variables T_{cum} resp. C_{cum}.

The model is applied to a case study taken from [7]. Results obtained with a simplified model without consideration of risk of extraordinary events, as well as a sensitivity analysis studying the effect of different assumptions, are presented in [12]. An example of the estimated total excavation time is depicted in Fig. 1(b).

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Direct Optimized Probabilistic Calculation - DOProC Method

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Direct Optimized Probabilistic Calculation - DOProC method [1, 3, 4] has been developed as an alternative for Monte Carlo in the assessment of structural reliability in probabilistic calculations. The original name of the method was Direct Determined Fully Probabilistic Calculation (DDFPC), which means that the calculation procedure for a certain task is clearly determined by its algorithm, while Monte Carlo generates calculation data for simulation on a random basis. It, however, followed from a number of consultations and discussions that the word ”determined” is somewhat misleading. The method requires high-performing information systems for complex tasks. Therefore, efforts have been made to optimize calculations in order to reduce the number of operations, keeping, at the same time, reliable calculation results. Chances of optimizing the calculation steps seem to be extensive. Having consulted the issue with experts in construction reliability (Šejnoha, Novák, Keršner, Teplý 2009), the name of the method was made more precise and reads now Direct Optimized Probabilistic Calculation - DOProC. Input random quantities (such as the load, geometry, material properties, or imperfections) are expressed as histograms in the calculations.

In the probabilistic calculations, all input random variables are combined with each other. The number of possible combinations is equal to the product of classes (intervals) of all input variables. With rather many input random variables, the number of combination is very high. Only a small portion of possible combinations results, typically, in failures. When DOProC method is used, the calculation takes too much time, because combinations are taken into account that does not contribute to the failure. Efforts to reduce the number of calculation operations have resulted into the development of algorithms that provide the numerical solution of the integral that defines formally the failure probability with rather many random variables:

$$p_f = \int_{D_f} f\left(X_1, X_2, ..., X_\mathrm{n}\right) \mathrm{d}X_1, \mathrm{d}X_2, ..., \mathrm{d}X_\mathrm{n}, \tag{1}$$

where D_f represents a failure area where $g(X) \leq 0$, $f\left(X_1, X_2, ..., X_\mathrm{n}\right)$ for the function of the combined density of probabilities of random quantities $X_1, X_2, ..., X_\mathrm{n}$.

The algorithms are implemented into the ProbCalc software - package of three program utilities [5, 6, 7], which is possible to download in lite versions on web pages [2].

The software is very useful for a lot of probabilistic calculations, for the assessment of structural reliability included. The random input variables can be expressed by means of histograms with parametric and non-parametric distribution created from sets of random quantities that have been measured or observed.

One part of theoretical science and practice according probabilistic concept of DOProC method is focused into the probabilistic calculation of fatigue crack propagation of steel structures and bridges under fatigue stress [8, 9, 10]. Solution leads to the probabilities of three basic random events in dependence on years of structure's operation and fatigue crack propagation. On the basis of that calculation for each individual year, determined by analysis of reliability function, the dependence of the failure probability on time of the bridge's operation is specified. When the limit reliability is known, it is possible to determine times of the structure's inspections using conditional probability.

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Spatial Reliability Assessment of Deteriorating Reinforced Concrete Slabs with Monitoring and Inspection Data

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Introduction

Concrete structures are subject to a variety of deterioration processes. A widespread deterioration mechanism is the corrosion of the reinforcement induced by chloride ions. The influencing factors (cover depth of the reinforcement, surface concentration of chloride ions and concrete properties) are subject to spatial random variability. Therefore, they should ideally be modeled by spatial random fields, which becomes particularly relevant, when inspection and monitoring data of the structure are taken into account. In this presentation, a method for Bayesian updating of the probability of corrosion based on such data is demonstrated – here: measurements of cover depth and chloride concentration at certain locations. This method was introduced by [2].

Description of the Method

Consider F to be the event of the initiation of the corrosion process at the reinforcement and let $\mathbf{X} = [X_1, X_2, ..., X_n]$ be the basic random variables with joint probability density function $f_{\mathbf{X}}(\mathbf{x})$ influencing the deterioration function. The event F is usually described by the failure domain $\Omega_F = \{g(\mathbf{x}) \leq 0\}$, in which $g(\mathbf{x})$ is the limit state function. The probability $\Pr(F)$ of F is calculated by means of structural reliability methods (SRM) as[1]

$$\Pr(F) = \int_{\Omega_F} f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\mathbf{x} \quad . \tag{1}$$

When observations Z of the process are available from measurements and inspections, these can be described by a domain Ω_Z in the space of $\mathbf{X}$. The probability of the event F can then be updated with the new information using the

[1]Note that the expression in equation 1 is a common abbreviation for the expression $\int\int ... \int_{\Omega_F} f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}x_1 \mathrm{d}x_2 ... \mathrm{d}x_n$.

definition of conditional probability

$$\Pr(F|Z) = \frac{\Pr(F \cap Z)}{\Pr(Z)} = \frac{\int_{\mathbf{x} \in \{\Omega_F \cap \Omega_Z\}} f_{\mathbf{x}}(\mathbf{x})\,\mathrm{d}\mathbf{x}}{\int_{\mathbf{x} \in \Omega_Z} f_{\mathbf{x}}(\mathbf{x})\,\mathrm{d}\mathbf{x}} \,. \tag{2}$$

The domain Ω_Z is also described by limit state functions $h(\mathbf{x}, \epsilon)$. Let $s(\mathbf{X})$ be any system characteristic to be assessed by a measurement with outcome s_m. As the measurement method is usually not exact, a measurement error ϵ must be considered, with probability distribution $f_\epsilon(\epsilon)$. The limit state function $h(\mathbf{x}, \epsilon)$ can be written as

$$h(\mathbf{x}, \epsilon) = s(\mathbf{x}) - s_\mathrm{m} + \epsilon \,. \tag{3}$$

In this case, the information is of the equality type, since the domain is defined as $\Omega_Z = \{h(\mathbf{x}, \epsilon) = 0\}$. However, SRM are suitable to calculate the probability of events that are defined by domains Ω of the inequality type, $\Omega_Z = \{h(\mathbf{x}, \epsilon) \leq 0\}$. For this reason, $h(\mathbf{x}, \epsilon) = 0$ is transformed to an equivalent inequality information following [2]. It is noted that information Z on the parameters $\mathbf{X}$ received from inspections or measurements can be described by a likelihood function $\mathcal{L}(\mathbf{x}) \propto \Pr(Z|\mathbf{X} = \mathbf{x})$. For the above example, this likelihood is

$$\mathcal{L}(\mathbf{x}) = f_\epsilon(s_\mathrm{m} - s(\mathbf{x})) \,. \tag{4}$$

Using the likelihood, an equivalent inequality event Z_e can be defined. To this end, a uniformly distributed random variable P in the interval $[0, 1]$ is introduced. The event $Z_e = \{P \leq c \cdot \mathcal{L}(\mathbf{x})\}$ is defined and the corresponding limit state function is

$$h_\mathrm{e}(\mathbf{x}, p) = p - c \cdot \mathcal{L}(\mathbf{x}) \,. \tag{5}$$

It is shown in [2] that the use of the equivalent inequality event $Z_e = \{h_\mathrm{e}(\mathbf{x}, p) \leq 0\}$ is identical to the use of the original event $\Omega_Z = \{h(\mathbf{x}, \epsilon) = 0\}$ in equation 3. To calculate the integrals in equation 2, importance sampling techniques are applied, since solution methods like first or second order reliability methods (FORM/SORM) may not be suitable in case of highly non-linear limit state functions. With this in mind, one finds

$$\Pr(F|Z) = \frac{\int_{\mathbf{x},p \in \{\Omega_F \cap \Omega_{Z_e}\}} f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\mathbf{x}\,\mathrm{d}p}{\int_{\mathbf{x},p \in \Omega_{Z_e}} f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\mathbf{x}\,\mathrm{d}p} \tag{6}$$

$$\approx \frac{\sum_{i=1}^{n_S} \mathrm{I}[h_e(\mathbf{x}_i, p_i) \leq 0]\mathrm{I}[g(\mathbf{x}_i) \leq 0] \frac{f_{\mathbf{X}}(\mathbf{x}_i)}{\psi(\mathbf{x}_i, p_i)}}{\sum_{i=1}^{n_S} \mathrm{I}[h_e(\mathbf{x}_i, p_i) \leq 0] \frac{f_{\mathbf{X}}(\mathbf{x}_i)}{\psi(\mathbf{x}_i, p_i)}} \,, \tag{7}$$

wherein $\psi(\mathbf{x}, p)$ is the sampling density of $\mathbf{X}$ and P, n_S is the number of generated samples, and $\mathrm{I}[\bullet]$ is the indicator function, which returns one if the condition $[\bullet]$ is fulfilled, and zero otherwise. In [2], it is shown that $\psi(\mathbf{x}, p)$ is given by a product ansatz $\psi(\mathbf{x}, p) = \psi_1(\mathbf{x}) \cdot \psi_2(p|\mathbf{x})$, where $\psi_1(\mathbf{x})$ is the sampling PDF of $\mathbf{X}$ and $\psi_2(p|\mathbf{x}) = \frac{1}{c \cdot \mathcal{L}(\mathbf{x})}$ represents an optimal conditional sampling density of P for all $p \in [0; c \cdot \mathcal{L}(\mathbf{x})]$ when $\mathbf{X} = \mathbf{x}$ is given.

Application to Corrosion Inspections

The ingress of chloride ions is commonly described using Fick's second law of diffusion. The concentration of chloride ions is thus written

$$C(z,t) = C_{\mathrm{S},i} \cdot \operatorname{erf}\left(\frac{W_i}{\sqrt{4D_i \cdot t}}\right) , \tag{8}$$

where $C_{\mathrm{S},i}$ is the surface chloride concentration, W_i is the cover thickness of the concrete, and D_i denotes the diffusion coefficient. They are combined through the error function $\operatorname{erf}(\bullet)$ and can thus be seen as demand S. To calculate the

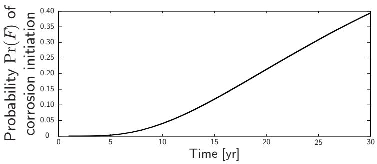

Figure 1: Probability $\Pr(F)$ of corrosion initiation over the time t (SORM solution), adopted from [1].

probability $\Pr(F)$ of corrosion initiation, the concentration $C(z,t) = S$ has to be compared to the resistance, the critical concentration $R = C_{\text{crit}}$, at which the corrosion process will be initiated. Thus, the failure domain Ω_F may be defined as

$$\Omega_F = \{R - S \leq 0\} = \{C_{\text{crit}} - C(z,t) \leq 0\} , \tag{9}$$

which now allows the calculation of equation 1. The probability $\Pr(F)$ of the failure event F – corrosion is just initiated – is calculated using SORM (figure 1). Since no information is available a-priori, $\Pr(F)$ is equal at each location i. The Bayesian updating following equation 7 is now performed using information Z from assessment of the structure. Here, after ten years of service of the parking deck, the concrete cover depth is continuously measured on the one hand, and the concentration of chloride ions at certain discrete locations j, which are shown in

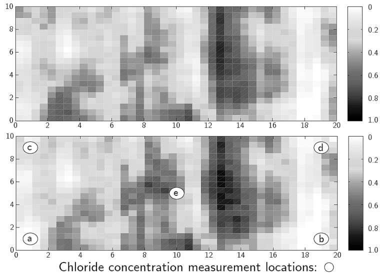

Figure 2: Probability $\Pr(F|Z)$ of corrosion initiation at time $t = 15$ [yr] conditional on measurement of the cover depth and on the measurement of the chloride concentration (a–e), executed at time $t = 10$[yr], adopted from [1].

figure 2, is assessed in various depths on the other hand. With this information, the probability of corrosion initiation conditional on cover depth measurement as well as chloride concentration measurement is computed for each point i on the surface (figure 2).

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A Risk Based Approach for the Robustness Assessment of Timber Roofs

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In the recent years, various collapses of wide span roofs occurred in Northern Europe during winter under high snow loads, several of them were built with timber elements (solid or glulam timber). As several studies highlighted [2, 4], such failures mostly originate from errors made during the design phase, followed by errors made during the execution, while failures due to material deficiencies or maintenance are relatively uncommon.

The goal of this study is to investigate the behavior of a wide span timber roof, whose secondary structure (purlins) is designed according to three different structural configurations, which were already the subject of a previous deterministic analysis carried out by Dietsch & Winter [1], and to compare the performance of the three different configurations with respect to reliability, robustness and risk.

In the first part of the study, reported in [5], the failure of primary beams is not considered and the risk assessment is performed by considering (probabilistically) all possible failure scenarios for purlin elements. The assessment accounts for the possibility of systematic errors (which are modeled by weakened sections that occur randomly in the secondary structure) in order to include the possibility of a systematic weakening of the structure, which can be due to errors in the production and/or construction process. The second part of the study deals with possible failure mechanisms of the primary beams and with the consequences of these failures on the entire system.

The risk associated with structural failure of the secondary system is considered to be proportional to the failed area of the roof A_F. Since there is no interest in computing absolute values of the risk, the risk can be defined as:

$$\text{Risk} = \mathrm{E}[A_F] = \int_0^{A_{\text{roof}}} a f_{A_F}(a)\mathrm{d}a, \tag{1}$$

where E[ ] denotes the expectation operation and $f_{A_F}(a)$ is the probability density function (PDF) of the failed area.

Computations are performed with Monte Carlo Simulations (MCS), that enable the evaluation of the full distribution of the damaged area, while the sensitivity of the results on the probabilistic model, is assessed by means of First-Order Reliability Method (FORM).

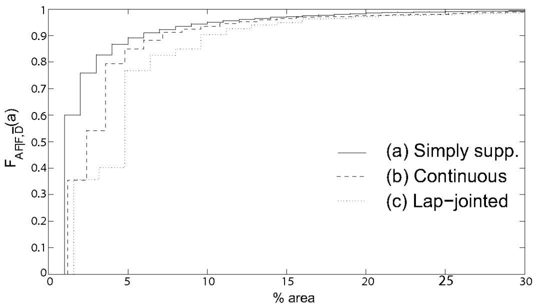

Figure 1: $F_{A_F|F,\overline{D}}(a)$ for the three systems without systematic errors.

Figure 1 shows the computed CDF of the failed area A_F conditional on the system having failed F and on the absence of systematic errors $\overline{D}$, $F_{A_F|F,\overline{D}}(a)$: a failure in the structural system with simply supported purlins (a) results in smaller damages than the other configurations. In the statically indeterminate configurations, progressive collapse mechanisms lead to a larger number of purlin failures once the first section has failed. EN 1991-1-7 includes a requirement that a failure should not lead to a failed area in excess of 15% of the total area. To check this "robustness" criterion, we calculate the probability that the failed area exceeds 15% of the total area, given a failure occurs, Table 1. Finally, the risk, which is defined as the expected size of the failed area $\mathrm{E}[A_F]$ in Eq. 1, is summarized in Table 2.

Purlin configuration	$1 - \mathrm{F}(A_F = 15\% \mid F)$
(a) Simply supp.	0.027
(b) Continuous	0.035
(c) Lap-Jointed	0.032

Table 1: Probability of the failed area exceeding 15% of the total area upon failure (for Pr(D)=0.1) in 50 years.

Purlin configuration	$\mathrm{E}[A_F]$, Pr(D) $= 1\%$	$\mathrm{E}[A_F]$, Pr(D) $= 10\%$
(a) Simply supp.	$1.3 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$
(b) Continuous	$0.8 \cdot 10^{-3}$	$0.9 \cdot 10^{-3}$
(c) Lap-Jointed	$0.8 \cdot 10^{-3}$	$0.9 \cdot 10^{-3}$

Table 2: Expected value of area failed $(\%)$, for two different values of the probability of systematic errors Pr(D).

From the robustness requirement, it could be argued that structural system configuration (a), consisting of simply supported purlins, is the optimal one, because a failure in this configuration leads to the smallest failed area (Figure 1) and it has the lowest probability of not fulfilling the 15%-area requirement (Table 1). However, the risk calculated for configuration (a) is higher than for configurations (b) and (c), which are statically indeterminate (Table 2). This is due to the fact that the probability of system failure is higher for configuration (a), even though the consequences are lower. Therefore, it is argued that despite the fact that configuration (a) is more robust, this study indicates that configurations (b) and (c) are more optimal.

The combined evaluation of the state of the system considering the interaction between beam and purlin is still object of current analysis. This analysis is important to understand the real behavior of the system and to evaluate the probability of progressive collapse of the timber roof.

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Coping With Uncertainty in Water Resources Problems Using Reliability Approach

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Surface and subsurface hydrologic systems are highly dynamic and variable. Material properties, such as hydraulic conductivity, can vary over 10 orders of magnitude with significant heterogeneity. Hydraulic stresses, such as rainfall, vary both spatially and temporally in all scales. However, the present stage of technology typically only allows local-scale data to be collected at selected locations, resulting in substantial uncertainty in the behavior of the surface water and groundwater systems. Even human water demands are uncertain. As a result, water resources professionals are constantly facing the challenge and risk of making decisions in the presence of uncertainties.

Many water resources problems are concerned with achieving specific performance goal(s). For example, the chemical concentration in a water production well is required to be smaller than allowable maximum contaminant level. A way to manage such problem is to formulate a solution using a reliability approach whereas failure is defined as not meeting the performance goal(s). This presentation will use two examples to illustrate the use of reliability approach to cope with uncertainties in water resources problems.

The first example involves using a reliability approach for probabilistic finite element analysis of contaminant transport. It focuses on evaluating the probability that the concentration at a compliance location exceeds a risk-based limit. Uncertainty boundary and initial conditions as wells as material properties are considered as spatially correlated random fields which are represented by a mesh of random variables and direct local-scale data. The values at the Gaussian points of the finite elements are calculated using optimal linear estimator. The reliability analysis is performed by importance sampling simulation centered at the design points obtained by the Hasofer-Lind first-order reliability method. In addition to the failure probability, importance sampling simulations generate other near-failure probability distributions. The sensitivity derivatives involving the transport equation are computed by the adjoint method. Conditioning on indirect data are incorporated using a Bayesian approach.

The results suggest that for obtaining a design point accurate enough for importance sampling simulation, first-order reliability method can be performed using a relatively coarse random variable mesh. Approximating a random variable space by an eigen-subspace can further improve the efficiency of FORM. To represent the local correlation structures in higher resolution in the subsequent importance sampling simulations, the mesh can be refined, particularly in the sensitive regions. Optimal linear estimator converts the design point for the refined mesh from the coarse mesh. In general, importance sampling simulations is significantly more efficient than the Monte Carlo simulation for the same level of accuracy.

The second example involves an adaptive reliability-based water resources management framework that utilizes stochastic optimization techniques to account for uncertainties associated water demand, water availability, and material properties. Many of these uncertainties are caused by uncertainties in prediction of future climate conditions. The framework was developed to manage water resources from groundwater production wells, stream flow withdrawal, regional reservoir, and a desalination plant, while protecting ecology and preventing seawater intrusion. The developed method maximizes the reliability of achieving the goal that eco-hydrologic condition is healthy.

The framework involves (1) a distribution system simulation model to represent the water supply operation, (2) a Monte Carlo simulation model to generate realizations of climatic events, water demand, available surface water quantity, and (3) integrated hydrologic modeling of groundwater-surface water system. A response model simulates how water supply operators adjust the optimized rates of groundwater extraction, surface water withdrawal, and reservoir inflow/outflow to meeting the water demand. The reliability optimization problem is solved using a differential evolutionary algorithm.

Delays in Construction Tasks

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Delays in construction are a very common and ancient problem. Different studies have tried to analyze the problem from several viewpoints. The aim of this paper is to show a distinct approach analyzing the curve of productivity vs. time of the one task, giving it a stochastic touch based on systematic sampling of semi random numbers generated by a spreadsheet using multiple simulations to identify the most reliable time within a percentage of confidence making use of the beta distribution because of its flexibility and closer description of construction problems. The paper also includes the method of virtual management momentum simulation or marginal differences which are the comparison between the graphs of production, speed of production and acceleration where it is possible to see the risk present in the curves because of their unpredictable behavior.

Inverse Reliability Analysis in Structural Design

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The presence of uncertainties in engineering systems has always been recognized in the analysis and design of structures. Uncertainties are involved in every part of system Structure – Load – Environment (e.g. material properties, geometrical imperfections, dead load, live load, wind, snow, humidity, corrosion rate, etc.). Traditional approaches simplified the problem by considering the uncertain parameters to be deterministic, and accounted for the uncertainties through the use of empirical safety factors. Safety factors are usually derived based on the past experience. But, they cannot absolutely guarantee required reliability level; they do not provide information on the influence of individual parameters on reliability. Also it is difficult (almost impossible) to design structures with uniform reliability levels among components.

To determine the values of design parameters related to particular limit states design (both ultimate and serviceability) "trial and error" procedure is generally used. Design parameters (material properties, geometry, etc.) are changed in order to satisfy specified limit states. The problem leads to a sort of optimization procedures.

The task to achieve target reliability levels, expressed by theoretical failure probabilities or reliability indexes, is more difficult. Reliability problem is generally described by limit state function and basic random variables. Design parameters can be deterministic or they can be associated to random variables described by statistical moments and probability distribution functions (PDF). They affect the failure probability – the measure which cannot be easily calculated and require a special approximation or simulation techniques (e.g. [1]).

Some sophisticated approaches were proposed under the name "inverse reliability methods" in the past. Reliability calculation is usually based on approximation procedures like FORM as these inverse techniques require repetitive calculation of reliability and calculation of reliability even by advanced simulation techniques of Monte Carlo type is generally extremely time-consuming.

Therefore FORM, originally proposed by Hasofer and Lind [2], is a feasible alternative despite of its potential inaccuracy when dealing with highly nonlinear safety margins.

Inverse reliability problem can be formulated as follows. Suppose the safety margin Z and the limit state function $g(\cdot)$ in the original basic space of random variables $\mathbf{X}$ is:

$$Z = g(\mathbf{X}). \tag{1}$$

Theoretical failure probability p_f is expressed as:

$$p_f = P(Z \leq 0). \tag{2}$$

In inverse reliability problem design variables can be deterministic or random ones. Therefore we include additionally to the vector of basic random variables $\mathbf{X} = X_1, X_2, \ldots, X_i, \ldots, X_n$ also the vector of design deterministic parameters $\mathbf{d} = d_1, d_2, \ldots, d_k, \ldots, d_p$ and the vector of design parameters of random variables $\mathbf{r} = r_1, r_2, \ldots, r_l, \ldots, r_q$. Note, that design parameters of random variable can be statistical moments of first and/or second order. To consider higher statistical moments as design parameters is mathematically possible but useless from practical point of view.

In case of multiple limit states we have several safety margins Z_j and target failure probabilities $p_{f,j}$, where $j = 1, 2, \ldots, m$. The inverse problem can be stated generally as:

Given: $p_{f,j}$
Find: $\mathbf{d}$ or/and $\mathbf{r}$
Subject to: $Z_j = g(\mathbf{X}, \mathbf{d}, \mathbf{r})_j = 0$ for $j = 1, 2, \ldots, m$.

The target failure probabilities $p_{f,j}$ can be substituted by reliability indexes β_j which makes the inverse reliability problem numerically more feasible to solve. One of the advanced inverse reliability methods is the artificial neural network (ANN) based method proposed by authors. The general methodology has been developed and applied by authors for material parameters identification [6, 4] and damage identification [5]. It is now extended and applied for inverse reliability problems too. ANNs were already used for inverse reliability problems by some authors [3]. The procedure suggested here is very efficient in the sense of small-sample simulation used for training ANN.

For inverse reliability analysis a double stochastic analysis is required. The first analysis is used to randomize design parameters in order to prepare training set for ANN, the second one is used to calculate reliability measure (reliability index or failure probability) for given realization of design parameters obtained from the first one.

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Optimization of the Target Reliability Level in Engineering

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The target reliability levels specified for various engineering works may efficiently reflect current requirements on the sustainability in construction and other technical fields, and significantly contribute to the need of meeting growing demands with limited resources. Recommendations offered in various national and international documents are often inconsistent, indicating target reliabilities in a broad range for different reference periods. As a rule no explicit link between the design working life and target reliability level is provided.

The contribution attempts to clarify the relationship between the design working life and the reliability index and to provide guidance for specification of the target reliability level for given consequences, design working life and discount rate. The theoretical study based on probabilistic optimization is supplemented by practical recommendations. It appears that the optimum reliability indexes depend primarily on the ratio of cost of failure (malfunctioning costs) and the cost per unit of structural parameter, less significantly on the design working life and discount rate.

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Conventional Probabilistic Models for Uncertainty Modelling in Civil Engineering

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Probabilistic models of basic variables used in studies of structural reliability are often significantly different. Obviously, the reliability studies may then lead to dissimilar results.

The present paper aims to summarize and propose conventional probabilistic models to enable an efficient comparison of reliability studies of various structural members made of different materials. Proposed models are intended to be used as prior theoretical models that could be accepted for general reliability studies of civil engineering structures. The models include frequently used probabilistic distributions of action and material properties of common structures executed under normal quality control.

The proposed models are applicable for time-invariant and selected time-variant basic variables. Recent scientific publications and working documents of international organisations including JCSS [1] and CIB documents are taken into account. An example of reliability analysis of a generic steel member exposed to different load conditions illustrates practical use of recommended models.

The proposed conventional models reflect normal conditions only. In the reliability analysis of a particular structure probabilistic models should be specified taking into account actual loading, structural conditions and relevant experimental data. The proposed models may be considered as prior models to be updated by available experimental data.

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Uncertainty Updating in Description of Coupled Transport Processes in Heterogeneous Materials

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Structural durability is highly influenced by heat and moisture transport. Proper estimation of the durability, therefore, needs a reliable and probabilistic simulation of transport processes. The reliability of such simulation is related to the suitable identification of material parameters. To include some expert knowledge (e.g. positivity or realistic bounds of parameters) together with experimental results, one can employ an updating procedure, where prior distributions based on expert's recommendations are updated by measurements in order to obtain more reliable a posteriori distributions [4]. This procedure usually involving Monte Carlo sampling is however very computationally demanding.

Here, the novel stochastic computational techniques like generalized polynomial chaos expansion (PCE) are used for much faster updating procedure by replacing the computationally expensive forward simulation via a FE-program by the PCE via a stochastic FE-computation [3]. This inexpensive PCE-forward model can then be used in the Bayesian updating as in the [1].

The presented uncertainty updating techniques are applied to the numerical model of coupled heat and moisture transport [2] for heterogeneous material, where the particular transport coefficients are not spatially constant and can be described by correlated random fields.

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Bayesian Identification for Non-Gaussian Parameters

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Inverse problems and identification procedures are known to lead to ill-posed problems in the sense of Hadamard when considered in a deterministic setting. In a probabilistic Bayesian setting, on the other hand, they are well-posed. In the simplest setting of a linear system and Gaussian randomness this leads to the well-known Kalman Filter (KF) procedures. They are also the simplest kind of linear Bayesian updates. Extensions to nonlinear or non-Gaussian settings which are based on linearisation like the extended Kalman Filter (EKF) are only of limited applicability. Without linearisation, they invariably involve some kind of sampling, e.g. in the form of ensemble Kalman Filter (EnKF), particle filters, or Markov chain Monte Carlo (MCMC) methods.

Here we cast the probabilistic identification problem in a functional approximation setting - the best known of which is the polynomial chaos expansion (PCE) - and the linear Bayes form of updating. In this way the identification process can be carried out completely deterministically. In the case where the original problem was a deterministic identification task it additionally provides a quantification of the remaining uncertainty in a Bayesian setting. But it can also be used as an identification procedure in an originally (frequent) probabilistic setting. We give numerical examples of both.

A Deterministic Filter for non-Gaussian Bayesian Estimation

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We present a fully deterministic method to compute sequential updates for stochastic state estimates of dynamic models from noisy measurements. It does not need any assumptions about the type of distribution for either data or measurement — in particular it does not have to assume any of them as Gaussian. It is based on a polynomial chaos expansion (PCE) of the stochastic variables of the model. We use a minimum variance estimator that combines an a priori state estimate and noisy measurements in a Bayesian way. For computational purposes, the update equation is projected onto a finite-dimensional PCE-subspace. The resulting Kalman- type update formula for the PCE coefficients can be efficiently computed solely within the PCE. As it does not rely on sampling, the method is deterministic, robust, and fast.

In this presentation we discuss the theory and practical implementation of the new method. The original Kalman filter is shown to be a second-order special case. In a first experiment, we perform a bi-modal identification using noisy measurements. Additionally, we provide numerical experiments by applying it to the well known Lorenz-84 model and compare it to a related method, the ensemble Kalman filter.

Parameter Identification Using the Skewed Kalman Filter

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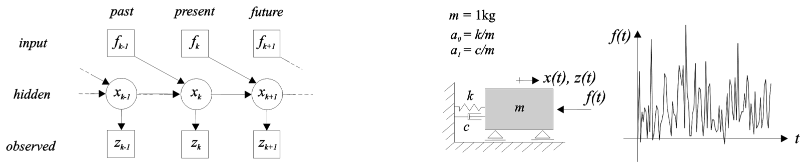
Introduction

In an Bayesian Approach the Kalman filter can be regarded as recursive Bayesian estimator and be described as Bayesian dynamic network as shown in Fig. 1(a) where $\mathbf{x}_k$, $\mathbf{f}_k$, $\mathbf{z}_k$ denote the hidden state, the system input and the incoming measurement, respectively. The linear dynamic system discretized in the time domain



(a) (b)

Figure 1: (a) Dynamic Bayesian network ; (b) SDOF system excited by a skewed process

follows a first order hidden Markov process where uncertainties in the system model and the measurement model are assumed to be Gaussian and modeled as uncorrelated white noise processes. As the assumption of linearity and Gaussianity is often violated an Bayesian approach of an extended skewed Kalman filter is derived which allows to consider a nonlinear dynamic system excited by wind which is described as process with skew-normal probability distributions.

Closed skew-normal distribution (CSN)

The skewed Kalman filter is based on the CSN which allows to model skewness while preserving the advantageous properties of the Gaussian distribution as the closure under conditioning, linear transformations and marginalization [1, 2, 3].

For $m \geq 1$, $n \geq 1$, $\boldsymbol{\mu} \in \mathbb{R}^n$, $\boldsymbol{\nu} \in \mathbb{R}^m$, an arbitrary matrix $\mathbf{D} \in \mathbb{R}^{m \times n}$ and positive definite covariance matrices $\mathbf{\Sigma} \in \mathbb{R}^{n\times n}$ and $\mathbf{\Delta} \in \mathbb{R}^{m\times m}$ the CSN of a n-dimensional vector $\mathbf{X} \in \mathbb{R}^n \sim \mathrm{CSN}_{n,m}(\boldsymbol{\mu}, \mathbf{\Sigma}, \mathbf{D}, \boldsymbol{\nu}, \mathbf{\Delta})$ is given by

$$p_{n,m}(\mathbf{x}) = C^{-1}\phi_n(\mathbf{x}; \boldsymbol{\mu}, \mathbf{\Sigma})\Phi_m(\mathbf{D}(\mathbf{x}-\boldsymbol{\mu}); \boldsymbol{\nu}, \mathbf{\Delta}) \tag{1}$$

with $C = \Phi_m(\mathbf{0}; \boldsymbol{\nu}, \mathbf{\Delta} + \mathbf{D}\mathbf{\Sigma}\mathbf{D}^{\mathrm{T}})$ where $\phi_n(*; \boldsymbol{\eta}, \mathbf{\Omega})$ and $\Phi_n(*; \boldsymbol{\eta}, \mathbf{\Omega})$ are the probability density function and the cumulative distribution function (CDF) of a n-dimensional normal distribution with mean vector vector $\boldsymbol{\eta} \in \mathbb{R}^n$ and covariance matrix $\mathbf{\Omega} \in \mathbb{R}^{n\times n}$. The matrix $\mathbf{D}$ regulates the skewness of the distribution

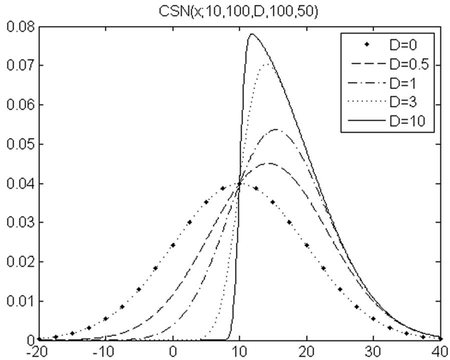


Figure 2: CSN distribution

and allows to vary continuously from the normal PDF $(\mathbf{D} = \mathbf{0})$ to a half normal distribution (Fig. 2), whereas the constraints $\boldsymbol{\mu}$ and $\mathbf{\Sigma}$ are location and scale parameters. The remaining parameters $\boldsymbol{\nu}$, $\mathbf{\Delta}$ and C ensure the closure of the CSN under conditioning, marginalization and under summation of independent CSN random variables [2].

Extended skewed Kalman Filter (EsKF)

The EsKF is a modification of the standard extended Kalman Filter (EKF) in order to introduce skewness to the measurement model. In [1] the linear state space model is modified by splitting up the observational model in a linear part and a skewed part

$$\mathbf{z}_k = \mathbf{G}_k\mathbf{x}_k + \mathbf{v}_k \equiv \underbrace{\mathbf{A}_k\mathbf{u}_k}_{\text{linear}} + \underbrace{\mathbf{B}_k\mathbf{s}_k}_{\text{skewed}} + \underbrace{\mathbf{v}_k}_{\text{noise}} \tag{2}$$

where $\mathbf{G}_k = [\mathbf{A}_k, \mathbf{B}_k]$ and $\mathbf{x}_k = [\mathbf{u}_k^{\mathrm{T}}, \mathbf{s}_k^{\mathrm{T}}]^{\mathrm{T}}$ with the vectors $\mathbf{s}_k \in \mathbb{R}^s$, $\mathbf{u}_k \in \mathbb{R}^u$ and the matrices of scalars $\mathbf{A}_k \in \mathbb{R}^{n\times s}$, $\mathbf{B}_k \in \mathbb{R}^{n\times u}$. Here the linear vector

$\mathbf{u}_k$ as well as the zero-mean measurement noise $\mathbf{v}_k$ is assumed to be Gaussian distributed and independent of the skewness vector $\mathbf{s}_k$ with a CSN distribution.

In the investigated identification problem, the method is used to determine the system response due to an unknown ambient load function with CSN distribution. Hence the linear vector corresponds to the homogeneous part of the system response while the skewed vector describes the steady state response due to a skewed process describing the load. In order to generate a skewed process the following lemma to be found in [3] is used: If $\mathbf{X} \in \mathbb{R}^n$ and $\mathbf{Y} \in \mathbb{R}^m$ are two random variables with joint normal distribution

$$\begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \end{bmatrix} \sim N_{n+m}\left(\begin{bmatrix} \boldsymbol{\mu} \\ \boldsymbol{\nu} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma} & -\boldsymbol{\Sigma}\mathbf{D}^{\mathrm{T}} \\ -\mathbf{D}\boldsymbol{\Sigma} & \boldsymbol{\Delta} + \mathbf{D}\boldsymbol{\Sigma}\mathbf{D}^{\mathrm{T}} \end{bmatrix} \right)$$

$$\mathbf{X}|\mathbf{Y} \leq \mathbf{D}\boldsymbol{\mu} \sim \mathrm{CSN}_{n,m}(\boldsymbol{\mu}, \boldsymbol{\Sigma}, \mathbf{D}, \boldsymbol{\nu}, \boldsymbol{\Delta}) \tag{3}$$

then the conditional distribution of $\mathbf{X}$ given $\mathbf{Y} \leq \mathbf{D}\boldsymbol{\mu}$ is skew normally distributed. Defining

$$\mathbf{u}_k = \mathbf{T}_k \mathbf{u}_{k-1} + \mathbf{B}_{u,k}\mathbf{w}_{u,k} \qquad \mathbf{y}_k = -\mathbf{L}_k \mathbf{y}_{k-1} + \mathbf{w}_{y,k} \tag{4}$$

where $\mathbf{L}_k \in \mathbb{R}^{s\times s}$, $\mathbf{T}_k \in \mathbb{R}^{u\times u}$ are matrices of scalars, $\mathbf{w}_{u,k}$ and $\mathbf{w}_{y,k}$ are independent Gaussian distributed noise vectors and $\mathbf{s}_k \sim \mathbf{y}_k|\mathbf{y}_{k-1} \leq \mathbf{D}_{y,k}\boldsymbol{\mu}_{y,k}$ follows a $\mathrm{CSN}(\boldsymbol{\mu}_{s,k}, \boldsymbol{\Sigma}_{ss,k}, \mathbf{D}_{s,k}, \boldsymbol{\nu}_{s,k}, \boldsymbol{\Delta}_{s,k})$ distribution, where

$$\begin{aligned} \boldsymbol{\mu}_{s,k} &= -\mathbf{L}_k\boldsymbol{\mu}_{y,k-1} + \boldsymbol{\mu}_{w_{y,k}} & \boldsymbol{\Sigma}_{ss,k} &= \mathbf{L}_k\boldsymbol{\Sigma}_{yy,k-1}\mathbf{L}_k^{\mathrm{T}} + \boldsymbol{\Sigma}_{w_{y,k}} \\ \mathbf{D}_{s,k} &= \boldsymbol{\Sigma}_{yy,k-1}\mathbf{L}_k^{\mathrm{T}}\boldsymbol{\Sigma}_{ss,k}^{-1} & \boldsymbol{\nu}_{s,k} &= \boldsymbol{\mu}_{y,k-1} - \mathbf{D}_{s,k}\boldsymbol{\mu}_{s,k} \\ \boldsymbol{\Delta}_{s,k} &= \boldsymbol{\Sigma}_{yy,k-1} - \mathbf{D}_{s,k}\boldsymbol{\Sigma}_{ss,k}\mathbf{D}_{s,k}^{\mathrm{T}} \end{aligned} \tag{5}$$

In contrast to the linear filter the prior time update of the state $\mathbf{x}_k = (\mathbf{u}_k, \mathbf{s}_k)^{\mathrm{T}}$ cannot be calculated directly as $\mathbf{s}_k$ has to be generated at each time step from the joint normal distribution $(\mathbf{y}_k, \mathbf{y}_{k-1})^{\mathrm{T}}$.

The filter is initialized by the multivariate normal distributed posterior PDF of $\mathbf{u}_{k-1}$, $\mathbf{y}_{k-1}|\mathbf{z}_{1:k-1}$ known from the previous time step. In the prediction step the prior estimate of the multivariate normal distributed variable $p(\mathbf{u}_k, \mathbf{y}_k, \mathbf{y}_{k-1}|\mathbf{z}_{1:k-1})$ conditional on the observations $\mathbf{z}_{1:k-1}$ up to time $k-1$ is determined using Eq. 4. After observing $\mathbf{z}_k$, the measurement likelihood $p(\mathbf{z}_k|\mathbf{u}_k, \mathbf{y}_k, \mathbf{y}_{k-1}, \mathbf{z}_{1:k-1})$ is obtained from the error $\mathbf{e}_k = \mathbf{z}_k - \mathbf{A}_k\overline{\boldsymbol{\mu}}_{u,k-1} - \mathbf{S}_k\mathrm{E}[\mathbf{s}_k|\mathbf{z}_{1:k-1}]$ between the predicted and the incoming measurement using the

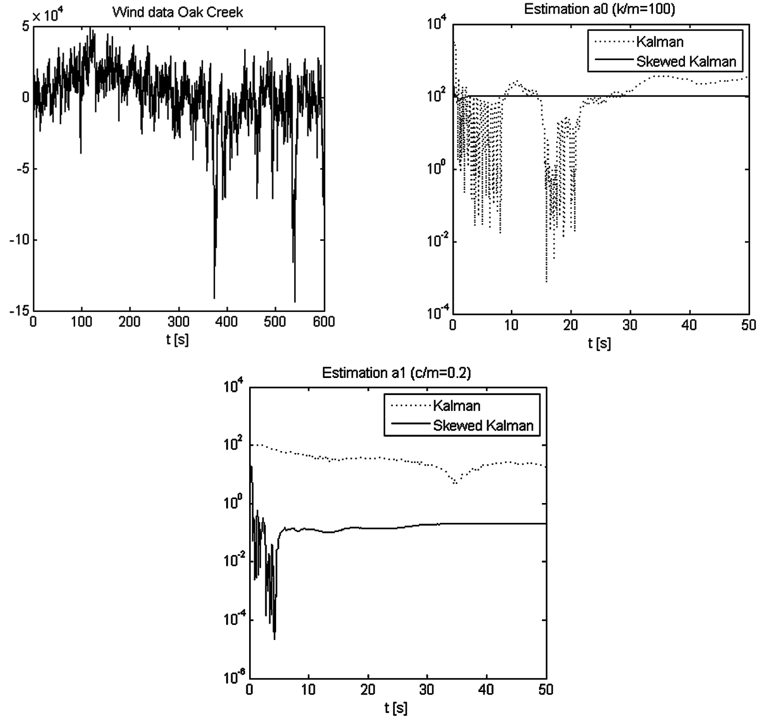


Figure 3: Wind data from Oak Creek wind farm site; identified values of the stiffness and damping parameters a_0 and a_1 using the EKF (red dashed) and EsKF (blue line)

conditional expectation $\mathrm{E}[\mathbf{s}_k|\mathbf{z}_{1:k-1}]$ of the priori PDF. Applying the Bayes' rule the posterior PDF of the updated parameters of the linear $\mathbf{u}_k|\mathbf{z}_{1:k}$ and the skewed part $\mathbf{y}_k|\mathbf{z}_{1:k}$ can be derived [1]. This leads to Eq. 6

$$
\begin{aligned}
\mathbf{y}_k|\mathbf{z}_{1:k} &\sim \begin{cases} \boldsymbol{\mu}_{u,k} = \mathbf{T}_k \boldsymbol{\mu}_{u,k-1} + \mathbf{B}_{u,k} \boldsymbol{\mu}_{w_{u,k}} + \overline{\boldsymbol{\Sigma}}_{uu,k} \mathbf{A}_k^{\mathrm{T}} \boldsymbol{\Sigma}_{zz,k}^{-1} \mathbf{e}_k \\ \boldsymbol{\Sigma}_{uu,k} = \overline{\boldsymbol{\Sigma}}_{uu,k} - \overline{\boldsymbol{\Sigma}}_{uu,k} \mathbf{A}_k^{\mathrm{T}} \boldsymbol{\Sigma}_{zz,k}^{-1} \mathbf{A}_k \overline{\boldsymbol{\Sigma}}_{uu,k} \end{cases} \\
\mathbf{u}_k|\mathbf{z}_{1:k} &\sim \begin{cases} \boldsymbol{\mu}_{y,k} = -\mathbf{L}_k \boldsymbol{\mu}_{y,k-1} + \boldsymbol{\mu}_{w_{y,k}} + \mathbf{C}_k \mathbf{B}_k^{\mathrm{T}} \boldsymbol{\Sigma}_{zz,k}^{-1} \mathbf{e}_k \\ \boldsymbol{\Sigma}_{yy,k} = \overline{\boldsymbol{\Sigma}}_{yy,k} - \mathbf{C}_k \mathbf{B}_k^{\mathrm{T}} \boldsymbol{\Sigma}_{zz,k}^{-1} \mathbf{S}_k \mathbf{C}_k \end{cases}
\end{aligned}
\tag{6}
$$

Finally the PDF of $\mathbf{s}_k|\mathbf{z}_{1:k}$ is obtained from the PDF of $\mathbf{y}_k$, $\mathbf{y}_{k-1}|\mathbf{z}_{1:k}$ by applying Eq. 5.

Numerical example

The damped SDOF system (Fig. 1(b)) is excited by a wind load with skewed PDF which is approximated by a CSN distribution. The wind field time series was downloaded from the internet database: "Database of Wind Characteristics" located at DTU, Denmark, (`www.winddata.com`) from the Oak Creek wind farm site. Using simulated noisy measurement data of the displacement the stiffness parameter $a_0 = k/m$ and damping parameter $a_1 = c/m$ are identified using the EKF (red dashed line) and the EsKF (blue line) as shown in Fig. 3. In the former case the system input was modeled as zero-mean white noise process. The Table 1 shows the initial values and the identified parameters after $t = 600$ s.

	true values	**initial values**	**initial error**	**identified parameters** EKF	EsKF	**identification error** EKF	EsKF
$a_0[\mathrm{s}^{-2}]$	100	150	50%	20.4	102.0	7960%	2%
$a_1[\mathrm{s}^{-1}]$	0.2	0.3	50%	1.180	0.202	98%	0.2%

Table 1: Initial values and identification result after $t = 600$ s

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Low-rank Response Surface with Application in Numerical Aerodynamic

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Nowadays the trend of numerical mathematics is often trying to resolve inexact mathematical models by very exact deterministic numerical methods. The reason of this inexactness is that almost each mathematical model of a real world situation contains uncertainties in the coefficients, right-hand side, boundary conditions, initial data as well as in the computational geometry. All these uncertainties can affect the solution dramatically, which is, in its turn, also uncertain.

We demonstrate the usage of very common black-box numerical methods (Monte Carlo and collocation methods) for quantification of uncertainties. Uncertain parameters are sampled many times (can be hundred tausends in MC) and then for each sample the corresponding deterministic problem is solved. The solution is a vector with millions degrees of freedom. After that, all obtained solutions are used for post-processing, e.g. for computing statistics of interest.

To make post-processing fast, we offer a data compression technique which allows us to compress (via QR decomposition) the solution data (PCE coefficients, realisations of the solution) on the fly with a log-linear complexity and log-linear storage requirement. Then the resulting low-rank representation is effectively used for approximation of statistical moments of interest.

As an application we take an example from numerical aerodynamic. We demonstrate how uncertainties in the input parameters (the angle of attack and the Mach number) and in the airfoil geometry propagate in the solution. The solution is approximated in the low-rank data format.

Competitive Comparison of Load Combination Models

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Civil engineering structures are, as a rule, exposed to time-variant actions such as imposed or climatic loads. Selection of an appropriate model for load combinations may be one of key issues of reliability studies.

Models for load combinations are often based on transformation of the time-variant case into a time-invariant one using the rule proposed by Turkstra [1] that is considered to be particularly useful for probabilistic calibrations. Despite numerous applications, accuracy of Turkstra's rule seems to be insufficiently analysed yet. An exceptional study by Wen [2] reveals that the rule leads to unconservative results if applied strictly as proposed. For "low" failure probabilities (approximately less than 0.001-0.01), good estimates are obtained while quite seriously unconservative results are reported when the failure probability is rather "high" (say in the range from 0.01 up to 0.1).

The submitted study attempts to re-investigate and improve the above findings considering the combination of two independent variable actions, different load ratios and various types of processes. Reliability of a generic structural member is analysed focusing on serviceability as well as ultimate limit states. Results obtained by Turkstra's rule are critically compared with those based on FBC processes, Ferry Borges and Castanheta [3], and rectangular wave renewal processes with intermittencies.

It is indicated that Turkstra's rule and the FBC processes provide sufficiently accurate results for a wide range of reliability problems while the analysis based on an upper bound on the failure probability for the intermittent processes yields in some cases rather conservative estimates. However, it appears that the theoretical models of basic variables may affect predicted reliability levels more significantly than the technique applied in time-variant reliability analysis.

ACKNOWLEDGEMENT

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Global Resistance Factors for non-Linear Analysis of Reinforced Concrete Structures

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The design value of a structural resistance can be determined directly from the mean value of the resistance using an appropriate global resistance (safety) factor. Probabilistic methods of the theory of structural reliability are applied to verify reliability of reinforced concrete members designed using the safety formats provided in the new *fib* Model Code – global safety factor method and method of estimate of coefficient of variation of resistance (ECOV). In numerical studies reliability of structural members exposed to bending, shear and compression is analysed. In addition the global factors are derived by probabilistic methods to achieve the target reliability level according to EN 1990 for basis of structural design.

It appears that the global resistance factors may depend on the type of concrete members, their reinforcement ratio and considered model uncertainties. In common cases (reliability index 3.8) the following global resistance factors related to mean values of basic variables may be considered: 1.4 for beams exposed to bending and 1.7 for beams exposed to shear and centrically loaded columns. The global safety method given in the Model Code, based on the mean yield strength of reinforcement and reduced value of concrete strength, hardly leads to a balanced reliability level for members subjected to different load effects. The recommended value 1.27 is in most cases conservative, but for lightly reinforced members may not yield an adequate reliability level. The ECOV method, based on the assumption of lognormally distributed resistance, leads to similar results as obtained by the probabilistic method. Further research should be primarily devoted to establishing uncertainties related to applications of finite element models including user and model factors as well as uncertainties of additional parameters of the models.

ACKNOWLEDGEMENT

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Uncertainty Modelling Using Software FReeT

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A large number of efficient stochastic analysis methods have been developed during last years. The common feature of all methods is the fact that they require a repetitive evaluation (simulations) of the response or limit state functions. The development of reliability methods is from the historical perspective certainly a struggle to decrease an excessive number of simulations. In spite of the increasing capabilities of computer hardware using a large number of simulations is still a problem when dealing with computationally demanding tasks.

The objective of the contribution is to present methods and software for efficient statistical, sensitivity and reliability assessment implemented in FReeT software [1]. The attention is given to those techniques that are developed for analyses of computationally intensive problems like nonlinear FEM. Sensitivity analysis is based on nonparametric rank-order correlation. Statistical correlation is imposed by the the simulated annealing [2]. The most interesting applications of FReeT software will be presented.

State-of-the-art probabilistic algorithms are implemented in FReeT to compute the probabilistic response and reliability. FReeT is a modular computer system for performing probabilistic analysis developed mainly for computationally intensive deterministic modelling such as FEM packages, and any user-defined subroutines. The main features of the software are (version 1.5):

Response/Limit state function

- Closed form (direct) using implemented Equation Editor (simple problems)
- Numerical (indirect) using user-defined DLL function prepared practically in any programming language
- General interface to third-parties software using user-defined *.BAT or *.EXE programs based on input and output text communication files

- Multiple response functions assessed in same simulation run

Probabilistic techniques

- Crude Monte Carlo simulation
- Latin Hypercube Sampling (3 alternatives)
- First Order Reliability Method (FORM)
- Curve fitting o Simulated Annealing
- Bayesian updating

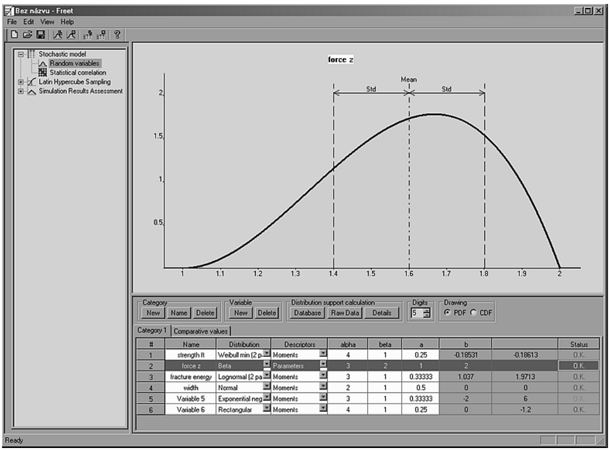

Figure 1: Window ”Random variables”

Stochastic model (inputs)

- Friendly Graphical User Environment (GUE)
- 30 probability distribution functions (PDF), mostly 2-parametric, some 3-parametric, two 4-parametric (Beta PDF and normal PDF with Weibullian left tail), Fig. 1.

- Unified description of random variables optionally by statistical moments or parameters or a combination
- PDF calculator o Statistical correlation (also weighting option)
- Categories and comparative values for PDFs
- Basic random variables visualization, including statistical correlation in both Cartesian and parallel coordinates

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Correlation Control in Monte Carlo Type Sampling: Theoretical Analysis and Performance Bounds

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The objective of this paper is twofold. Firstly to deliver theoretical bounds for performance of simulation techniques of Monte Carlo type measuring the ability to fulfill prescribed correlation matrices. Secondly, we study the performance in correlation control of recently proposed procedure for sampling from a multivariate population within the framework of Monte Carlo simulations [10, 11, 12] (especially Latin Hypercube Sampling). In particular, we study the ability of the method to fulfill the prescribed marginals and correlation structure of a random vector for various sample sizes. Two norms of correlation error are defined, one very conservative and related to extreme errors, other related to averages of correlation errors. We study behavior of Pearson correlation coefficient for Gaussian vectors and Spearman rank order coefficient (as a distribution-free correlation measure).

The paper starts with theoretical results on performance bounds for both correlation types in cases of desired uncorrelatedness. Firstly, the correlation errors (distance between the target and the actual correlation matrices) are studied for the case of random ordering of samples. The results for both correlation errors are based on the fact that a random correlation coefficient for a pair of random variables approximately follows Gaussian distribution. The errors for random vectors are then extended to the multivariate situations. These errors are understood to be the upper bounds on the mean performance of any algorithm because no algorithm should perform worse than just a random permutations (shuffling) of samples.

Lower bounds on the error are initially obtained by analyzing attainable values of correlation as they follow from analysis of the correlation estimation formulas.

For the cases when the sample size is less than the number of random variables, it is shown that the correlation matrix must be singular. Studies on spectral prop-

erties of these matrices helped us to derive lower bounds on the mean square error of the correlation in a closed form. Matrices that fulfill this optimality are shown to be non-unique and from all the possible solutions the one with minimal absolute correlation is shown to be preferable. The paper also proposes a simple mechanical model for dynamical simulations that is used to automatically deliver the lower bound on the matrix errors. Moreover, they deliver the whole optimal matrices. As an alternative, the task is also redefined as optimization problem and a numerical procedure of imposing a spectrum is used to solve the problem, too. The typical convergence of the correlation error for a fixed number of variables and increasing sample size is shown in Fig. 1.

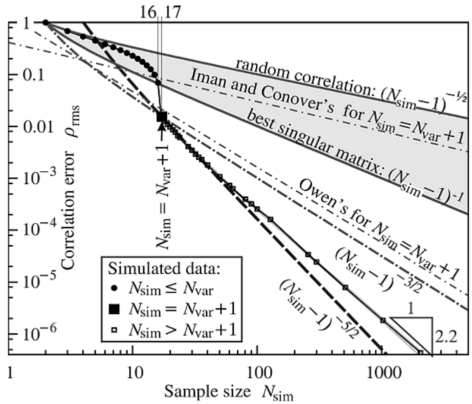

Figure 1: Typical performance plot obtained for $N_{\mathrm{var}} = 16$. Simulated data (averages are denoted by symbols and a solid line, minima and maxima by thin solid lines) are compared to theoretical bounds and results of other techniques.

As for the performance for cases when the sample size exceeds the number of random variables, the following observations have been made. It is shown that, under some circumstances, a very high rate of convergence can theoretically be achieved. These rates are compared to performance of other developed techniques for correlation control, namely the Cholesky orthogonalization as applied by Iman and Conover [3, 4]; and Owen's method [8] using Gram-Schmidt orthogonalization. We show that the proposed technique based on combinatorial optimization [10, 11, 12] yields much better results than the other known techniques. When correlated vectors are to be simulated, the proposed technique exhibits nearly the same excellent performance as in the uncorrelated case provided

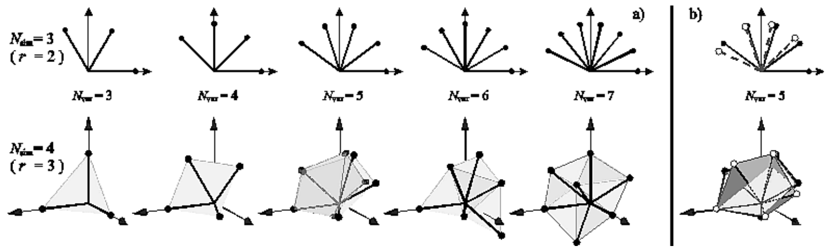

Figure 2: Visualization of optimal singular correlation matrices. a) $\mathbf{R}^{\mathrm{M}}$ (solid circles). All of these solutions are also $\mathbf{M}$ matrices except for $N_{\mathrm{var}} = 5$ and $N_{\mathrm{sim}} = 4$ (visualized with solid boxes). b) Examples of optimal correlation matrices $\mathbf{R}$ (empty circles) used compared with those from (a). Top row: $N_{\mathrm{sim}} = 3$ (dimension $r = 2$). Bottom row: $N_{\mathrm{sim}} = 4$ (dimension $r = 3$).

the desired vector exists. It is shown that the technique provide much wider range of acceptable correlations than the wide-spread Nataf [7] model [5] (known also as the Li-Hammond model [6] or the NORTA model [1]) and that it is also much more flexible than the Rosenblatt model [2, 9].

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A Multi-Scale Framework for Stochastic Analysis of Multi-Phase Material

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Modelling uncertainties in material is a common and open problem. The influence of inclusions inside of material, independent of the intention of their existance, needs to be understood in order to predict the strength of complex structures. The computational analysis of such effects implies many challenging problems for methods and models. In this work we present a proceeding which uses image data to derive a stochastic model for determining mechanical properties of multi-phase materials. In addition to the determination of such properties from images, we focus our work on highly efficient stochastic analysis methods like the Stochastic Galerkin Method [1, 2].

The presented approach incorporates measurements of geometrical material-phase distributions in micro-scale which are deduced from sequences of high-resolution cross-section images. On these images a segmentation algorithm is applied which maps local image properties to identify the local material phase.

These segmented images motivate a categorisation of similar areas, in our case we find fibre-layers of certain fibre orientation and matrix-layers which separate them. We use this information to build up categories for a decomposition of our input data. The introduction of categories helps in bring our analysis from micro-scale to a higher one [3]. This decomposition is also important, because the statistics of the geometry of void-inclusions is significantly influenced by the fibre orientation in their vicinity. The further analysis described is therefore applied on each of these categories.

We use a three dimensional polynomial ansatz for representing the geometrical distribution of phases in each block of our measured input. The block-size in this analysis step is chosen to be similar to the element-size which will be used in the analysis of coupons. This means that we use about 10^4 volume elements of our segmented input data for projecting them on 3D legendre ansatz polynomials.

The coefficients carry major information about each data block and provide their spacial coherency.

These coefficients are representing the phase distribution in each block in a relatively high dimensional space (in our application we end up with 196 dimensions). This information is highly redundant and needs to be compressed in a proper way. We do that by projecting the coefficient vectors onto an optimal orthogonal basis provided by a singular value decomposition. Our further stochastical analysis can now be made on a small set of variables with managable stochastics. The rate of compression from the input data to the coefficients we finally consider is about $1 : 10000$, while the most significant properties are still represented.

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Microstructurally-Informed Random Field Description: Case Study on Chaotic Masonry

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The goal of this contribution is to explore three numerical approaches to homogenization of inhomogeneous structures with moderate meso/macro-lengthscale ratio. The methods investigated include a representative of perturbation methods [1], the Karhunen-Loève expansion technique coupled with direct Monte-Carlo simulations and a solver based on the Hashin-Shtrikman variational principles [2]. In all cases, parameters of the underlying random field of material properties are directly derived via image analysis of the structure in question, by employing the tools of statistical description of random microstructures [3]. Added value as well as limitations of individual schemes are illustrated by a case study of elastic response of an irregular masonry panel.

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