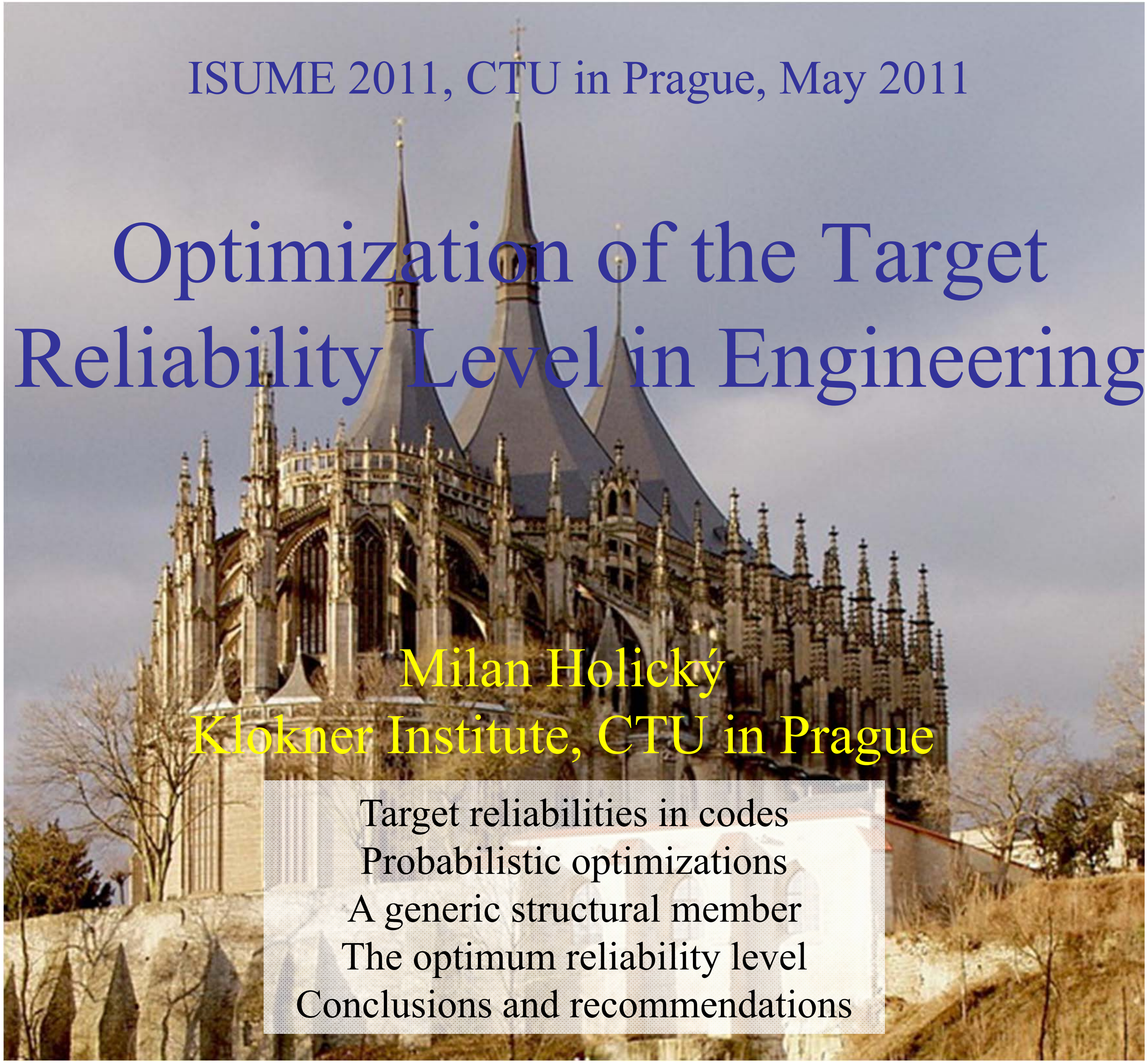

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Optimization of the Target Reliability Level in Engineering

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Target reliabilities in codes
Probabilistic optimizations
A generic structural member
The optimum reliability level
Conclusions and recommendations

Designer's questions

- What is the appropriate reliability level to be used for a structure having a given working design life different from 50 years and
  (a) negligible failure consequences (green houses)
  (b) very high failure consequences (exhibition halls, podiums)
- What are the relevant partial factors to be used in design?

This study attempts to provide correct answers and recommendations.

Structures of a short life: green houses, pavilions, podiums




Structures of a long design working life and great consequences: bridges, power plants

Target reliability indexes β in codes

Reliability classification in accordance with EN 1990, 2002

Reliability classes	Consequences of structural failure	Reliability index β for reference period		Examples of buildings and civil engineering works
		1 year	50 years	
RC3 – high	High	5,2	4,3	Bridges, public buildings
RC2 – normal	Medium	4,7	3,8	Residences and offices
RC1 – low	Low	4,2	3,3	Agricultural buildings

Target reliability indices β (life-time) in accordance with ISO 2394. 1998

Relative costs of safety measures	Consequences of failure			
	small	some	moderate	great
High	0	1,5	2,3	3,1
Moderate	1,3	2,3	3,1	3,8
Low	2,3	3,1	3,8	4,3

Target reliability indices β (annual rates) in accordance with JCSS, 2001

Relative costs of safety measures	Minor consequences of failure	Moderate consequencess of failure	Large consequences of failure
Lrage	β= 3,1 ($p \approx 10^{-3}$)	β= 3,3 ($p \approx 5\times10^{-4}$)	β= 3,7 ($p \approx 10^{-4}$)
Normal	β= 3,7 ($p \approx 10^{-4}$)	β= 4,2 ($p \approx 10^{-5}$)	β= 4,4 ($p \approx 5\times10^{-6}$)
Small	β= 4,2 ($p \approx 10^{-5}$)	β= 4,4 ($p \approx 5\times10^{-6}$)	β= 4,7 ($p \approx 10^{-6}$)

Target reliability indexes β in codes

for the reference period 50 years (life time in ISO) and 1 year,
"moderate" (ISO) or "normal" (JCSS) relative costs of safety measures

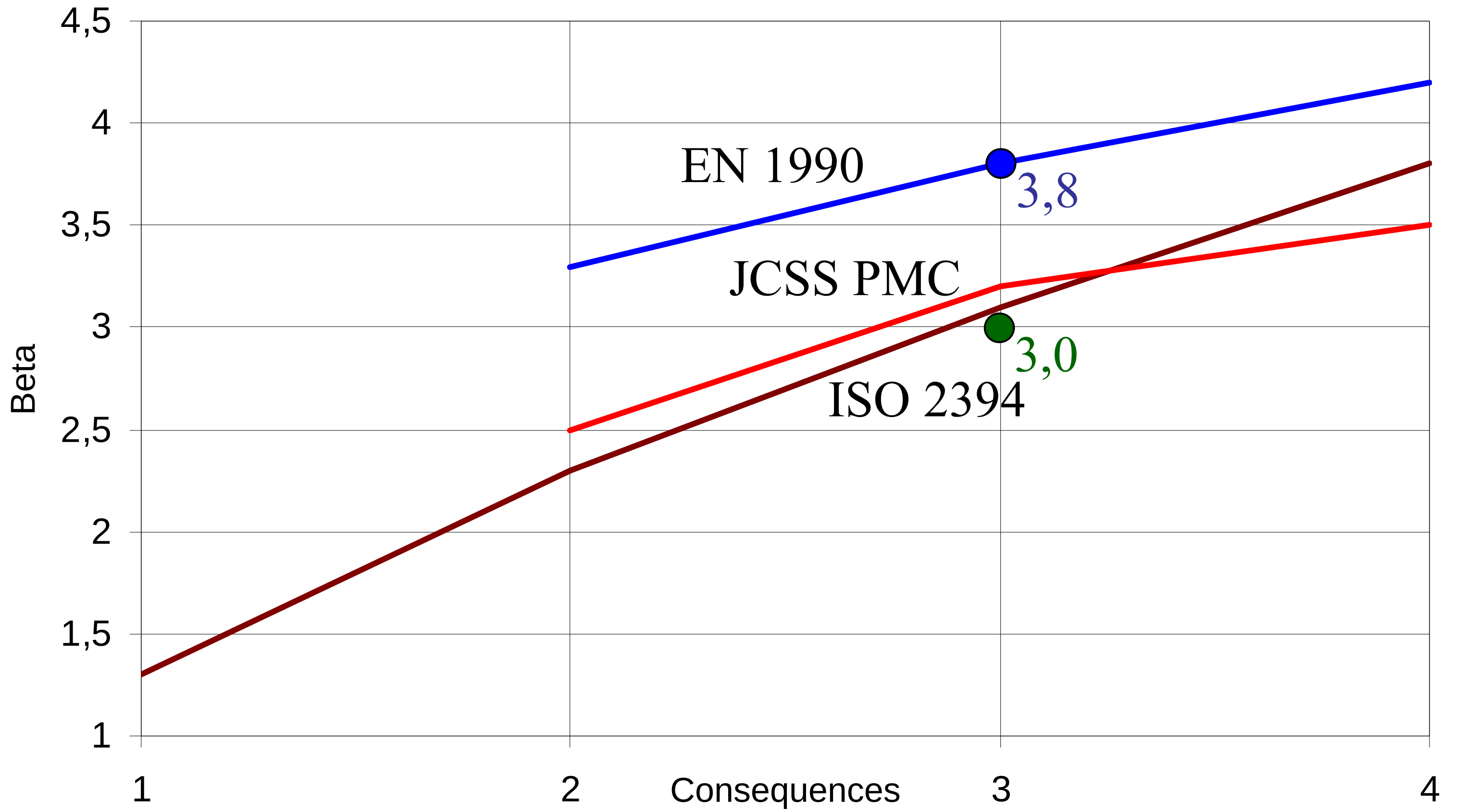
Codes	Consequences			
EN 1990, 2002 ISO 9324, 1998 JCSS PMC, 2001	small	low some minor	normal moderate moderate	high great large
EN – 50 years	-	3,3	3,8	4,2
ISO – life time *	1,3	2,3	3,1	3,8
JCSS – 50 years **	-	2,5	3,2	3,5
EN – 1 year	-	4,2	4,7	5,2
ISO – 1year ***	2,9	3,5	4,1	4,7
JCSS – 1 year	-	3,7	4,2	4,4

* For "moderate" relative costs of safety measures

** Recalculated from the annual rates for "normal" relative costs of safety measures

*** Recalculated from the life time rates for "normal" relative costs of safety measures

Target reliability indexes β for the reference period of 50 years



EN		low	normal	high
ISO	small	some	moderate	great
JCSS		minor	moderate	large

Bases of probabilistic optimization

1. Annual failure probability $p(x)$ depends on a structural parameter x (e.g. cross section area) considered as the decision parameter

2. Failure probability $P_f(x,i)$ at the year i and $P_{fn}(x)$ within n years

$$P_f(x,i) = p(x)\,(1 - p(x))^{i-1} \qquad P_{fn}(x) = 1 - (1 - p(x))^n \approx n\,p(x)$$

3. The basic objective function as the total cost

$$C_{tot}(x,q,n) = C_f \sum_{i=1}^{n} P_f(x,i)\,Q(q,i) + C_0 + x\,C_1$$

Failure costs (C_f), initial costs (C_0), marginal costs (C_1)

4. The discount factor at the year i considered as

$$Q(q,i) = 1/(1+q)^i$$

The optimum structural parameter x_{opt}

The necessary conditions for the optimum x_{opt}

$$\frac{\partial C_{\mathrm{tot}}(x,q,n)}{\partial x} = C_{\mathrm{f}} \sum_{i=1}^{n} Q(q,i) \left[\frac{\partial P_{\mathrm{f}}(x,i)}{\partial x} \right]_{x=x_{\mathrm{opt}}} + C_1 = 0$$

$$\sum_{i=1}^{n} Q(q,i) \left[\frac{\partial P_{\mathrm{f}}(x,i)}{\partial x} \right]_{x=x_{\mathrm{opt}}} = -\frac{C_1}{C_{\mathrm{f}}}$$

The optimum depends on the cost ratio C_{f}/C_1, n and q

$$x_{\mathrm{opt}}(C_{\mathrm{f}}/C_1, n, q)$$

Simplification using the standardized costs

The total cost expressed as

$$C_{\text{tot}}(x,q,n) = C_{\text{f}}\, p(x)\, PQ\,(x,q,n) + C_0 + x\, C_1$$

The time factor $PQ(x,q,n,)$ is

$$PQ\,(x,q,n) = \frac{1 - \left[\dfrac{(1-p(x))}{(1+q)}\right]^n}{1 - \dfrac{(1-p(x))}{(1+q)}}$$

The standardized costs $\kappa_{\text{tot}}(x,q,n)$ as a transformed total costs

$$\kappa_{tot}(x,q,n) = \frac{C_{tot}(x,q,n) - C_0}{C_1} = p(x)\, PQ(x,q,n)\frac{\mathrm{C_f}}{C_1} + x$$

Failure probability of a generic member

The limit state function of a generic structural member

$$Z(x) = x\,f - (G+Q)$$

The structural parameter x considered as deterministic quantity close to 1.

Probabilistic models of basic variables (annual extremes of Q)

Variables	Distribution	The mean	Standard deviation	Coefficient of variation
f	Lognormal	100	10	0,10
G	Normal	40	4	0,10
Q	Gumbel	10	5	0,50

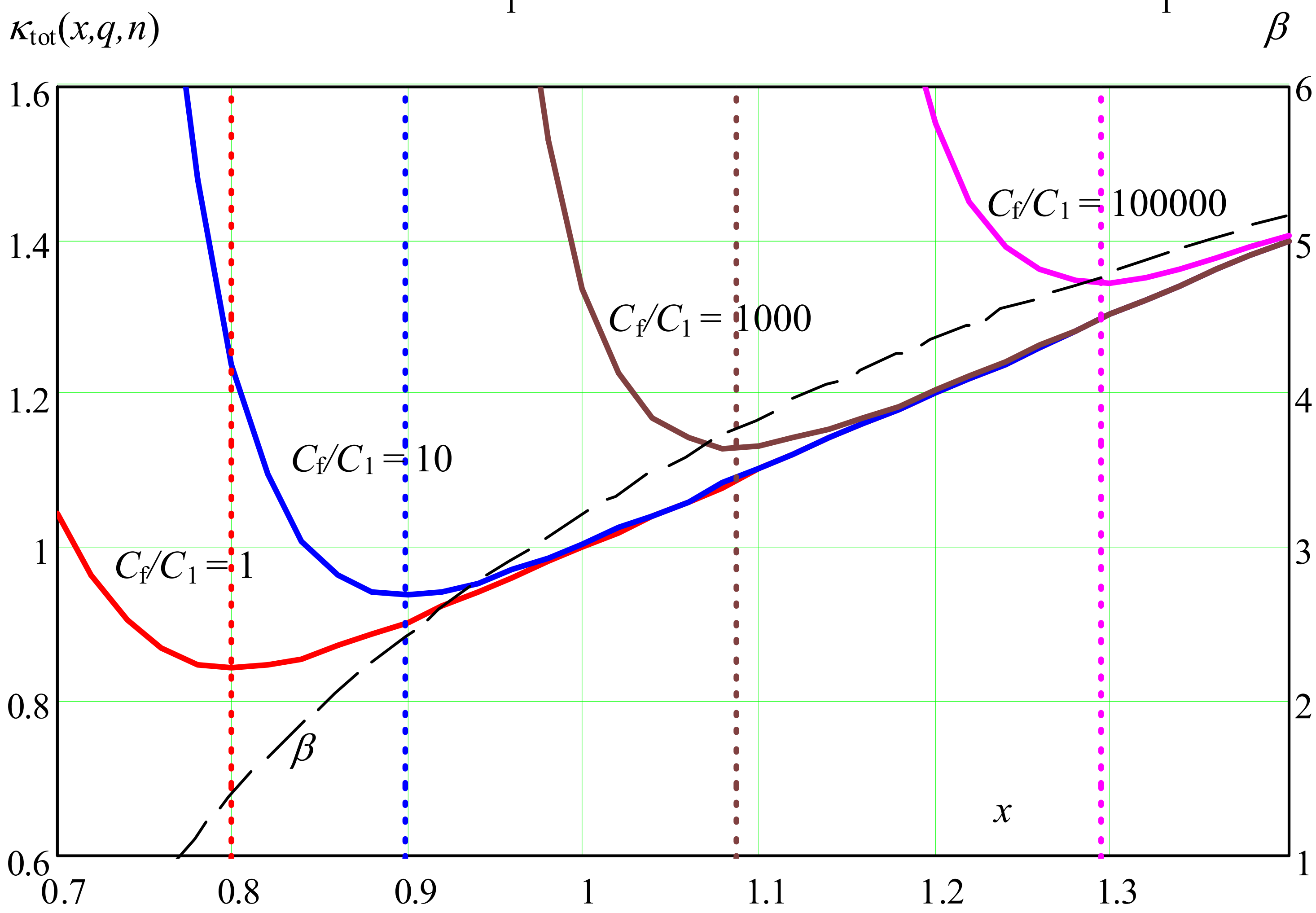
Annual failure probability $p(x)$ approximated by the normal distribution

$$p(x) = \Phi_{Z(x)}(Z(x) = 0)$$

The skewness of $Z(x)$ is around 0,1 only.

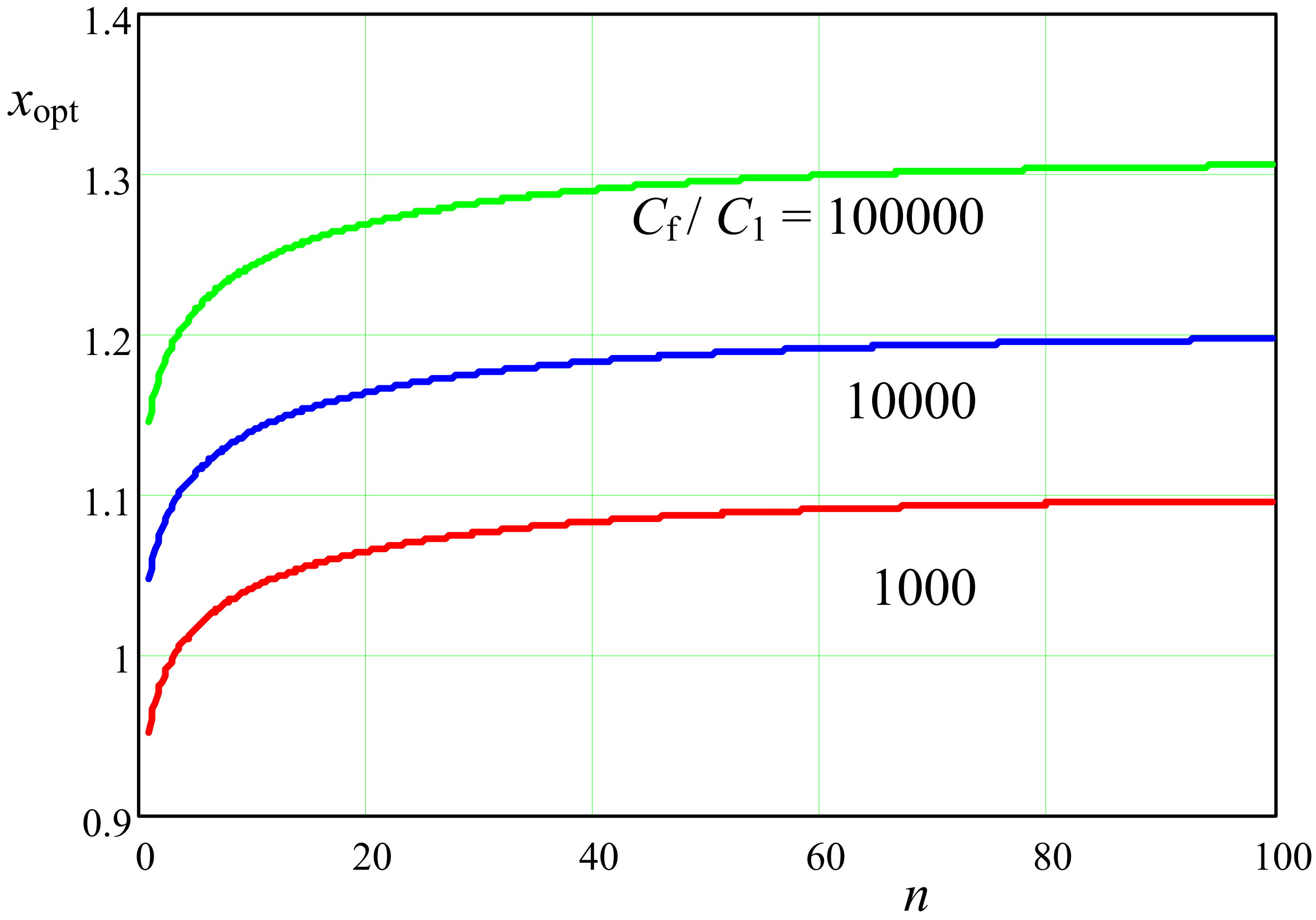
Standardized costs

$$\kappa_{tot}(x,q,n) = \frac{C_{tot}(x,q,n) - C_0}{C_1} = p(x)\ PQ(x,q,n)\frac{C_f}{C_1} + x$$



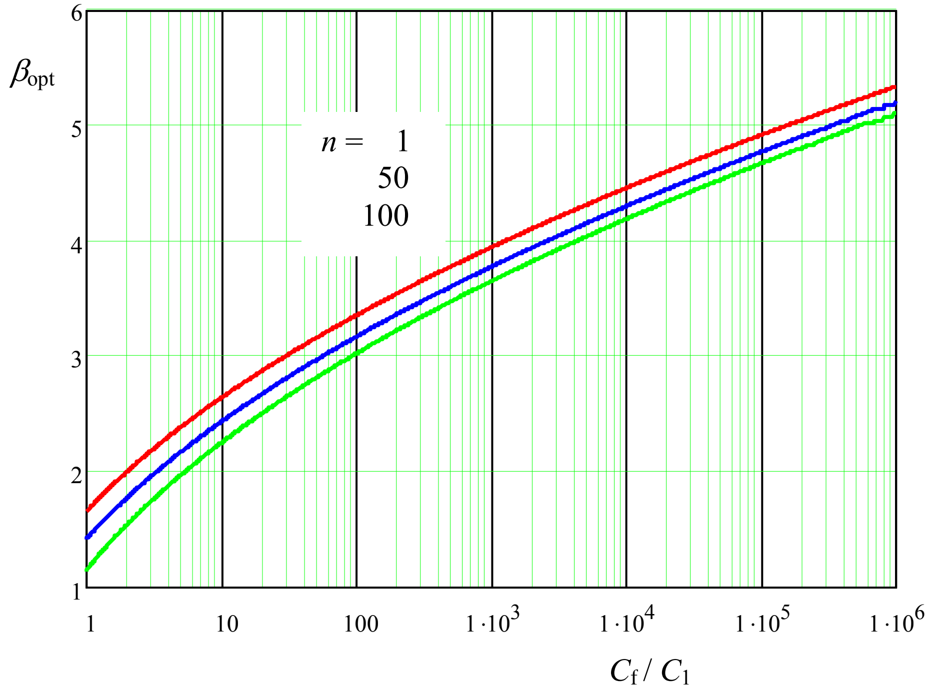
The optimum structural parameter x_{opt}

The discount rate q = 0,03

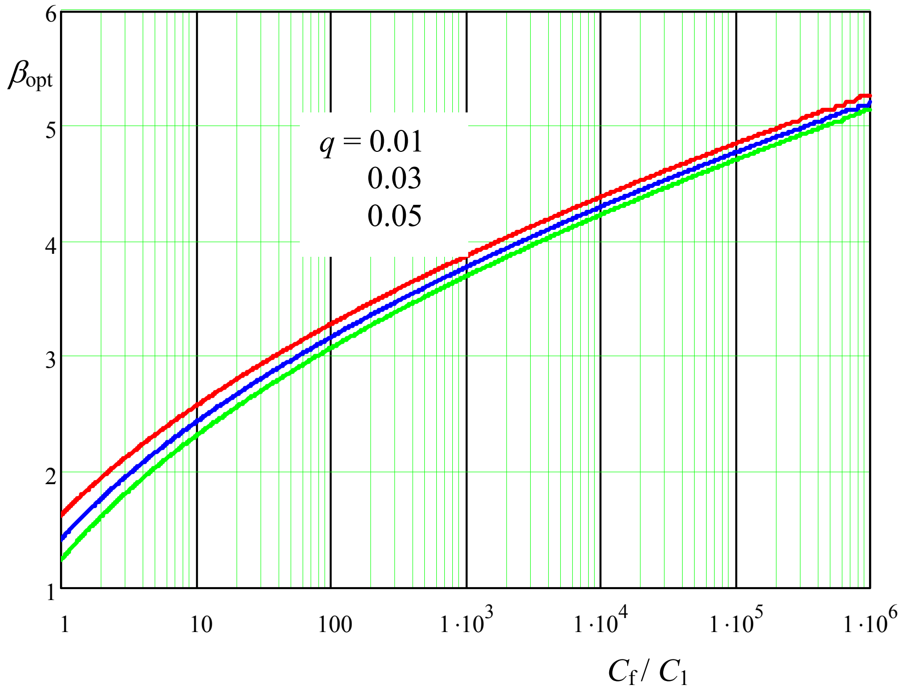


Variation of the reliability index β_{opt}

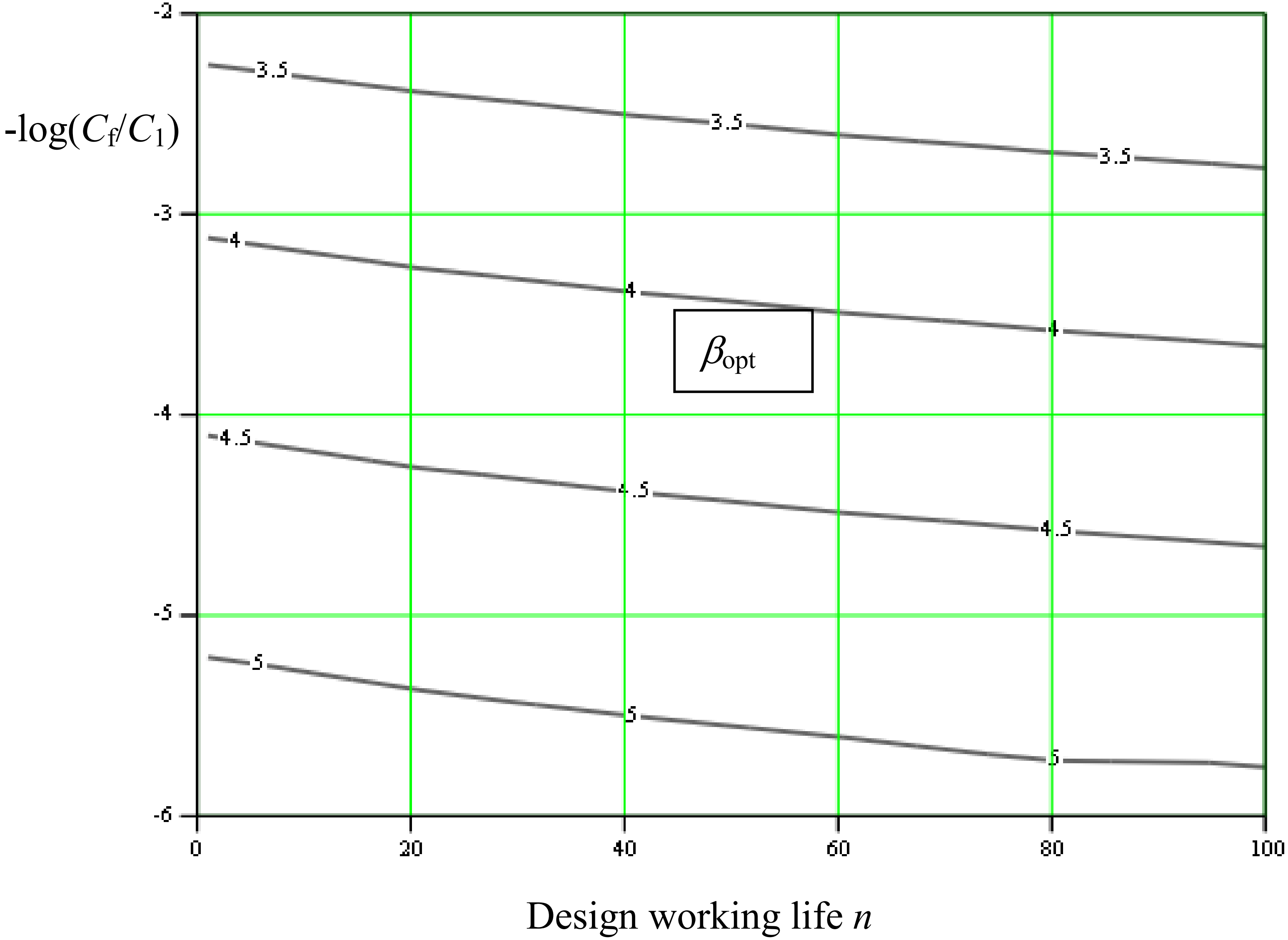
q= 0,03



n= 50

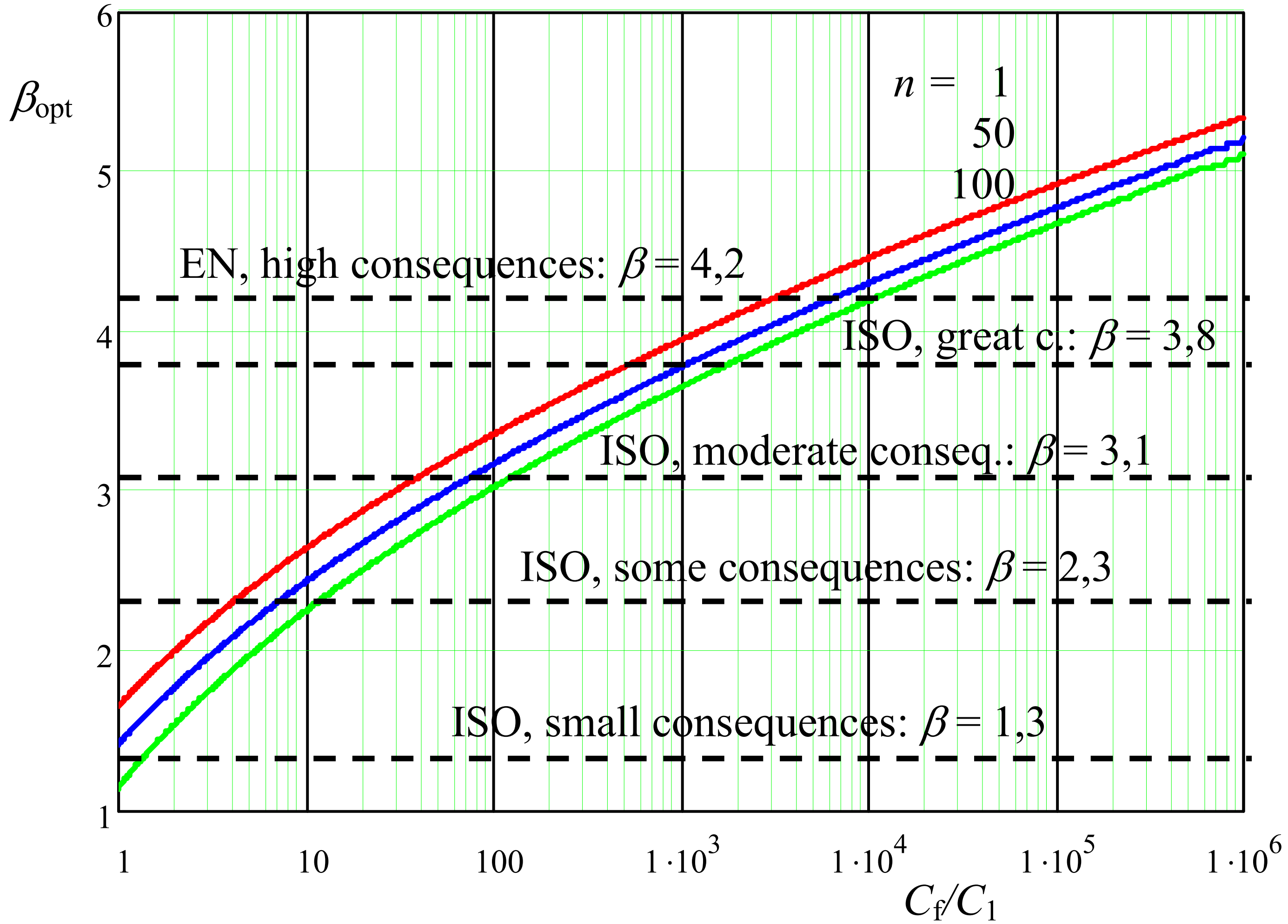


Contour lines for β_{opt}, $q = 0{,}03$



Variation of the reliability index β_{opt}
for discount rate $q = 0{,}03$

---- target β for 50 years (EN) or life-time (ISO)



Conclusions

- Present codes do not provide clear link between the design working life and the target reliability level.
- The same reliability level is approximately achieved considering the reference period 1 and the target reliability index 4,7 or the reference period 50 years and the reliability index 3,8.
- The optimum reliability level
  - depends on the ratio of cost of structural failure and marginal cost per unit of a structural parameter (relative safety measures),
  - less significantly depends on the design working life and on
  - the discount rate.
- The target reliability index may differ from the optimum value when the cost ratio, the design working life and the discount rates are difficult to assess. A conservative index estimated for an appropriate upper bound of the cost ratio and for lower bounds of the design working life and discount rate is then recommended.

Answers to the designer's questions

- The target reliability level should be primarily specified on the basis of the cost ratio of failure consequences and relative costs of safety measures.
- The design working life and discount rate seem to affect the optimum reliability and the target reliability level less significantly.
- Partial factors should be derived from the specified reliability index and, for time dependent basic variables, also from the design working life.

Recommendations

- Recommendations for codified design based on the partial factor method may be summarized as follows:
  - the characteristic values of basic variables are defined independently of the design working life and discount rate;
  - the design values are derived on the basis of appropriate reliability index, design working life and discount rate;
  - the partial factors are determined considering the specified design values and characteristic values of basic variables.
- Further investigations should consider costs of maintenance during the design working life and advanced models for consequences including societal, economic and ecological aspetcs.

Thank you for your attention