Direct Optimized Probabilistic Calculation - DOProC Method

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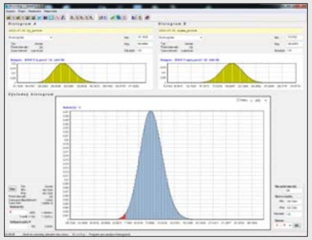


Contents of Presentation

- Direct Optimized Probabilistic Calculation - **DOProC** Method
    - Principles of DOProC method
    - Optimizing techniques of the probabilistic calculation
    - Software utilization

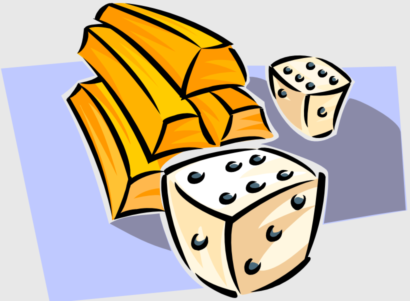
- Using DOProC method for the calculation of the fatigue crack propagation
    - Probabilistic Calculation of Fatigue Crack Propagation
        - from the edge
        - from the surface
    - Determining inspection time of the structure using conditioned probability

Essential of DOProC Characteristics

- Should be effectively used for the assessment of structural reliabilities and/or for other probabilistic calculations.
- Input random variables (load, geometry, material properties, imperfections) are expressed by the empirical or parametric distributions in histograms.
- Reliability function under analysis can be expressed analytically or using DLL library.
- Error of calculation is done only by discretization of input and output variables and numerical error.
- The number of intervals (classes) of each histogram is extremely important for the number of needed numerical operations and required computing time.
- The number of numerical operations can be reduced using optimizing techniques of the probabilistic calculation.

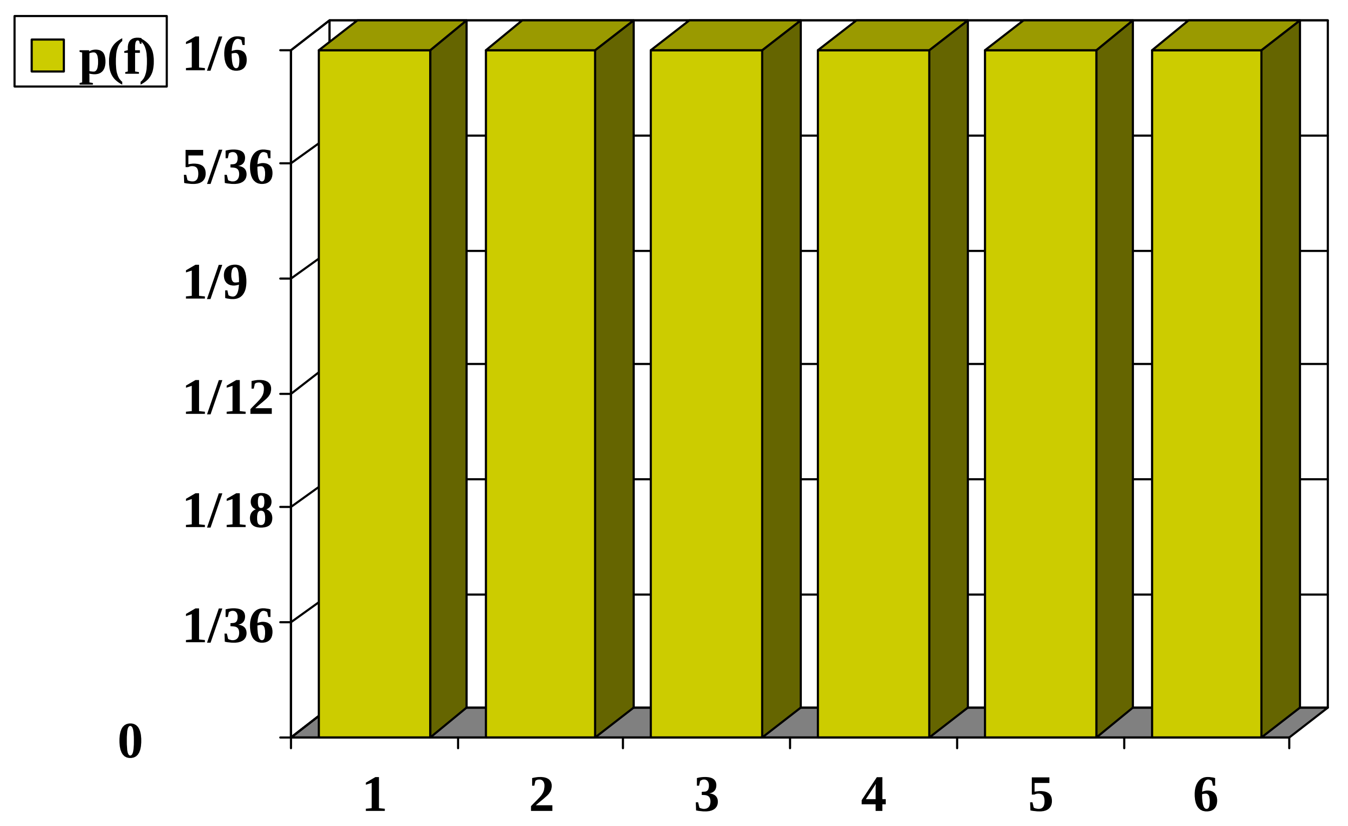
Statistics of Dice Throw

Throw of a single dice - all outcomes are equally probable.



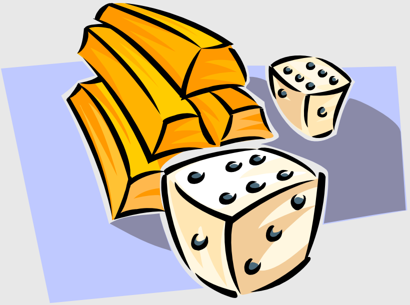
$$p_1 = \frac{1}{n}$$

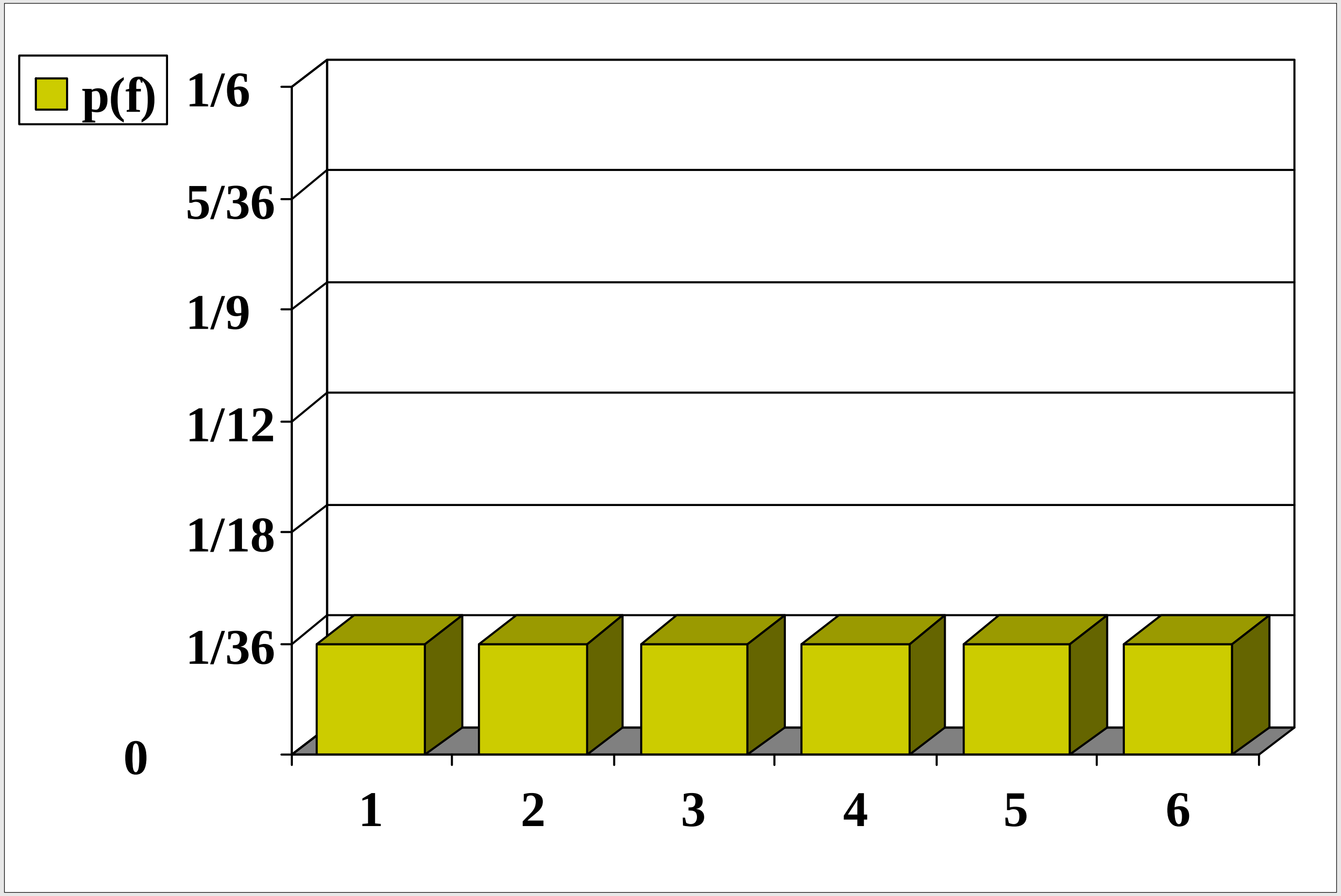
$$p_1 = \frac{1}{6}$$



Statistics of Two Dices Throw

Probabilities of identical numbers obtained by the throw of two dices.



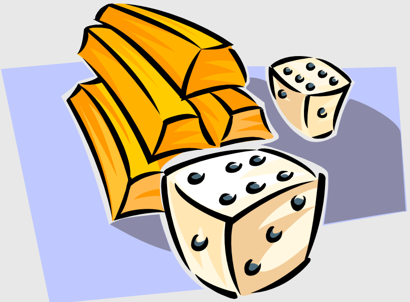


$$p = p_1 . p_2$$

$$p_1 = \frac{1}{36}$$

Statistics of Two Dices Throw

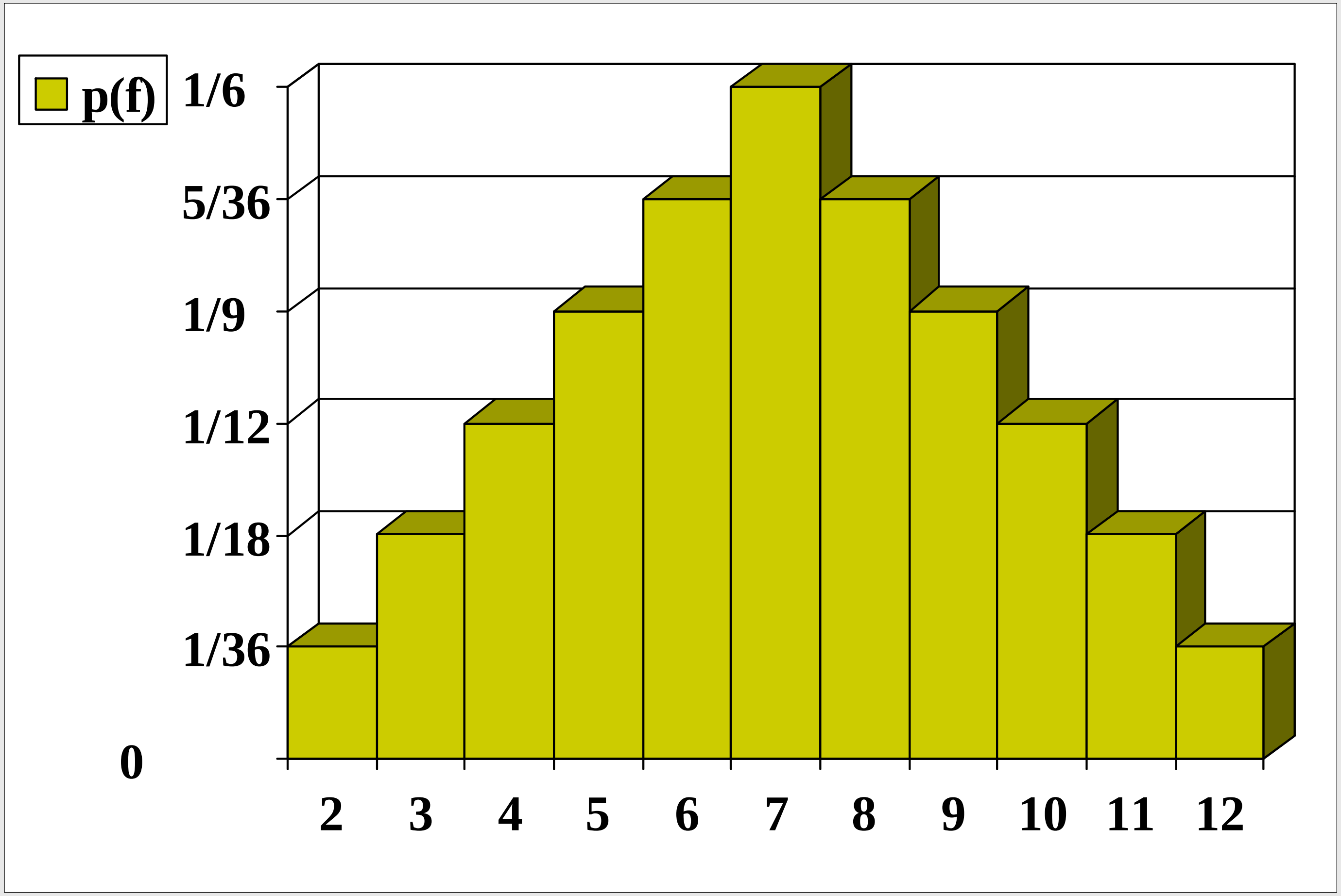
The different possibilities for the **total** of the numbers on two dices.



$$p(2) = \frac{1}{36}$$

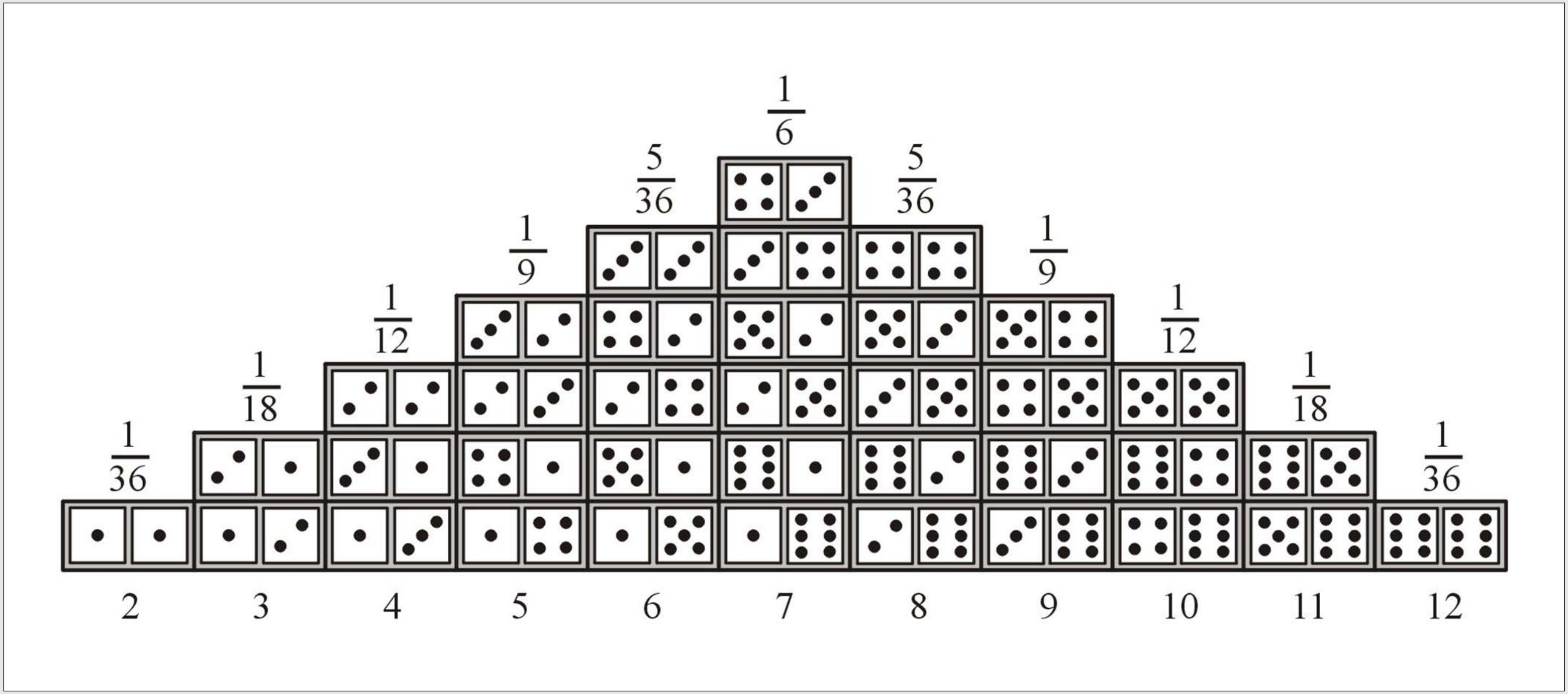
$$p(3) = \frac{1}{36} + \frac{1}{36}$$

$$p(4) = \dots$$



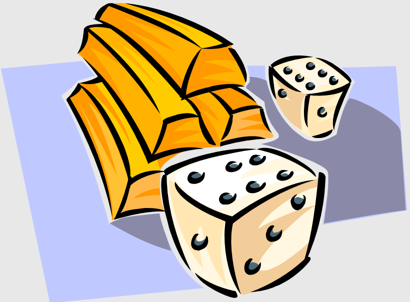
Statistics of Two Dices Throw

The different possibilities for the **total** of the numbers on two dices - are not equally probable - more ways to get some numbers than others.



Statistics of Two Dices Throw

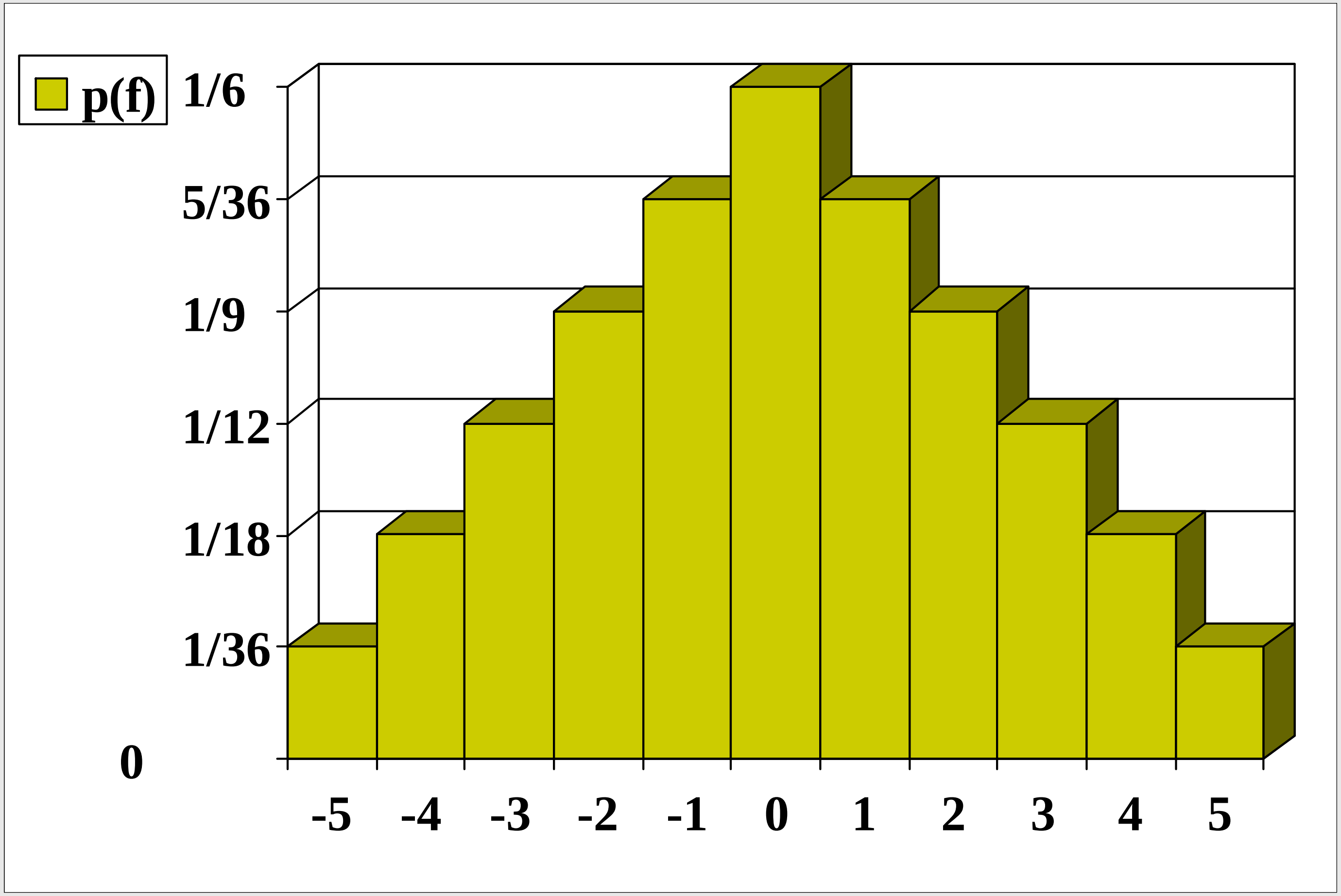
The different possibilities for the **algebraic difference** of the numbers on two dices.



$$p(5)=\frac{1}{36}$$

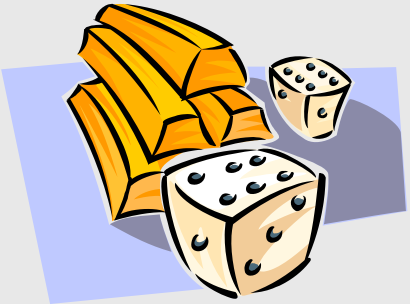
$$p(4)=\frac{1}{36}+\frac{1}{36}$$

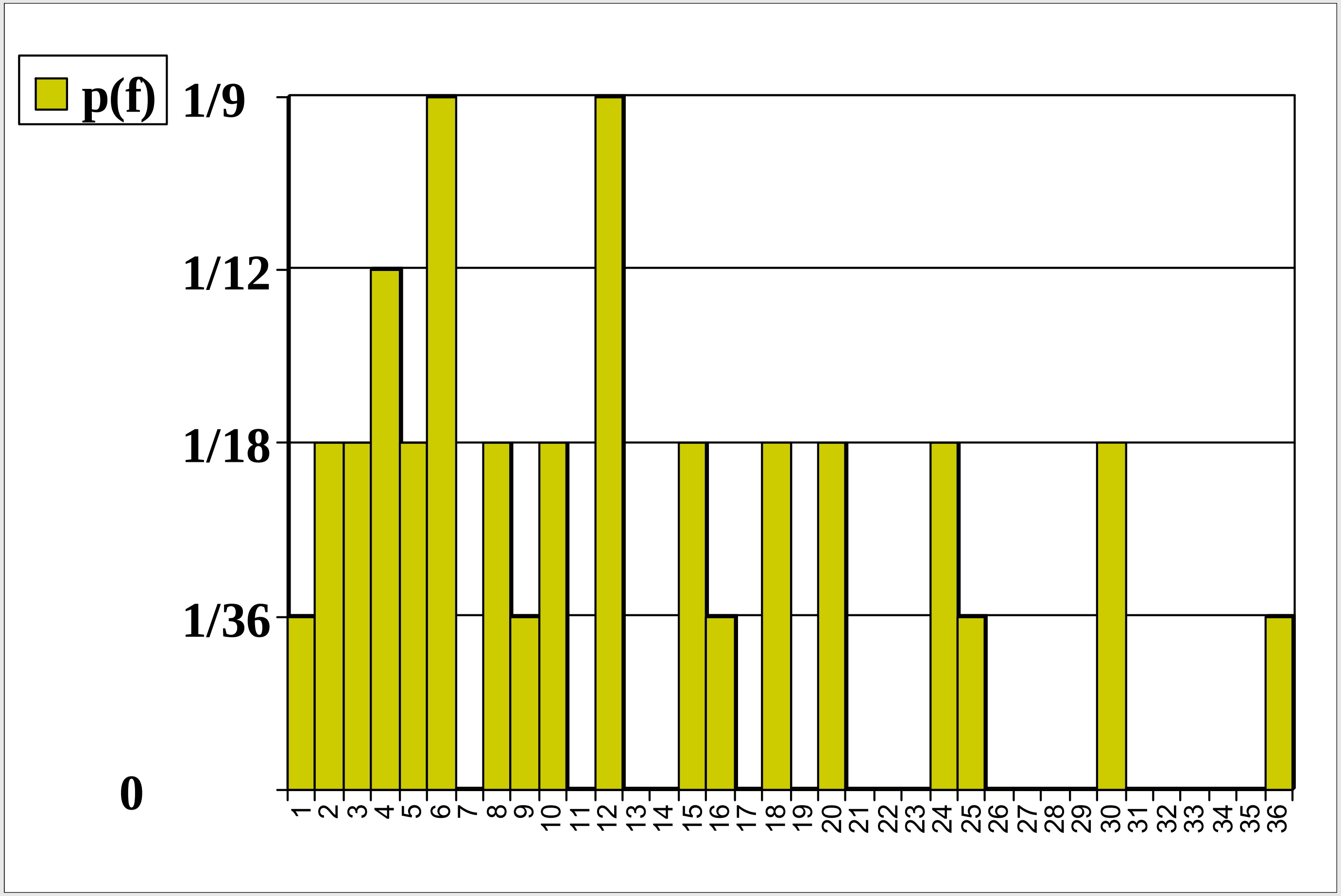
$$p(3)=\dots$$



Statistics of Two Dices Throw

The different possibilities for the **arithmetic product** of the numbers on two dices.



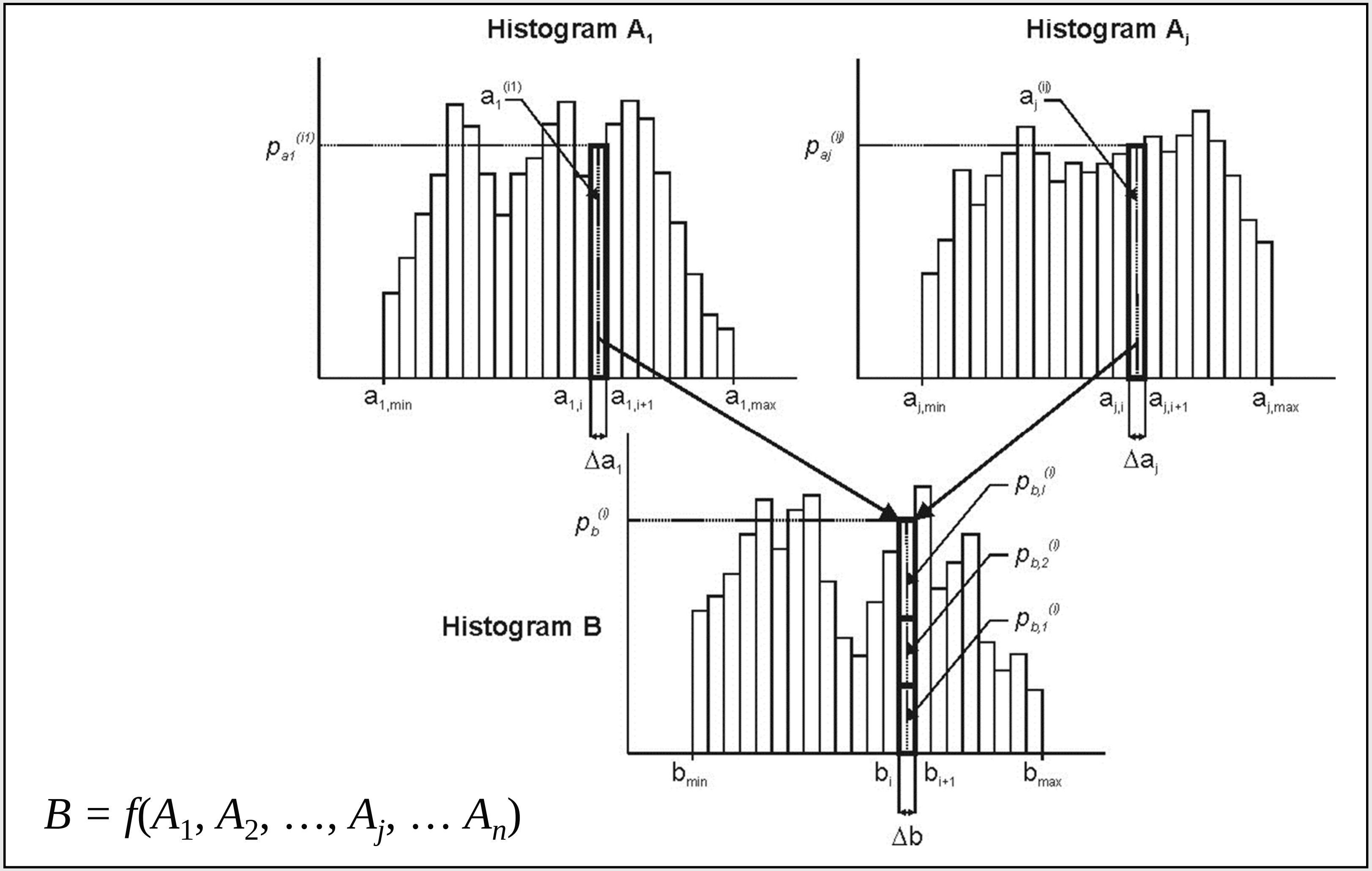


$$p(1)=\frac{1}{36}$$

$$p(2)=\frac{1}{36}+\frac{1}{36}$$

$$p(3)=\ldots$$

Principle of Numerical Calculation



$$B = f(A_1, A_2, \ldots, A_j, \ldots A_n)$$

Optimizing Techniques in DOProC

- Grouping of input random variables, which can be expressed by the common histogram.
- Interval optimizing - decreasing the number of intervals in input variable histograms.
- Zonal optimizing - each histogram is divided into areas (zones) depending on their share in the result (failure).
- Trend optimization – using correct or incorrect trend of input variable on the result.
- Grouping of partial calculations results.
- Parallelization of the calculation – calculation is proceeded on number of processors.
- Combination of the mentioned optimizing techniques.

Grouping of Input Random Variables

Let be $B = A_1 + A_2 + A_3 + A_4 + \ldots + A_N$

whereas in each histogram are n classes (e.g. $n = 256$, $N = 10$)

All allowable combinations are

$$P_0 = n^N = 256^{10} = 1{,}20893.10^{24}$$

The same result is possible to get step-by-step counting of both histograms. Then is

$$P^*_0 = (\,N - 1\,)\,n^2 = 9{,}256^2 = 589824$$

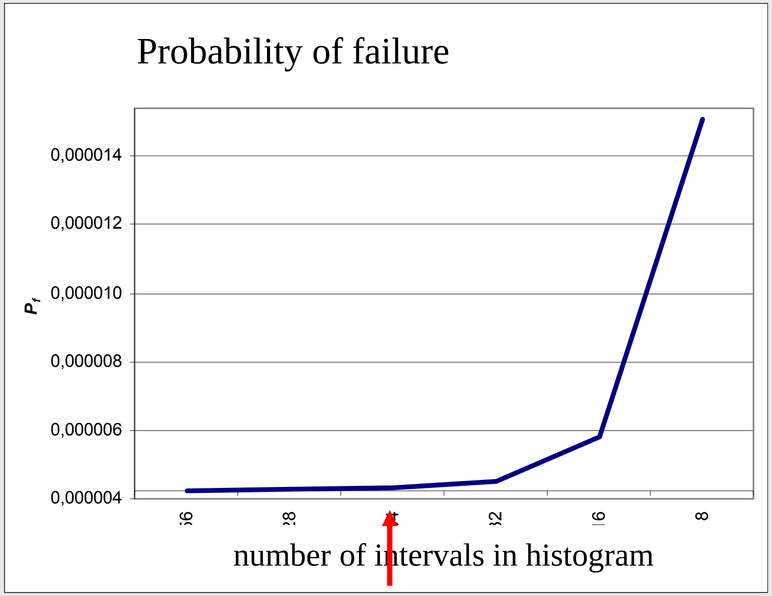
and ratio $P^*_0 / P_0 = (\,N - 1\,)\,n^{(N-2)} = 9{,}256^{-8} = 4{,}87891.10^{-19}$

If the creation of common histograms is correct – grouping of input random variables is very rational procedure.

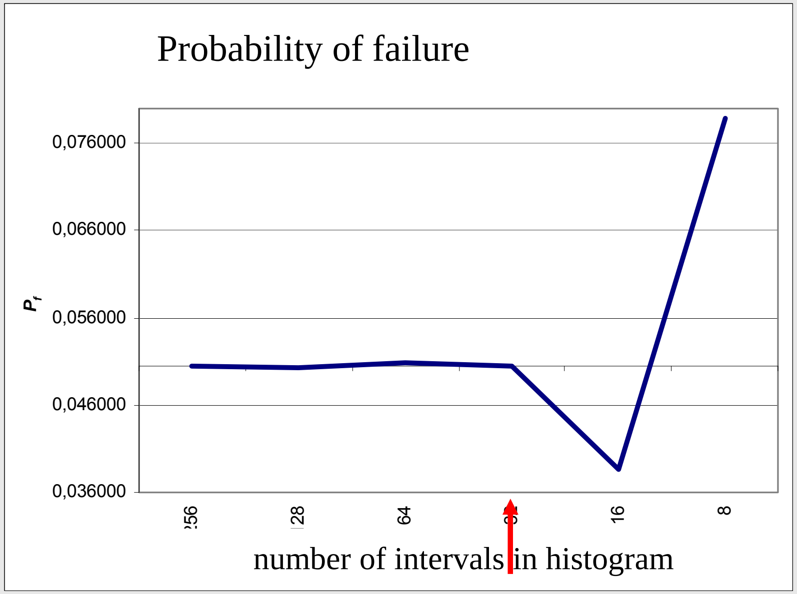
Interval Optimizing

Sense of **interval optimization** is

- number classes minimizing in histograms
- decreasing number of numerical operations and minimizing of computing time



Sufficient number of classes (intervals) of histogram



Sufficient number of classes (intervals) of histograms

Zonal Analysis and Optimizing

Each histogram is divided into areas (zones – „the zonal optimizing“) depending on their share in the failure, whatever are the values of the other variables:

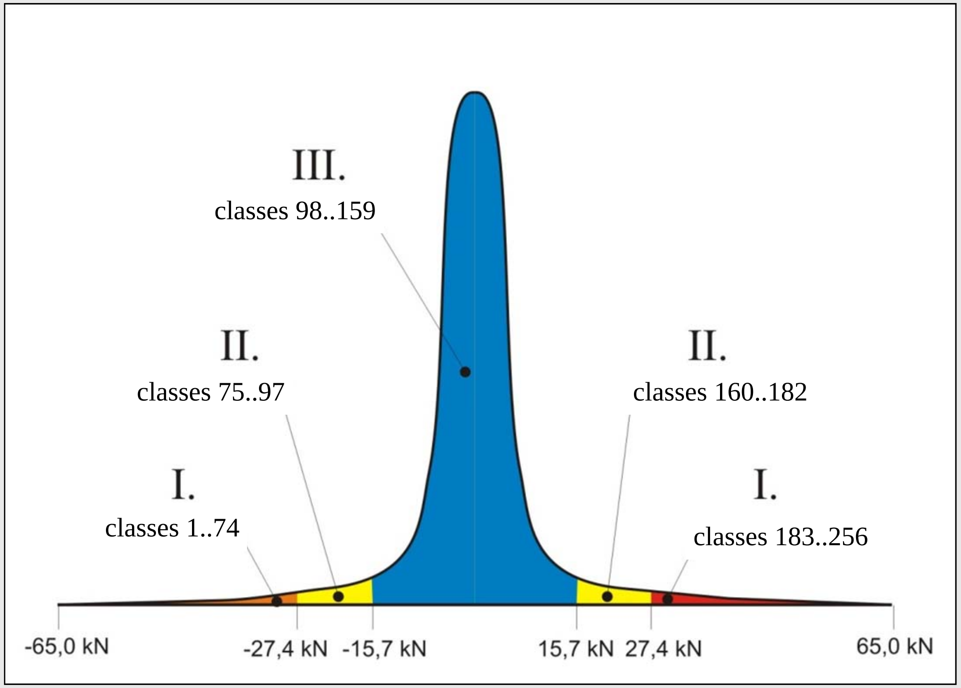
p_f =0 always

p_{f2} only in some events

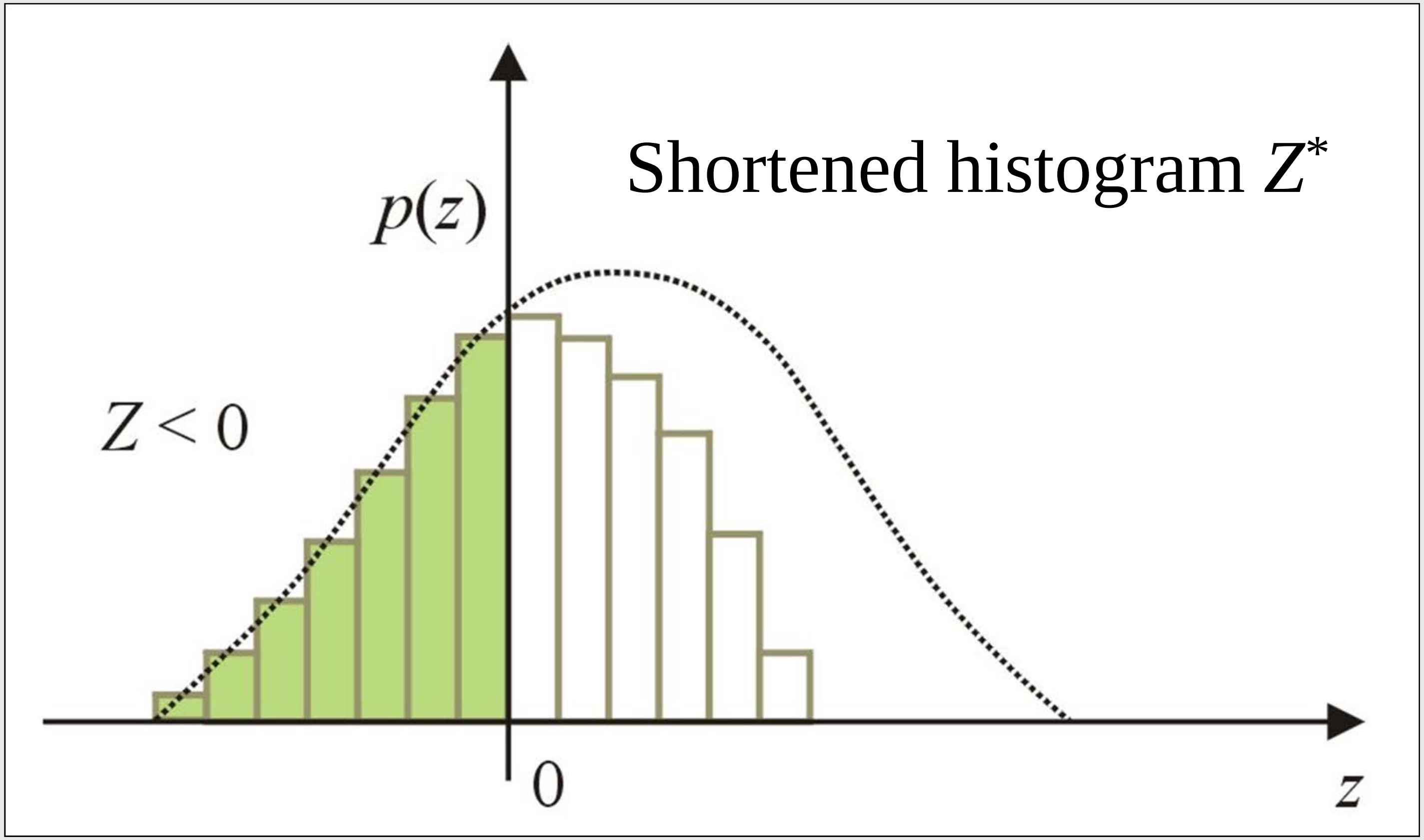
p_{f1} always

- 1st zone – the failure occurs always
- 2nd zone – the failure may occur depending on values of the other variables
- 3rd zone – the failure does not occur

$$p_f = p_{f1} + p_{f2}$$



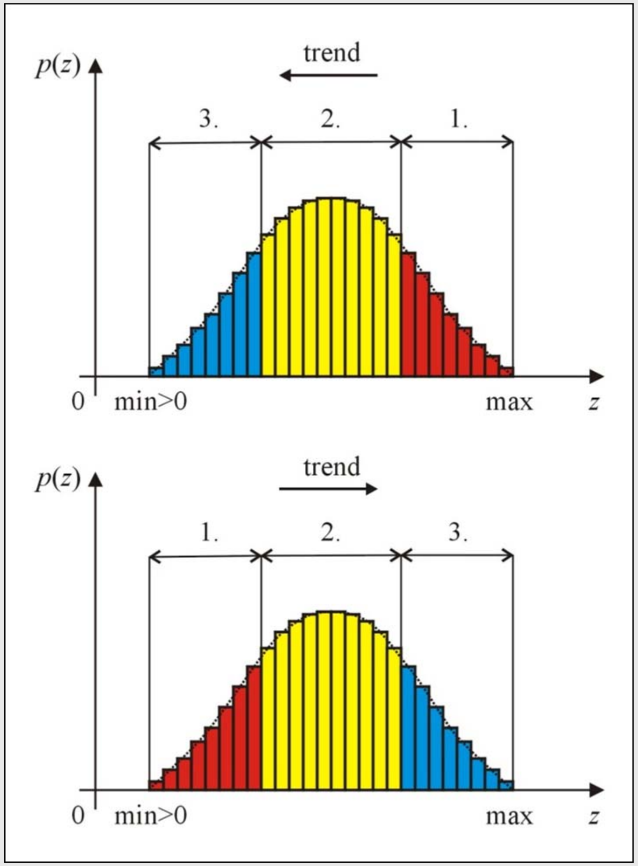
Zonal Analysis and Optimizing



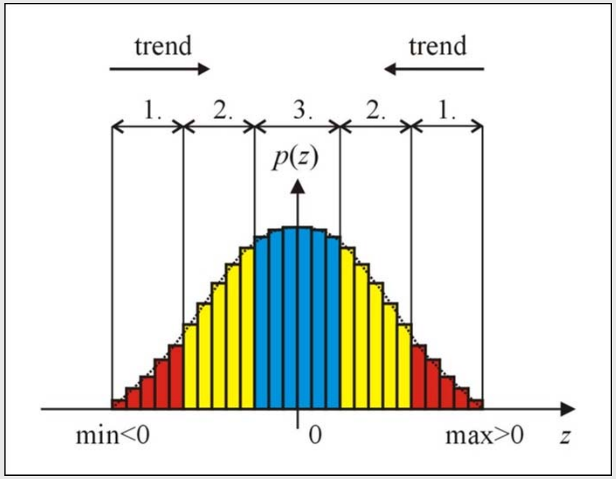
Resulting histogram of reliability function RF using DOProC method in action zonal optimizing – so-called „shortened histogram“ Z^*

Trend Analysis and Optimizing

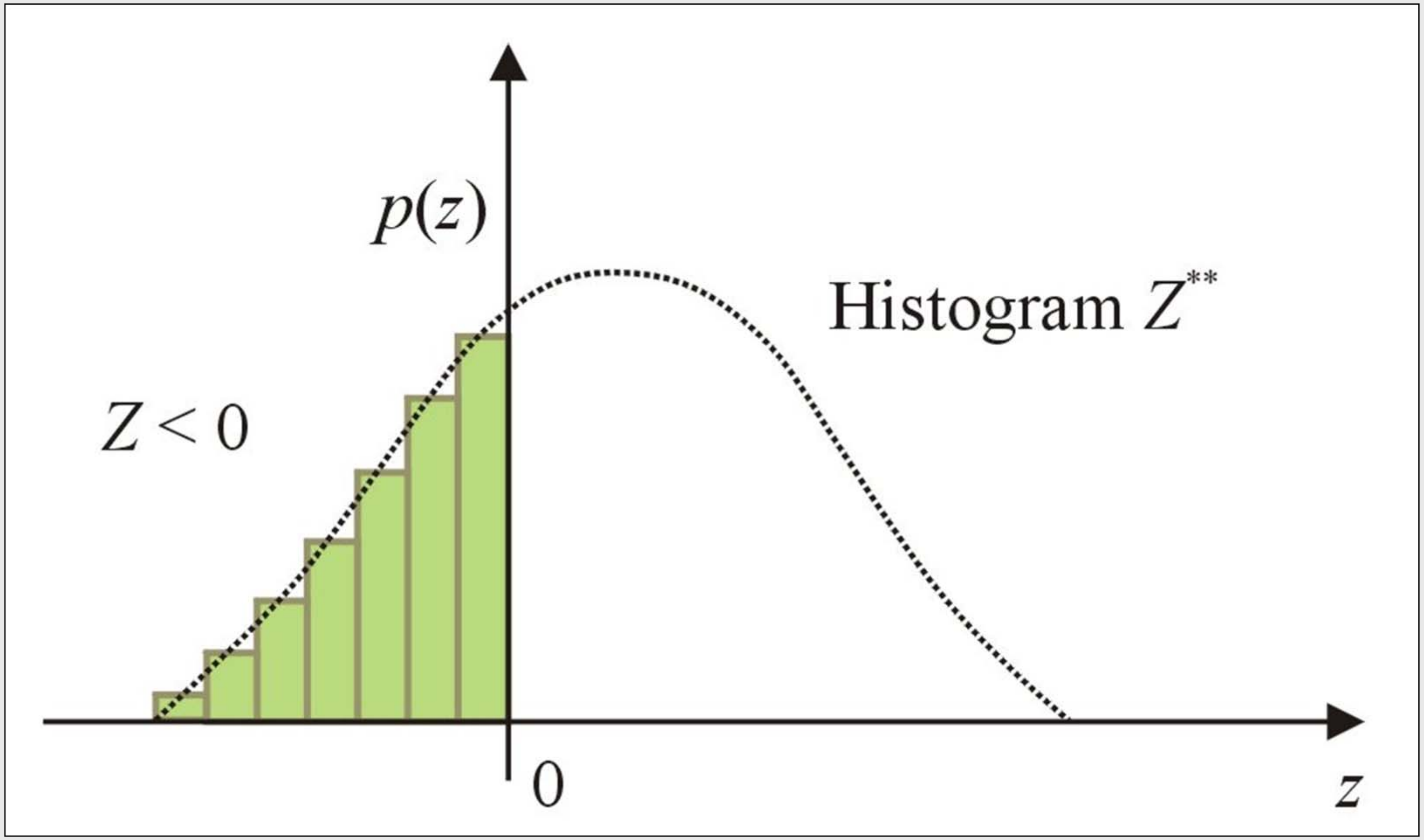
- Monotonous histograms:
  - zones in histograms are changing in one direction.



- Non-monotonous histogram:
  - zones in histograms are not changing only in one direction,
  - histograms have two same zones at least.



Trend Analysis and Optimizing



Resulting histogram of realibility function RF using DOProC method in action of trend optimizing - histogram Z^{**}

Grouping of Partial Calculations Results

Is analogy of input variables grouping.

If e.g. :

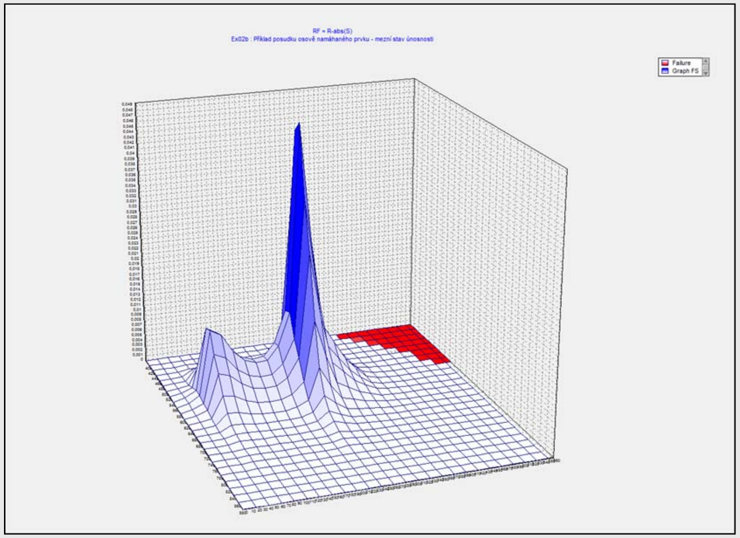
$$RF = R - f(A_1, A_2, A_3, \dots A_N)$$

then is often useful proceed independently calculation

$$S = f(A_1, A_2, A_3, \dots A_N)$$

and following

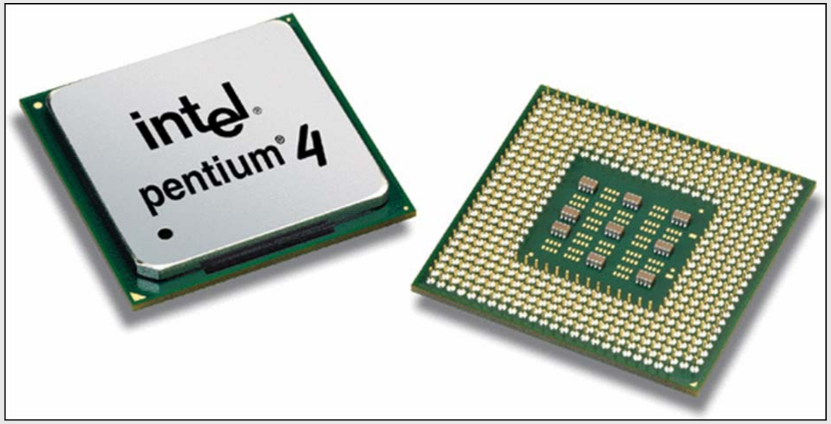
$$RF = R - S$$



Parallelization and Combination of the Optimizing Techniques

DOProC method is able to:

- combine the mentioned optimizing techniques,
- parallelize the calculation
  (still tested on computers with two processors).

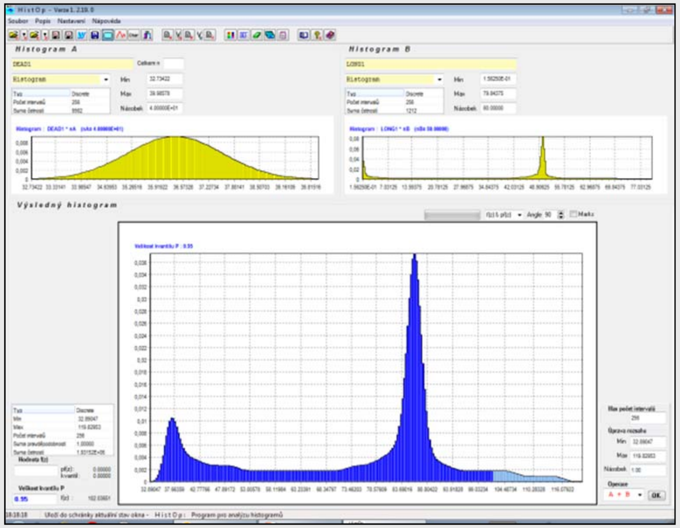


Statistically Dependent Input Variables

- Statistically independent random variables are entered into probabilistic calculation using DOProC method for now.
- Some of input variables are statistically dependent however, e.g. cross-section characteristics, strength properties etc.
- Statistically dependent input variables can enter into calculation using DOProC method indirect as function of useful independent input quantities.
- Problematic of statistically dependent input variables was still take care of cross-section characteristics of rolled shapes especially.

Program System ProbCalc

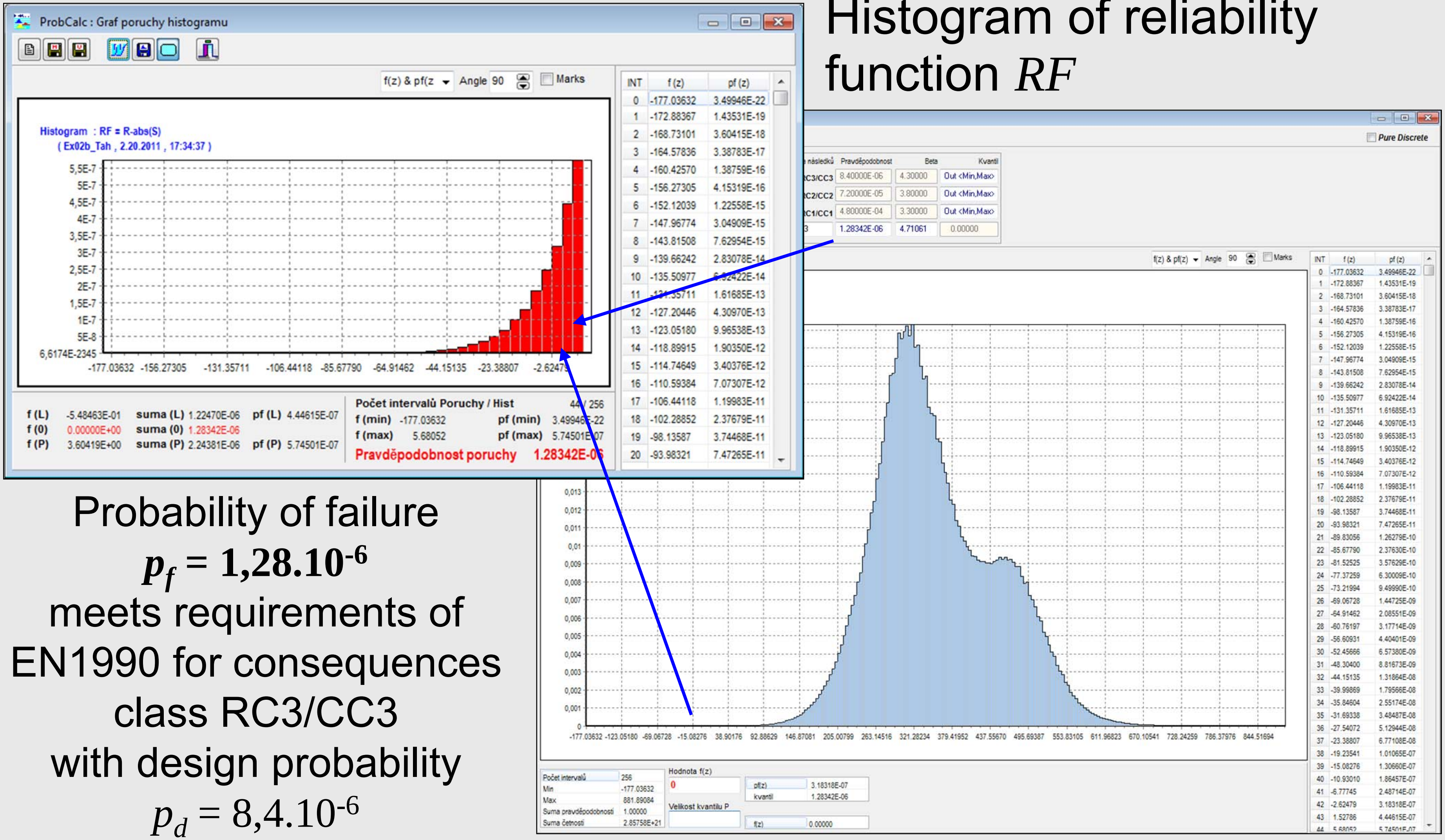
Implementation of DOProC method was created by three software utilities:



- **HistAn**:
  - utility for histogram analysis
- **HistOp**:
  - utility for basic arithmetic operations with 1 or 2 histograms
- **ProbCalc**:
  - served for probabilistic structural reliability assessment and for other probabilistic problems.
  - computing model can be defined using so-called calculator (text mode) or DLL library (machine code).
  - all of optimizing techniques were implemented.

Program Utility ProbCalc

Histogram of reliability function *RF*



f (L)	-5.48463E-01	suma (L)	1.22470E-06	pf (L)	4.44615E-07
f (0)	0.00000E+00	suma (0)	1.28342E-06		
f (P)	3.60419E+00	suma (P)	2.24381E-06	pf (P)	5.74501E-07

Počet intervalů Poruchy / Hist			44 / 256
f (min)	-177.03632	pf (min)	3.49946E-22
f (max)	5.68052	pf (max)	5.74501E-07

Pravděpodobnost poruchy 1.28342E-06

a následků	Pravděpodobnost	Beta	Kvantil
RC3/CC3	8.40000E-06	4.30000	Out <Min,Max>
RC2/CC2	7.20000E-05	3.80000	Out <Min,Max>
RC1/CC1	4.80000E-04	3.30000	Out <Min,Max>
3	1.28342E-06	4.71061	0.00000

INT	f (z)	pf (z)
0	-177.03632	3.49946E-22
1	-172.88367	1.43531E-19
2	-168.73101	3.60415E-18
3	-164.57836	3.38783E-17
4	-160.42570	1.38759E-16
5	-156.27305	4.15319E-16
6	-152.12039	1.22558E-15
7	-147.96774	3.04909E-15
8	-143.81508	7.62954E-15
9	-139.66242	2.83078E-14
10	-135.50977	6.92422E-14
11	-131.35711	1.61685E-13
12	-127.20446	4.30970E-13
13	-123.05180	9.96538E-13
14	-118.89915	1.90350E-12
15	-114.74649	3.40376E-12
16	-110.59384	7.07307E-12
17	-106.44118	1.19983E-11
18	-102.28852	2.37679E-11
19	-98.13587	3.74468E-11
20	-93.98321	7.47265E-11
21	-89.83056	1.26279E-10
22	-85.67790	2.37630E-10
23	-81.52525	3.57629E-10
24	-77.37259	6.30009E-10
25	-73.21994	9.49990E-10
26	-69.06728	1.44725E-09
27	-64.91462	2.08551E-09
28	-60.76197	3.17714E-09
29	-56.60931	4.40401E-09
30	-52.45666	6.57380E-09
31	-48.30400	8.81673E-09
32	-44.15135	1.31864E-08
33	-39.99869	1.79566E-08
34	-35.84604	2.55174E-08
35	-31.69338	3.48487E-08
36	-27.54072	5.12944E-08
37	-23.38807	6.77108E-08
38	-19.23541	1.01065E-07
39	-15.08276	1.30660E-07
40	-10.93010	1.86457E-07
41	-6.77745	2.48714E-07
42	-2.62479	3.18318E-07
43	1.52786	4.44615E-07
44	5.68052	5.74501E-07

Počet intervalů	256
Min	-177.03632
Max	881.89084
Suma pravděpodobnosti	1.00000
Suma četnosti	2.85758E+21

Hodnota f(z): 0

Velikost kvantilu P:

pf(z)	3.18318E-07
kvantil	1.28342E-06
f(z)	0.00000

Probability of failure

p_f = **1,28.10^{-6}**

meets requirements of

EN1990 for consequences

class RC3/CC3

with design probability

$p_d = 8{,}4.10^{-6}$

Usage of ProbCalc

- Probabilistic assessment of load combinations,
- Probabilistic reliability assessment of cross-sections and systems of statically (in)definite load-bearing constructions,
- Probabilistic aproach to assessment of mass concrete and fibrous concrete mixtures,
- Reliability assessment of arch supports in underground and mining workings,
- Reliability assessment of load-bearing constructions under impact loads,
- Probabilistic calculation of fatigue crack progression in steel structures and bridges.

Fatigue Crack Propagation

- Fatigue crack propagation, with possibility of their predict in time from the beginning of the variable load effect, is the example of calculation, where the probabilistic approach is required.
- Quantity of uncertainties in load effect and structural resistance.
  - **Load effect**: stochastic response to variable traffic load effect in stress range form in locations sensitive to fatigue damage
  - **Structural resistance**:
    - material and geometric characteristic,
    - changes of structural resistance according to different fatigue crack sizes in time,
    - the most difficult is definition of hypothetic initial crack size, solved worldwide (also detectable and acceptable fatigue crack size according to required reliability).

Calculation of Crack Propagation

Linearly elastic fracture mechanics model (known Paris-Erdogan law)

Resistance of structure R

$$R_{(a_d)} = \int_{a_0}^{a_{ad}} \frac{\mathrm{d}a}{\left(\sqrt{\pi . a} . F_{(a)}\right)^m} \qquad R_{(a_{ac})} = \int_{a_0}^{a_{ac}} \frac{\mathrm{d}a}{\left(\sqrt{\pi . a} . F_{(a)}\right)^m}$$

where	a_0	is initial fatigue crack size
	a_d	is detectable fatigue crack size
	a_{ac}	is acceptable fatigue crack size
	$F_{(a)}$	is calibration function represents the course of crack propagation

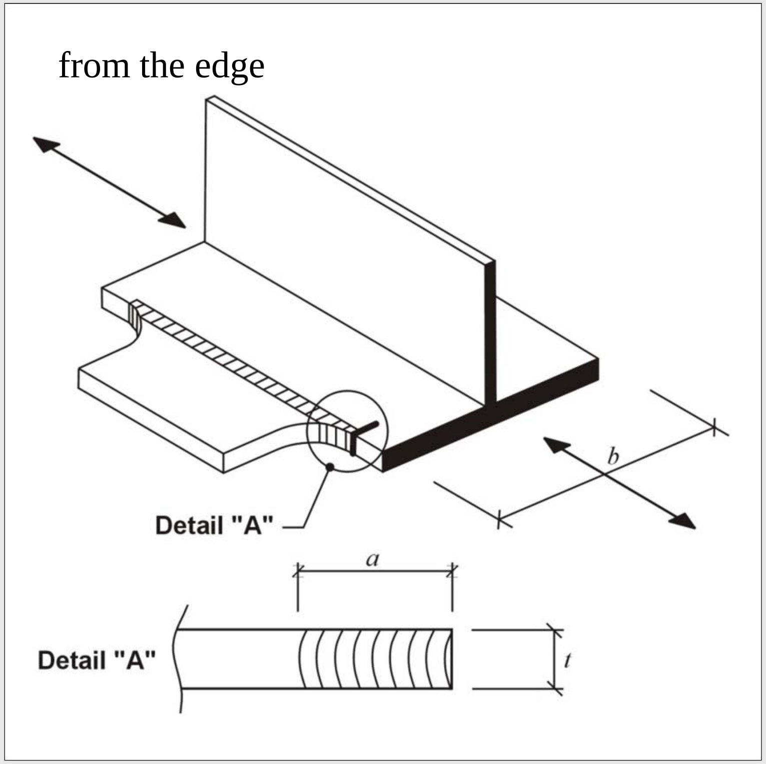
Cumulated effect of loads S

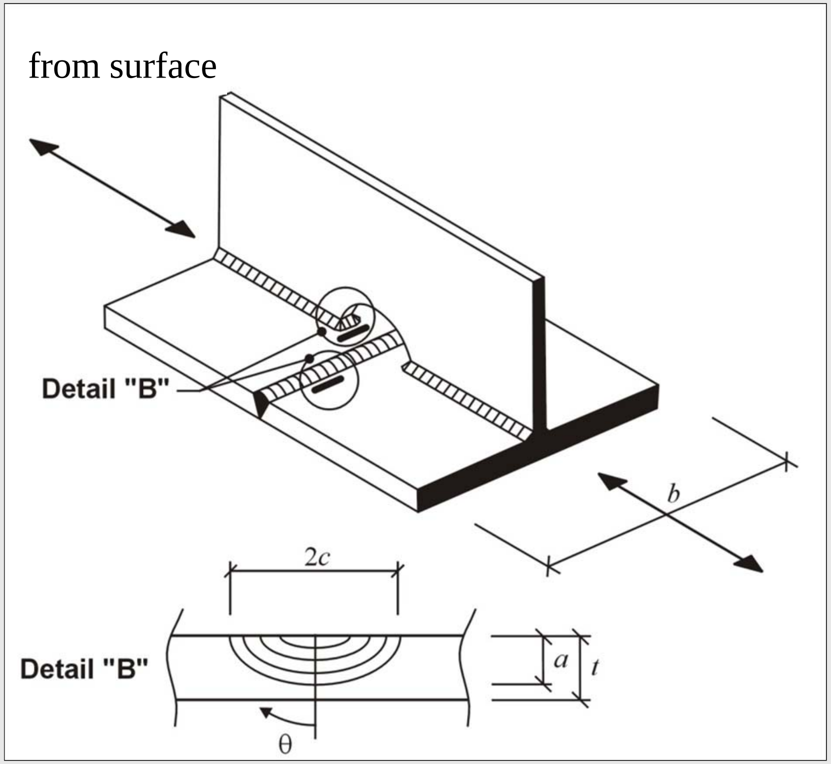
$$S = \int_{N_0}^{N} C . \Delta\sigma^m . \mathrm{d}N = C . \Delta\sigma^m . \left(N - N_0\right)$$

where	N	is number of cycles oscillation of the stress range $\Delta\sigma$ during the fatigue crack propagation from the a_0 to a_d or a_{ac}
	N_0	is number of cycles in time of fatigue crack initialization
	C, m	are material characteristics

Places of Fatigue Damage Danger’s Concentration

Crack’s propagations from the edge or from the surface are possible to monitor according to initial crack position.

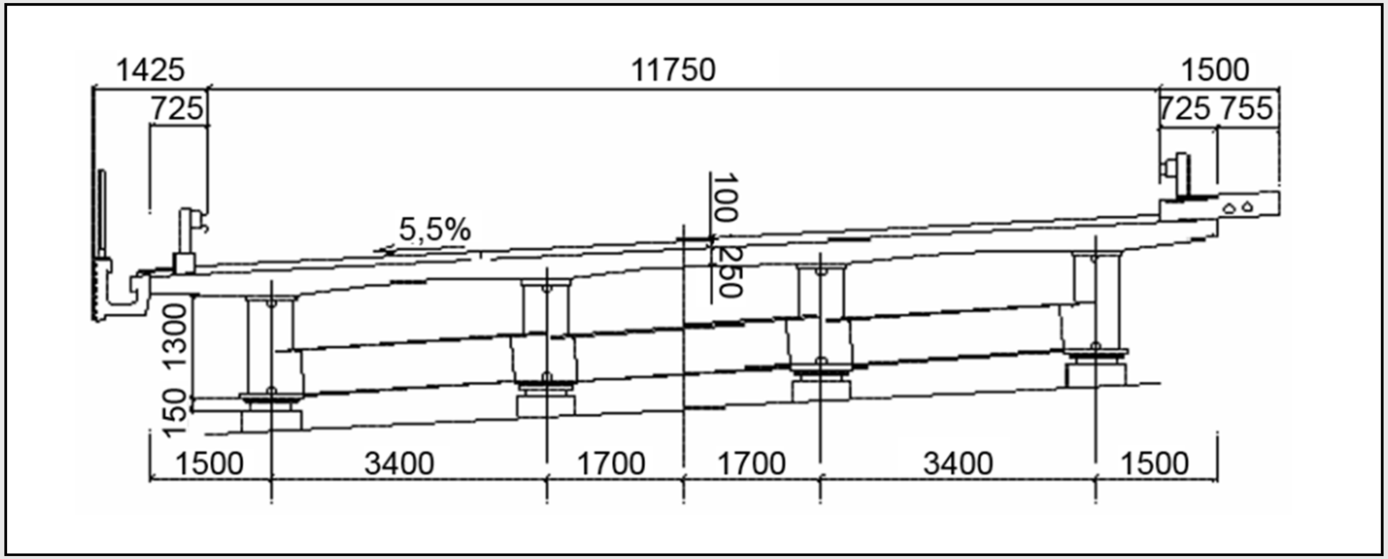

from the edge


from surface

Relliability Assesment of Steel Bridge’s Flange

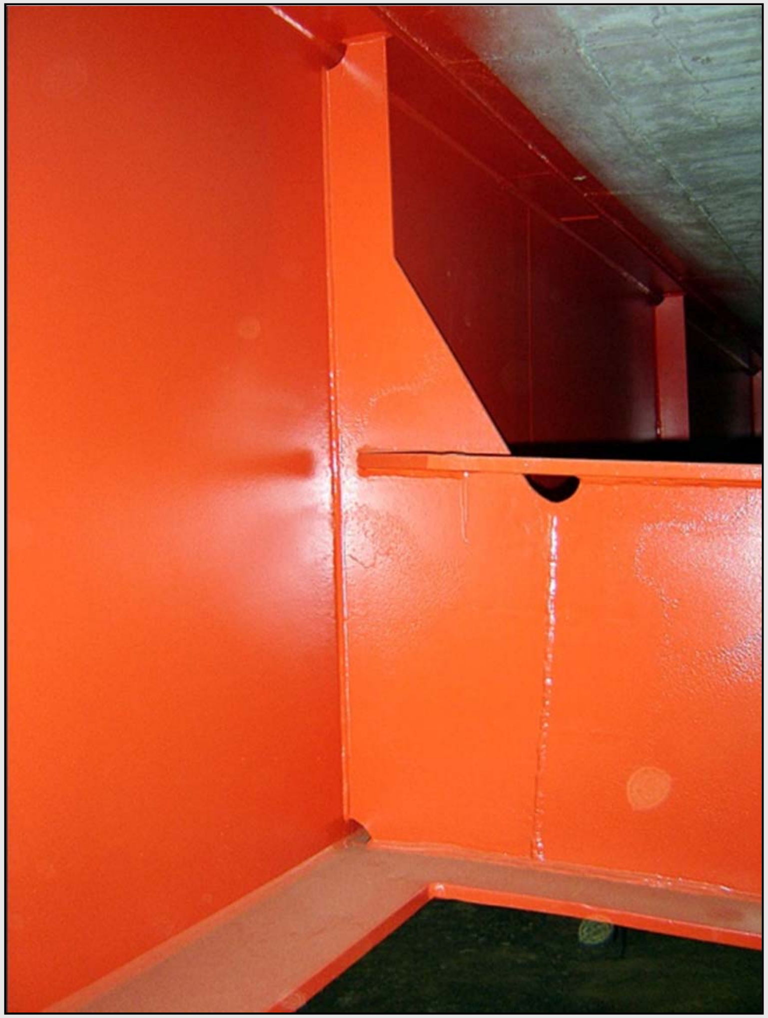
Assessment of steel bridge’s flange was selected for application of theoretical solution after performed studies.

Look to the reviewed road bridge
Photo: Ing. Jaroslav Odrobiňák, Ph.D.



Bridge’s crosswise cut

Detail of Solved Steel Bridge's Flange



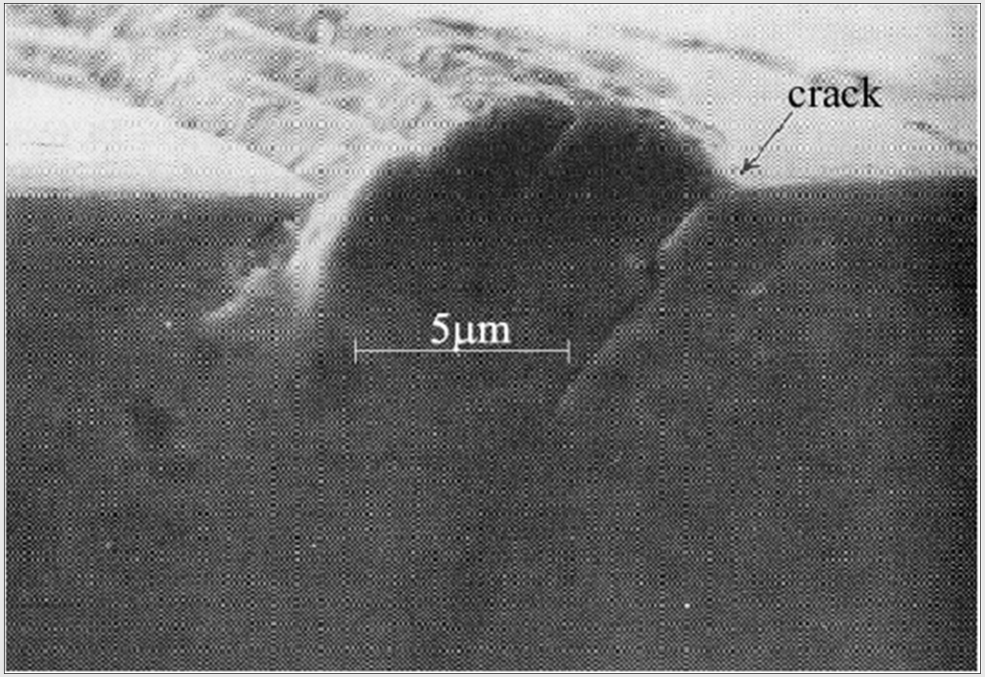
Detail of solved flange in tension sensitive to fatigue damage, photo: Ing. J. Odrobiňák, Ph.D.

Calibration Function

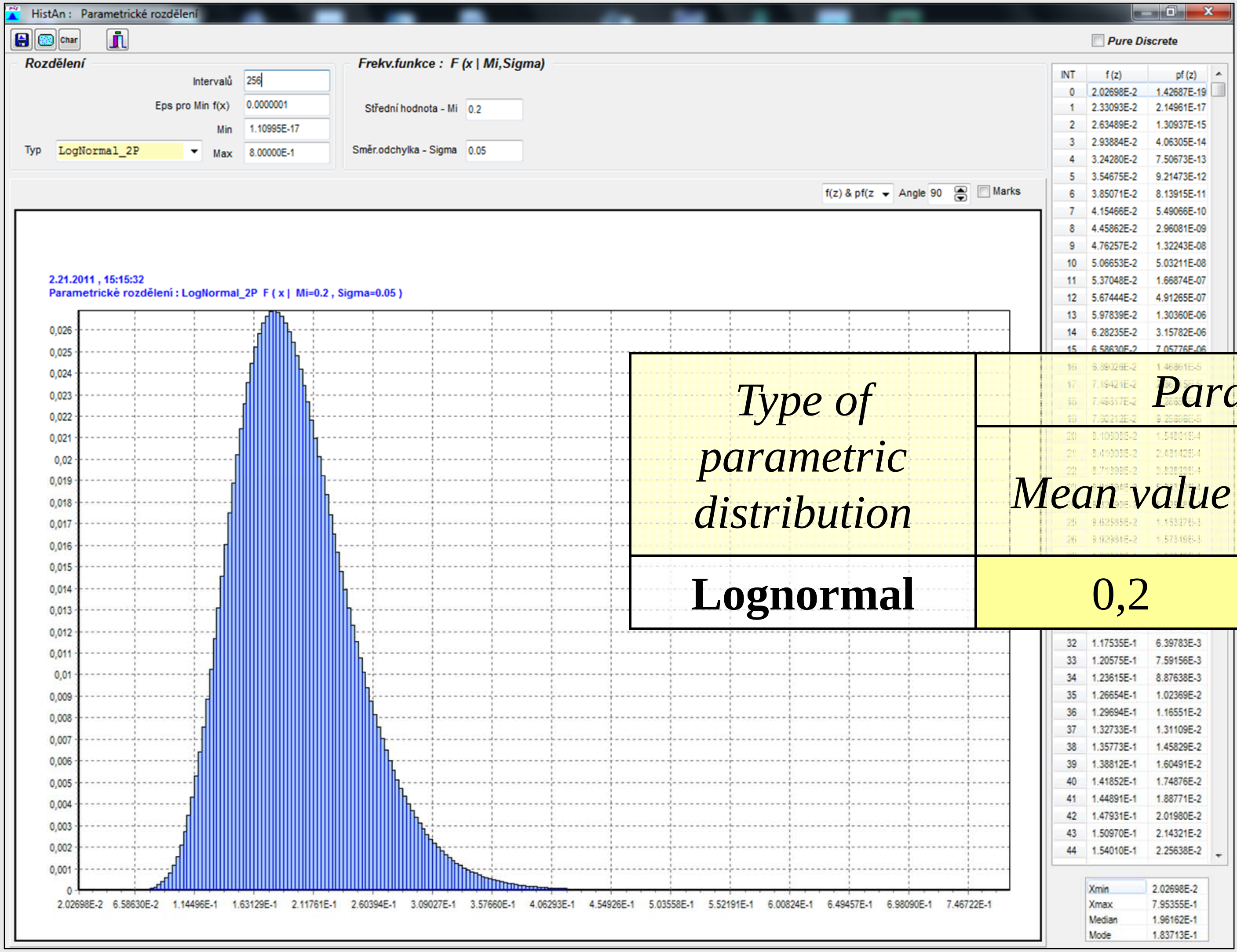
The propagation of the fatigue crack from the edge can be expressed by means of a calibration function:

$$F_{(a)} = 1{,}12 - 0{,}231\left(\frac{a}{b}\right) + 10{,}55\left(\frac{a}{b}\right)^2 - 21{,}72\left(\frac{a}{b}\right)^3 + 30{,}39\left(\frac{a}{b}\right)^4$$

where a is fatigue crack size,
b is width of the flange



Input Variables



HistAn : Parametrické rozdělení

☐ *Pure Discrete*

Rozdělení

Intervalů: 256

Eps pro Min f(x): 0.0000001

Min: 1.10995E-17

Max: 8.00000E-1

Typ: `LogNormal_2P`

Frekv.funkce : F (x | Mi,Sigma)

Střední hodnota - Mi: 0.2

Směr.odchylka - Sigma: 0.05

f(z) & pf(z — Angle: 90 — ☐ Marks

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Parametrické rozdělení : LogNormal_2P F (x | Mi=0.2 , Sigma=0.05)

INT	f (z)	pf (z)
0	2.02698E-2	1.42687E-19
1	2.33093E-2	2.14961E-17
2	2.63489E-2	1.30937E-15
3	2.93884E-2	4.06305E-14
4	3.24280E-2	7.50673E-13
5	3.54675E-2	9.21473E-12
6	3.85071E-2	8.13915E-11
7	4.15466E-2	5.49066E-10
8	4.45862E-2	2.96081E-09
9	4.76257E-2	1.32243E-08
10	5.06653E-2	5.03211E-08
11	5.37048E-2	1.66874E-07
12	5.67444E-2	4.91265E-07
13	5.97839E-2	1.30360E-06
14	6.28235E-2	3.15782E-06
15	6.58630E-2	7.05776E-06
32	1.17535E-1	6.39783E-3
33	1.20575E-1	7.59156E-3
34	1.23615E-1	8.87638E-3
35	1.26654E-1	1.02369E-2
36	1.29694E-1	1.16551E-2
37	1.32733E-1	1.31109E-2
38	1.35773E-1	1.45829E-2
39	1.38812E-1	1.60491E-2
40	1.41852E-1	1.74876E-2
41	1.44891E-1	1.88771E-2
42	1.47931E-1	2.01980E-2
43	1.50970E-1	2.14321E-2
44	1.54010E-1	2.25638E-2

Xmin	2.02698E-2
Xmax	7.95355E-1
Median	1.96162E-1
Mode	1.83713E-1

Initial size of fatigue crack a_0 [mm]

Type of parametric distribution	*Parameters*	
	Mean value	*Standard deviation*
Lognormal	0,2	0,05

Probability of crack occurrence in time t

Using fully probabilistic calculation is possible to solve probability of these defined events:

- Probability of crack **undetection** in time t, crack size $a(t)$ is less than detectable size a_d :

$$P\left(U_{(t)}\right) = P\left(a_{(t)} < a_d\right)$$

- Probability of crack **detection** in time t, crack size $a_{(t)}$ is less than acceptable size a_{ac} :

$$P\left(D_{(t)}\right) = P\left(a_d \le a_{(t)} < a_{ac}\right)$$

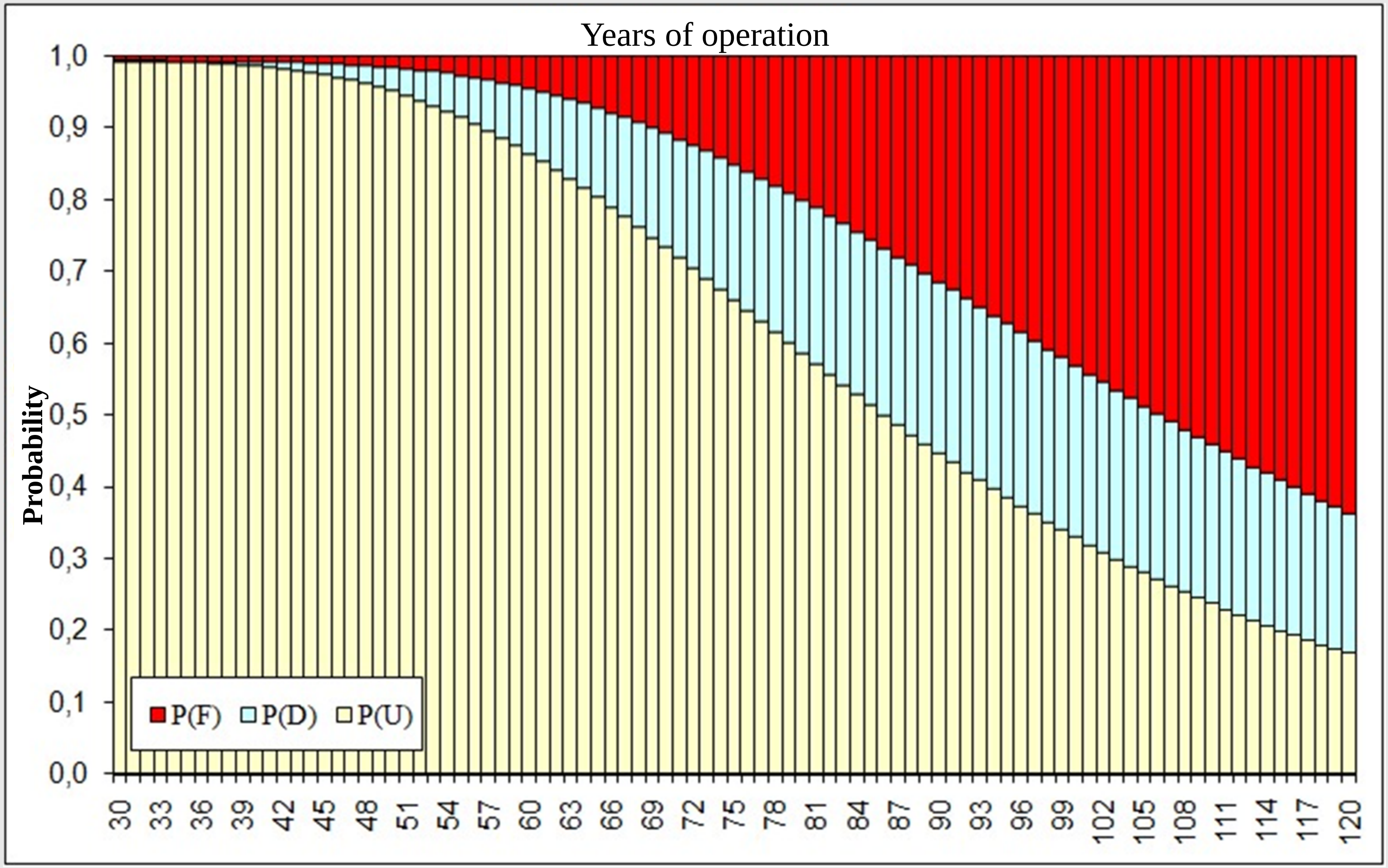
- Probability of crack **detection** in time t, crack size $a_{(t)}$ is equal or greater than acceptable size a_{cr} :

$$P\left(F_{(t)}\right) = P\left(a_{(t)} \ge a_{ac}\right)$$

All of these three events creates full space of event, which can come in time t, can be applied:

$$P\left(U_{(t)}\right) + P\left(D_{(t)}\right) + P\left(F_{(t)}\right) = 1$$

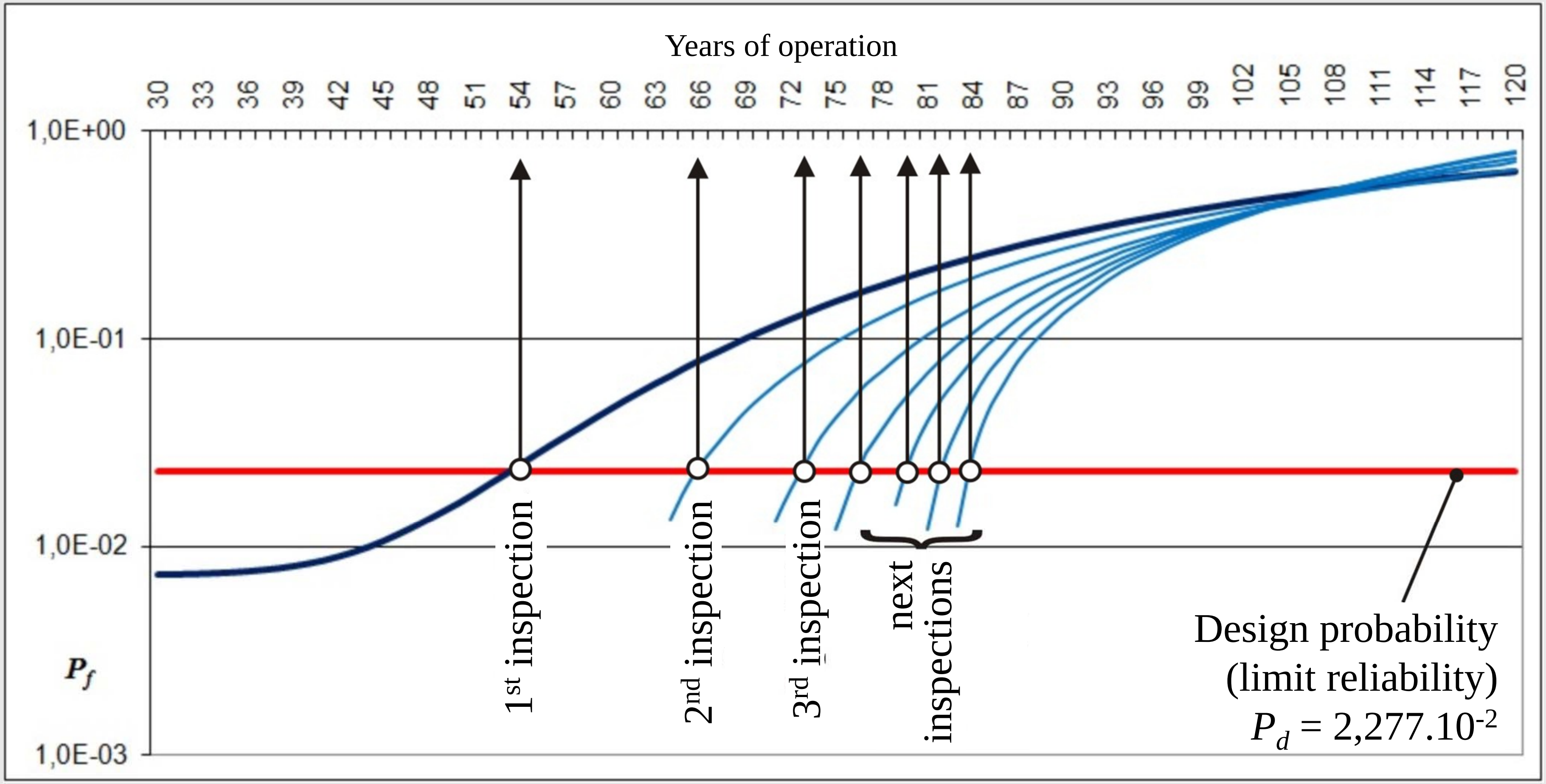
Probability of crack occurrence in time *t*



Probability of U, D and F
events in dependence on years of operation of the bridge

Determining Inspection Time

Fatigue crack from the edge



Dependence of failure probability P_f on years of operation of the bridge

Conclusions

- Software under development for DOProC method is able to compute various probabilistic applications.
- Software includes a number of optimizing techniques to minimalize computing time.
- See http://www.fast.vsb.cz/popv for details and download lite version of ProbCalc.
- Using DOProC method is possible to do probabilistic calculation e.g. fatigue crack progresion in steel structures and bridges and define the time of inspections using conditioned probability.

Thanks for your attention!