Direct Optimized Probabilistic Calculation - DOProC Method

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Contents of Presentation

- Direct Optimized Probabilistic Calculation - DOProC Method
  - Principles of DOProC method
  - Optimizing techniques of the probabilistic calculation
  - Software utilization

- Using DOProC method for the calculation of the fatigue crack propagation
  - Probabilistic Calculation of Fatigue Crack Propagation
    - from the edge
    - from the surface
  - Determining inspection time of the structure using conditioned probability
Essential of DOProC Characteristics

- Should be effectively used for the assessment of structural reliabilities and/or for other probabilistic calculations.
- Input random variables (load, geometry, material properties, imperfections) are expressed by the empirical or parametric distributions in histograms.
- Reliability function under analysis can be expressed analytically or using DLL library.
- Error of calculation is done only by discretization of input and output variables and numerical error.
- The number of intervals (classes) of each histogram is extremely important for the number of needed numerical operations and required computing time.
- The number of numerical operations can be reduced using optimizing techniques of the probabilistic calculation.
Statistics of Dice Throw

Throw of a single dice - all outcomes are equally probable.

\[ p_1 = \frac{1}{n} \]

\[ p_1 = \frac{1}{6} \]
Statistics of Two Dices Throw

Probabilities of identical numbers obtained by the throw of two dices.

\[ p = p_1 \cdot p_2 \]

\[ p_1 = \frac{1}{36} \]
Statistics of Two Dices Throw

The different possibilities for the total of the numbers on two dices.

\[
p(2) = \frac{1}{36}
\]

\[
p(3) = \frac{1}{36} + \frac{1}{36}
\]

\[
p(4) = \ldots
\]
Statistics of Two Dices Throw

The different possibilities for the total of the numbers on two dices - are not equally probable - more ways to get some numbers than others.
Statistics of Two Dices Throw

The different possibilities for the *algebraic difference* of the numbers on two dices.

The probabilities for each algebraic difference are as follows:

\[
p(5) = \frac{1}{36}
\]

\[
p(4) = \frac{1}{36} + \frac{1}{36}
\]

\[
p(3) = \ldots
\]
Statistics of Two Dices Throw

The different possibilities for the arithmetic product of the numbers on two dices.

\[ p(1) = \frac{1}{36} \]
\[ p(2) = \frac{1}{36} + \frac{1}{36} \]
\[ p(3) = \ldots \]
Principle of Numerical Calculation

\[ B = f(A_1, A_2, \ldots, A_j, \ldots A_n) \]
Optimizing Techniques in DOProC

- Grouping of input random variables, which can be expressed by the common histogram.
- Interval optimizing - decreasing the number of intervals in input variable histograms.
- Zonal optimizing - each histogram is divided into areas (zones) depending on their share in the result (failure).
- Trend optimization – using correct or incorrect trend of input variable on the result.
- Grouping of partial calculations results.
- Parallelization of the calculation – calculation is proceeded on number of processors.
- Combination of the mentioned optimizing techniques.

Direct Optimized Probabilistic Calculation - DOProC Method
Grouping of Input Random Variables

Let be \( B = A_1 + A_2 + A_3 + A_4 + \ldots + A_N \)

whereas in each histogram are \( n \) classes (e.g. \( n = 256, N = 10 \))

All allowable combinations are

\[
P_0 = n^N = 256^{10} = 1,20893.10^{24}
\]

The same result is possible to get step-by-step counting of both histograms. Then is

\[
P^*_0 = (N - 1) n^2 = 9,256^2 = 589824
\]

and ratio

\[
P^*_0 / P_0 = (N - 1) n^{(N - 2)} = 9,256^{-8} = 4,87891.10^{-19}
\]

If the creation of common histograms is correct – grouping of input random variables is very rational procedure.
Interval Optimizing

**Sense of interval optimization** is

- number classes minimizing in histograms
- decreasing number of numerical operations and minimizing of computing time

**Direct Optimized Probabilistic Calculation - DOProC Method**
Zonal Analysis and Optimizing

Each histogram is divided into areas (zones – “the zonal optimizing“) depending on their share in the failure, whatever are the values of the other variables:

- **1st zone** – the failure occurs always
- **2nd zone** – the failure may occur depending on values of the other variables
- **3rd zone** – the failure does not occur

\[ p_f = p_{f1} + p_{f2} \]

\[ p_{f1} \text{ always} \]
\[ p_{f2} \text{ only in some events} \]
\[ p_f = 0 \text{ always} \]

Direct Optimized Probabilistic Calculation - **DOProC** Method
Zonal Analysis and Optimizing

Resulting histogram of reliability function $RF$ using DOProC method in action zonal optimizing – so-called „shortened histogram“ $Z^*$
Trend Analysis and Optimizing

- **Monotonous histograms:**
  - zones in histograms are changing in one direction.

- **Non-monotonous histogram:**
  - zones in histograms are not changing only in one direction,
  - histograms have two same zones at least.
Resulting histogram of reliability function $RF$ using DOProC method in action of trend optimizing - histogram $Z^{**}$
Grouping of Partial Calculations Results

Is analogy of input variables grouping.
If e.g.:

\[ RF = R - f(A_1, A_2, A_3, \ldots A_N) \]

then is often useful proceed independently calculation

\[ S = f(A_1, A_2, A_3, \ldots A_N) \]

and following

\[ RF = R - S \]
Parallelization and Combination of the Optimizing Techniques

DOProC method is able to:

- **combine** the mentioned optimizing techniques,
- **parallelize** the calculation (still tested on computers with two processors).
Statistically Dependent Input Variables

- Statistically independent random variables are entered into probabilistic calculation using DOProC method for now.
- Some of input variables are statistically dependent however, e.g. cross-section characteristics, strength properties etc.
- Statistically dependent input variables can enter into calculation using DOProC method indirect as function of useful independent input quantities.
- Problematic of statistically dependent input variables was still take care of cross-section characteristics of rolled shapes especially.
Program System ProbCalc

Implementation of DOProC method was created by three software utilities:

- **HistAn:**
  - utility for histogram analysis

- **HistOp:**
  - utility for basic arithmetic operations with 1 or 2 histograms

- **ProbCalc:**
  - served for probabilistic structural reliability assessment and for other probabilistic problems.
  - computing model can be defined using so-called calculator (text mode) or DLL library (machine code).
  - all of optimizing techniques were implemented.
Program Utility ProbCalc

Probability of failure

\[ p_f = 1,28 \times 10^{-6} \]

meets requirements of

EN1990 for consequences
class RC3/CC3

with design probability

\[ p_d = 8,4 \times 10^{-6} \]
Usage of ProbCalc

- Probabilistic assessment of load combinations,
- Probabilistic reliability assessment of cross-sections and systems of statically (in)definite load-bearing constructions,
- Probabilistic approach to assessment of mass concrete and fibrous concrete mixtures,
- Reliability assessment of arch supports in underground and mining workings,
- Reliability assessment of load-bearing constructions under impact loads,
- Probabilistic calculation of fatigue crack progression in steel structures and bridges.
Fatigue Crack Propagation

- Fatigue crack propagation, with possibility of their predict in time from the beginning of the variable load effect, is the example of calculation, where the probabilistic approach is required.
- Quantity of uncertainties in load effect and structural resistance.
  - **Load effect**: stochastic response to variable traffic load effect in stress range form in locations sensitive to fatigue damage
  - **Structural resistance**:
    - material and geometric characteristic,
    - changes of structural resistance according to different fatigue crack sizes in time,
    - the most difficult is definition of hypothetic initial crack size, solved worldwide (also detectable and acceptable fatigue crack size according to required reliability).
Calculation of Crack Propagation

Linearly elastic fracture mechanics model (known Paris-Erdogan law)

Resistance of structure $R$

$$R(a_d) = \int_{a_0}^{a_d} \frac{da}{(\sqrt{\pi a F(a)})^m}$$

where $a_0$ is initial fatigue crack size
$a_d$ is detectable fatigue crack size
$a_{ac}$ is acceptable fatigue crack size
$F(a)$ is calibration function represents the course of crack propagation

Cumulated effect of loads $S$

$$S = \int_{N_0}^{N} C.\Delta\sigma^m .dN = C.\Delta\sigma^m .(N - N_0)$$

where $N$ is number of cycles oscillation of the stress range $\Delta\sigma$ during the fatigue crack propagation from the $a_0$ to $a_d$ or $a_{ac}$
$N_0$ is number of cycles in time of fatigue crack initialization
$C, m$ are material characteristics
Places of Fatigue Damage
Danger’s Concentration

Crack’s propagations from the edge or from the surface are possible to monitor according to initial crack position.
Reliability Assessment of Steel Bridge’s Flange

Assessment of steel bridge’s flange was selected for application of theoretical solution after performed studies.

Look to the reviewed road bridge
Photo: Ing. Jaroslav Odrobiňák, Ph.D.

Bridge’s crosswise cut
Detail of Solved Steel Bridge’s Flange

Detail of solved flange in tension sensitive to fatigue damage, photo: Ing. J. Odrobiňák, Ph.D.

Probabilistic Calculation of Fatigue Crack Propagation
Calibration Function

The propagation of the fatigue crack from the edge can be expressed by means of a calibration function:

\[ F(a) = 1,12 - 0,231 \left( \frac{a}{b} \right) + 10,55 \left( \frac{a}{b} \right)^2 - 21,72 \left( \frac{a}{b} \right)^3 + 30,39 \left( \frac{a}{b} \right)^4 \]

where \( a \) is fatigue crack size, \( b \) is width of the flange.
### Input Variables

<table>
<thead>
<tr>
<th>Type of parametric distribution</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lognormal</td>
<td>Mean value</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
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</table>

Initial size of fatigue crack $a_0$ [mm]
Probability of crack occurrence in time $t$

Using fully probabilistic calculation is possible to solve probability of these defined events:

- **Probability of crack undetection** in time $t$, crack size $a(t)$ is less than detectable size $a_d$:
  \[
P(U(t)) = P(a(t) < a_d)
  \]

- **Probability of crack detection** in time $t$, crack size $a(t)$ is less than acceptable size $a_{ac}$:
  \[
P(D(t)) = P(a_d \leq a(t) < a_{ac})
  \]

- **Probability of crack detection** in time $t$, crack size $a(t)$ is equal or greater than acceptable size $a_{cr}$:
  \[
P(F(t)) = P(a(t) \geq a_{ac})
  \]

All of these three events creates full space of event, which can come in time $t$, can be applied:

\[
P(U(t)) + P(D(t)) + P(F(t)) = 1
\]
Probability of crack occurrence in time $t$

Probability of $U$, $D$ and $F$ events in dependence on years of operation of the bridge.
Determining Inspection Time

Fatigue crack from the edge

Years of operation

Design probability (limit reliability)

\[ P_d = 2.277 \times 10^{-2} \]

Dependence of failure probability \( P_f \) on years of operation of the bridge
Conclusions

- Software under development for DOProC method is able to compute various probabilistic applications.
- Software includes a number of optimizing techniques to minimalize computing time.
- See http://www.fast.vsb.cz/popv for details and download lite version of ProbCalc.
- Using DOProC method is possible to do probabilistic calculation e.g. fatigue crack progression in steel structures and bridges and define the time of inspections using conditioned probability.
Thanks for your attention!