Spatial Reliability Assessment of Deteriorating Concrete Slabs with Inspection Data

– ISUME 2011 –

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Failure Probability

A-Priori Failure Probability

- Joint probability density

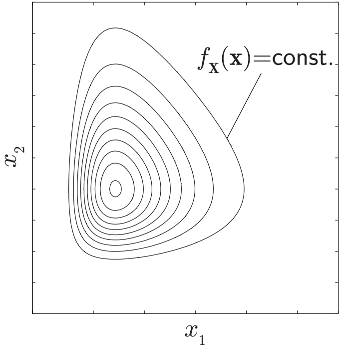
$$f_{\mathbf{X}}(\mathbf{x}) \quad , \quad \mathbf{X} = [X_1, X_2, ..., X_n]^{\mathrm{T}}$$

- Failure domain

$$\Omega_F = \{R(\mathbf{x}) - S(\mathbf{x}) \leq 0\} = \{g(\mathbf{x}) \leq 0\}$$

- Failure probability

$$\Pr(F) = \int_{\Omega_F} f_{\mathbf{X}}(\mathbf{x})\, \mathrm{d}\mathbf{x}$$



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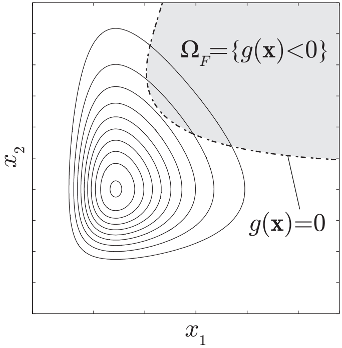

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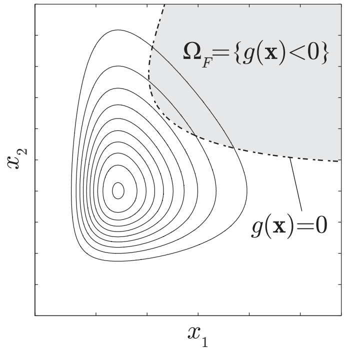

How can we get Information?

Information Z from

- Assessment
- Measurement
- Inspection
- Monitoring

$\rightarrow$ Expressed through mathematical functions

Information described by Domains Ω

- Information Z given by

  $\Omega_Z = \{h(\mathbf{x}) \leq 0\}$

  (**Inequality** Type)

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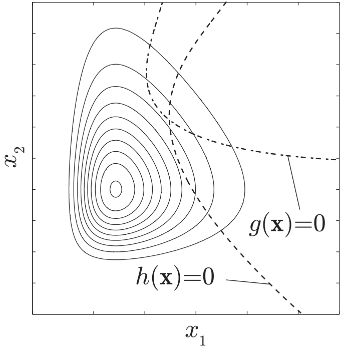

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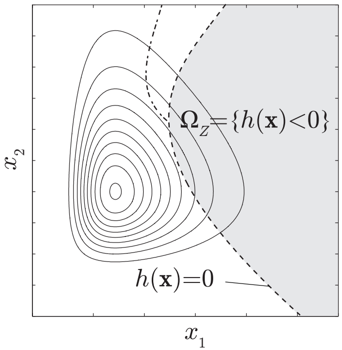

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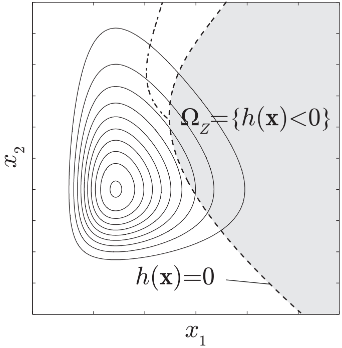

Updated Failure Probability

- Conditional probability

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- Probability update

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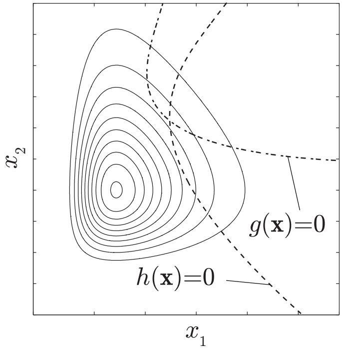

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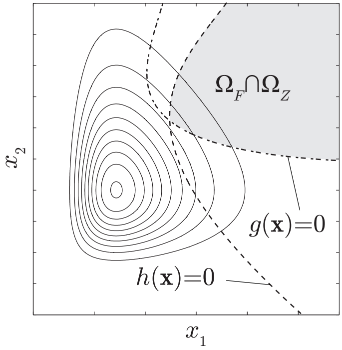

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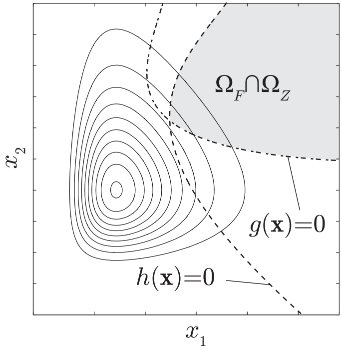

Measurement: Equality Information

- Measurement of system characteristic $S(\mathbf{X})$

  $h(\mathbf{X}, \epsilon_\mathsf{M}) = s(\mathbf{X}) - s_\mathsf{M} + \epsilon_\mathsf{M}$

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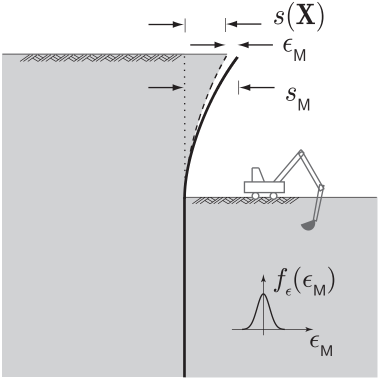

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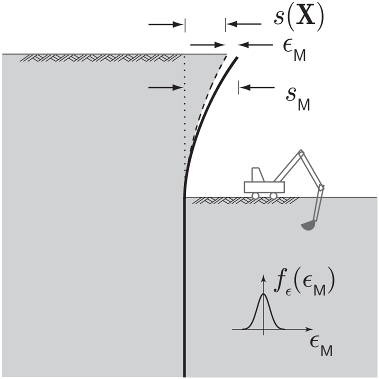

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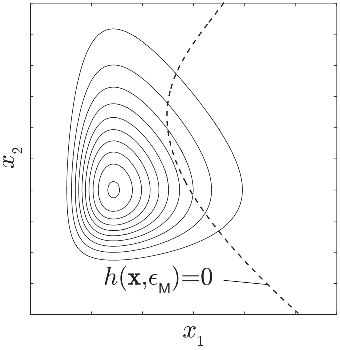

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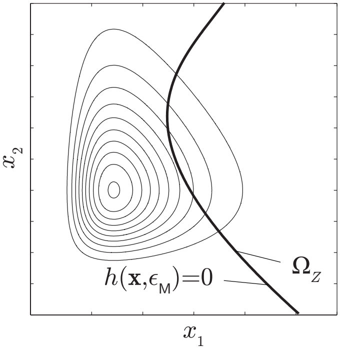

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How to Update?

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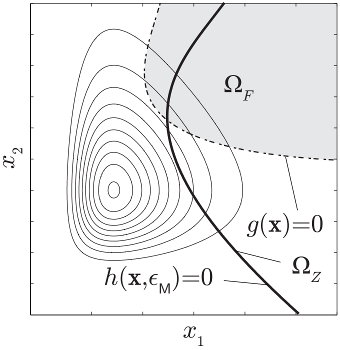

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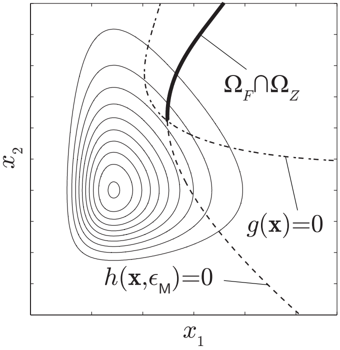

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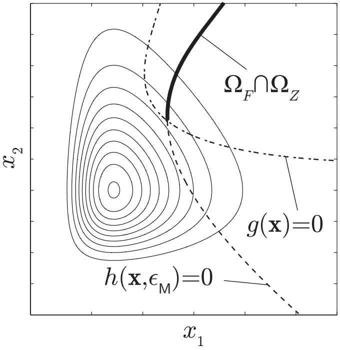

Summary on Information

What about the Update?

Résumé

- **Inequality** type information
  → any SR method **suitable** for probability update
- **Equality** type information
  → common SR methods **not suitable** for probability update

Introduction of Likelihood

- Likelihood describes information Z

$$\mathcal{L}(\mathbf{x}|Z) = a \cdot \Pr(Z|\mathbf{X}=\mathbf{x}) \propto \Pr(Z|\mathbf{X}=\mathbf{x})$$

- Express likelihood by error ϵ_{M}

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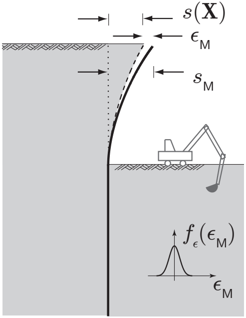

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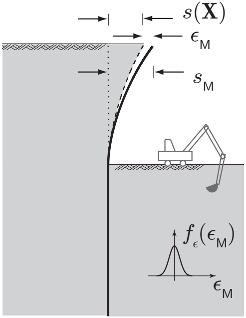

Preparations are necessary

- Introduce standard uniform RV U

$$f_U(u) = 1 \quad \forall \quad u \in [0; 1]$$

- Introduce constant c

$$0 \leq c \cdot \mathcal{L}(\mathbf{x}|Z) \leq 1$$

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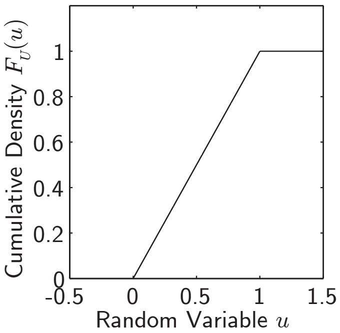

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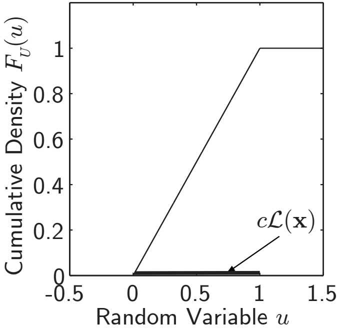

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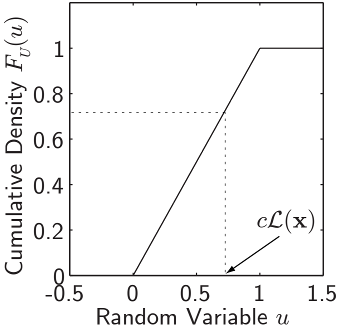

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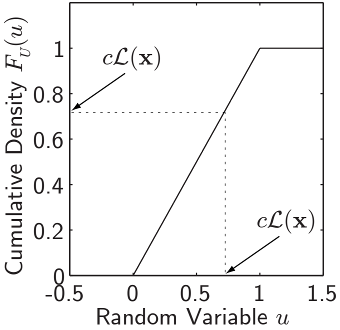

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Probability Update

Integration of $f_{\mathbf{X}}(\mathbf{x})$ by Importance Sampling

- Importance sampling solution

$$\Pr(F|Z) \approx \frac{\sum_{i=1}^{n_S} \mathrm{I}[h_e(\mathbf{x}_i, u_i) \leq 0]\mathrm{I}[g(\mathbf{x}_i) \leq 0] \frac{f_{\mathbf{X}}(\mathbf{x}_i)}{\psi(\mathbf{x}_i, u_i)}}{\sum_{i=1}^{n_S} \mathrm{I}[h_e(\mathbf{x}_i, u_i) \leq 0] \frac{f_{\mathbf{X}}(\mathbf{x}_i)}{\psi(\mathbf{x}_i, u_i)}}$$

- Optimal sampling density [Straub, 2010]

$$\psi(\mathbf{x}, u) = \psi_1(\mathbf{x}) \cdot \psi_2(u|\mathbf{x}) = \psi_1(\mathbf{x}) \cdot \frac{1}{c \cdot \mathcal{L}(\mathbf{x}|Z)}$$

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Application to Corrosion

- *Fick*'s law of diffusion (1D)

$$\frac{\mathrm{d}C(z,t)}{\mathrm{d}t} = D\frac{\partial^2 C(z,t)}{\partial z^2}$$

- Chloride concentration

$$C(z,t) = C_{\mathsf{S}}\left(1 - \mathrm{erf}\left(\frac{z}{2\sqrt{D \cdot t}}\right)\right)$$

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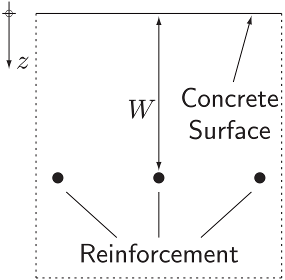
$$\Omega_F = \{C_{\mathsf{crit}} - C(W,t) \leq 0\}$$



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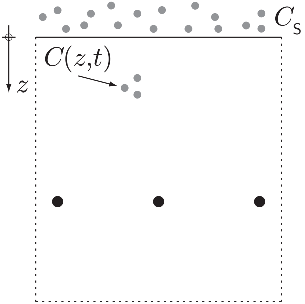

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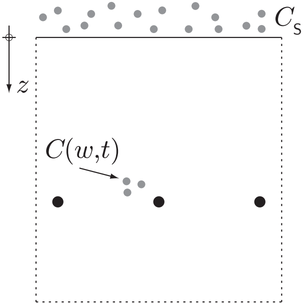


Application to Corrosion

RV	Dimension	Distribution	Parameters
W	[mm]	LogNormal	$\mu_W = 40.0$ $\sigma_W = 8.0$
D	[mm^2/yr]	LogNormal	$\mu_D = 20.0$ $\sigma_D = 10.0$
C_S	[m.-% cem.]	Normal	$\mu_{C_\mathsf{S}} = 3.1$ $\sigma_{C_\mathsf{S}} = 1.23$
C_crit	[m.-% cem.]	Normal	$\mu_{C_\mathsf{crit}} = 0.8$ $\sigma_{C_\mathsf{crit}} = 0.1$

Table: The Random variables of the corrosion model.

Application to Corrosion

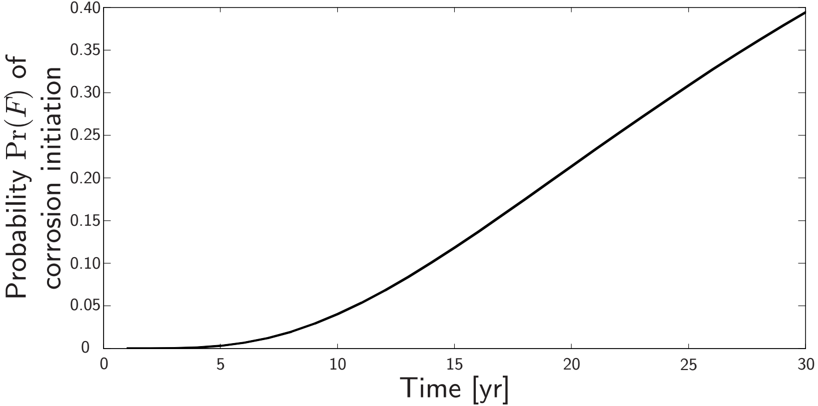


Figure: A-priori corrosion probability of the reinforcement.

Application to Corrosion

- Measurement of cover depth

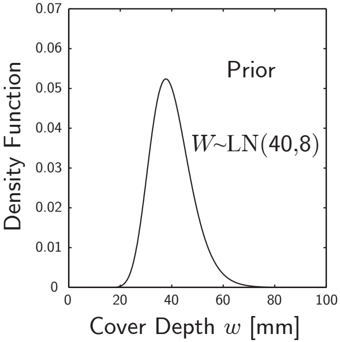
  $\mathbf{w}_\mathsf{M} = [w_1, w_2, ..., w_{800}]$

- Likelihood of measurement

  $\mathcal{L}(\mathbf{w}|\mathbf{w}_\mathsf{M}) = f_{\mathbf{W}_\mathsf{M}|\mathbf{W}}(\mathbf{w}_\mathsf{M}|\mathbf{w})$

- *Bayesian* update

  $f''_{\mathbf{W}}(\mathbf{w}) \propto \mathcal{L}(\mathbf{w}|\mathbf{w}_\mathsf{M}) f_{\mathbf{W}}(\mathbf{w})$



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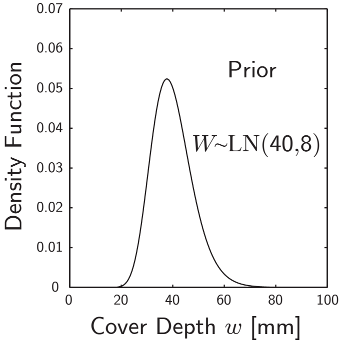

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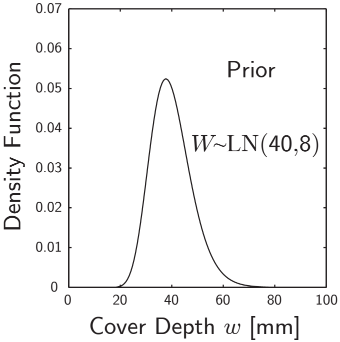

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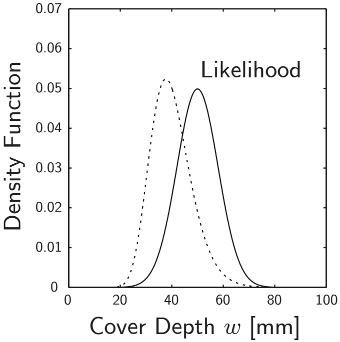

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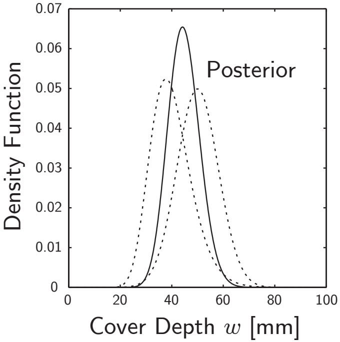

Application to Corrosion

- Measurement of chloride concentration

$$c_{Z_{\mathsf{M}},j}(\mathbf{X},t) = C_{\mathsf{S},j} \cdot \left(1 - \operatorname{erf}\left(\frac{z_{\mathsf{M}}}{\sqrt{4Dt}}\right)\right)$$

- Likelihood (Error $\epsilon \sim \mathrm{N}(0,\sigma_\epsilon)$)

$$\mathcal{L}_j(\mathbf{x}) = \frac{1}{\sigma_\epsilon\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{c_{z_{\mathsf{M}}}(\mathbf{x},t) - c_{\mathsf{M},j}(z_{\mathsf{M}},t)}{\sigma_\epsilon}\right)^2\right)$$

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Application to Corrosion

A-Posteriori Failure Probability

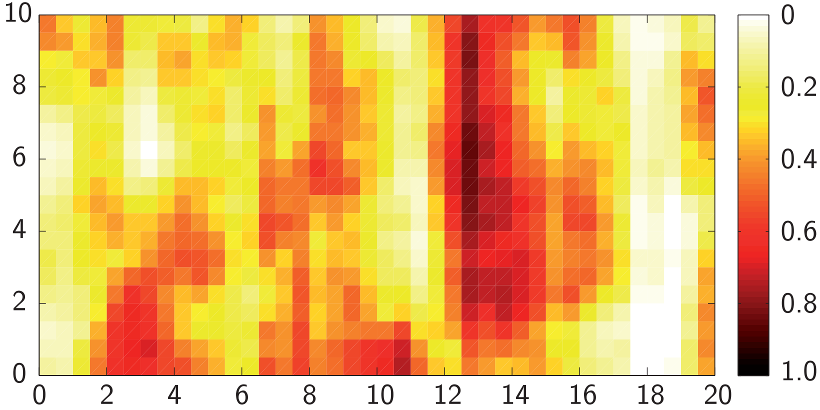

Figure: Corrosion probability ($t = 15$ [yr]) conditional on measurement results (cover depth & concentration).

Application to Corrosion

A-Posteriori Failure Probability

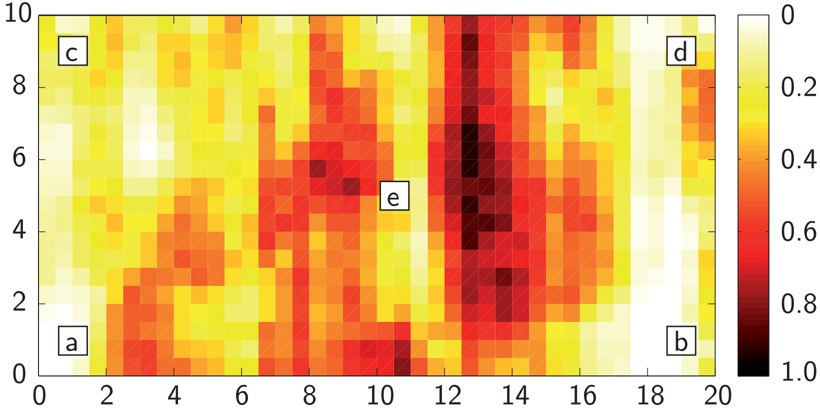

Figure: Corrosion probability ($t = 15$ [yr]) conditional on measurement results (cover depth & concentration).

To Conclude...

What was learned

- **Efficient method** available for updating failure probability using **equality information**
- Several **different information** can be taken into account

Some Literature

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Thank you for your attention!

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