INVERSE RELIABILITY ANALYSIS IN STRUCTURAL DESIGN

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Introduction

Inverse analysis in structural design – determination of values of "design parameters" (proportions, reinforcement, material properties, etc.) to satisfy particular limit state (both ultimate and serviceability).

Uncertainties and randomness:

- partial safety factor design (semi-probabilistic design)

  $E_d(X_{i,k}, \gamma_i, \ldots) \leq R_d(X_{j,k}, \gamma_j, \ldots)$

  Inverse analysis – analytical

  – "trial and error" method

  Design parameters are deterministic.

- fully probabilistic design

Introduction and motivation

- fully probabilistic design

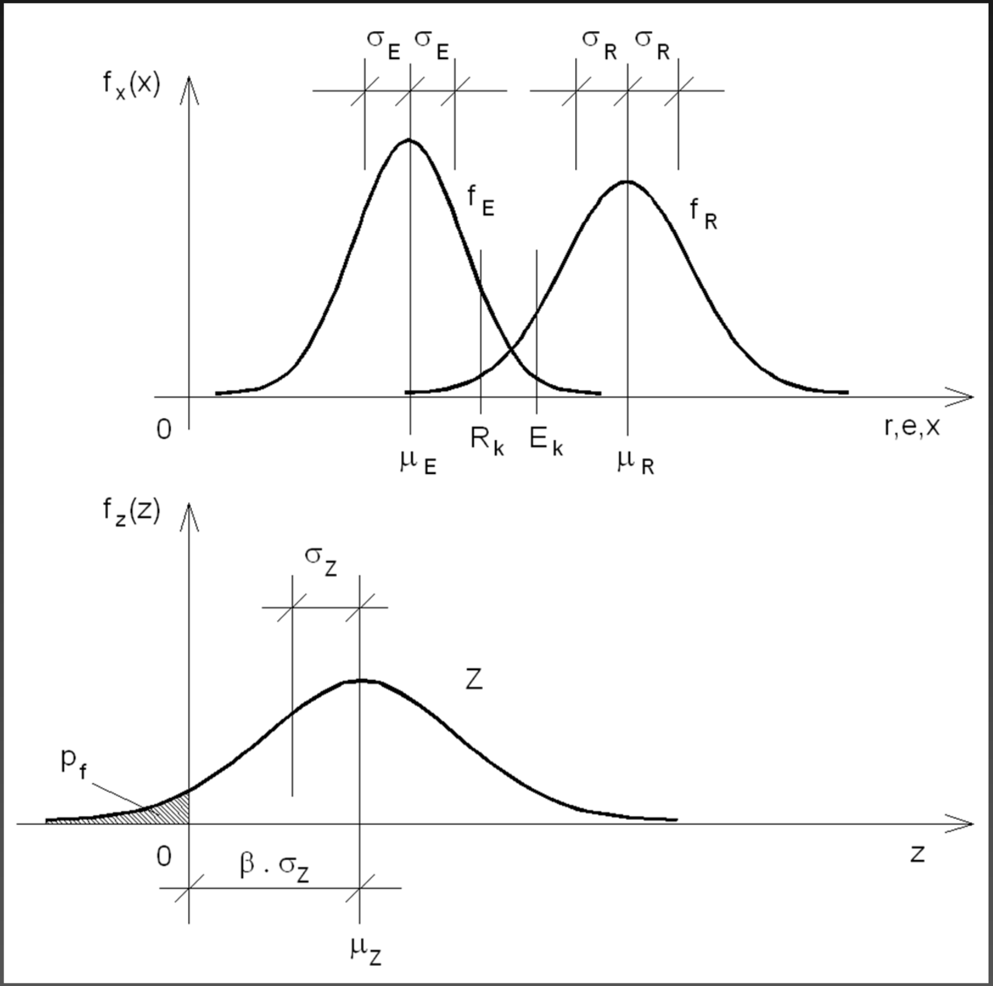
Safety margin: $Z = R - E$

generally: $Z = g(\mathbf{X})$

g(.) … limit state function

Reliability measures:

- Failure probability: $p_f = P(Z < 0)$
- Reliability index: $\beta = -\Phi^{-1}(p_f)$



Inverse analysis – advanced methods, e.g. ANN + stochastic analysis

Design parameters are deterministic or random variables (statistical moments).

Inverse reliability problem formulation

- basic random variables: $\mathbf{X} = X_1, X_2, \ldots, X_j, \ldots, X_n$
- deterministic design parameters: $\mathbf{d} = d_1, d_2, \ldots, d_k, \ldots, d_p$
- design parameters of random variables: $\mathbf{r} = r_1, r_2, \ldots, r_l, \ldots, r_q$
- safety margins Z_j with target failure probabilities $p_{f,j}$

Inverse problem:

Given: $p_{f,j}$

Find: **d** or/and **r**

Subject to: $Z_j = g(\mathbf{X}, \mathbf{d}, \mathbf{r})_j = 0$

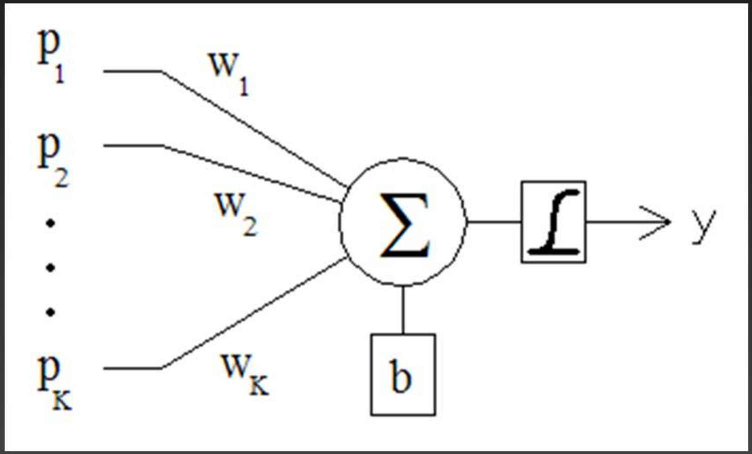
for $j = 1, 2, \ldots, m$.

Design parameters alternatives:

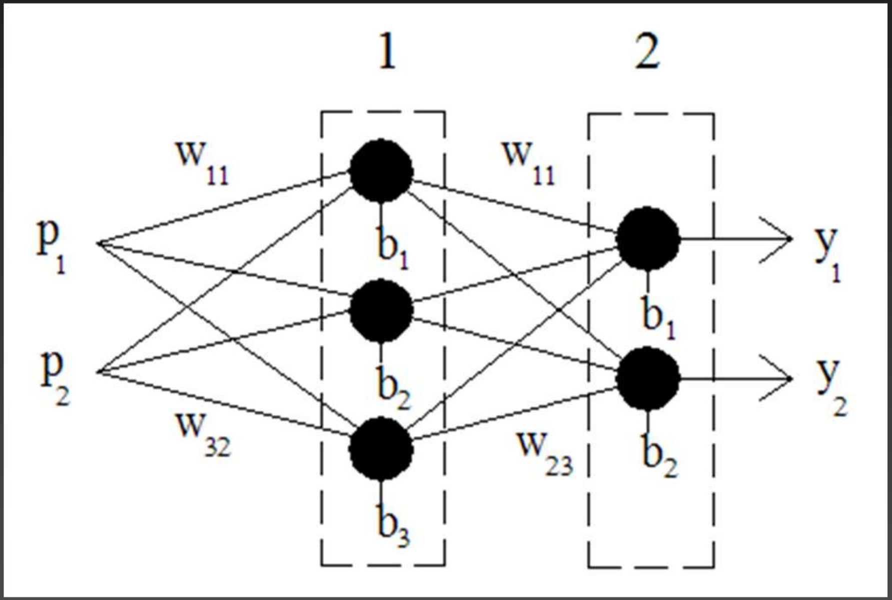
Variable	Deterministic	Random	
		Mean	Std.
d_k	?	–	–
r_l	–	?	prescribed
r_l	–	prescribed	?
r_l	–	?	?

Feed-forward multilayer network (backpropagation type)

NEURON:



NEURAL NETWORK:



(input of ANN, hidden layers, output layer)

Output from 1 neuron:

$$y = f(x) = f\left(\sum_k (w_k \cdot p_k) + b\right)$$

k – number of input impulse (1,...,K)

w_k – weight coefficient of connecting path from *k*-th neuron of previous layer

p_k – impulse from *k*-th neuron previous layer

b – bias of neuron

f – transfer function of neuron

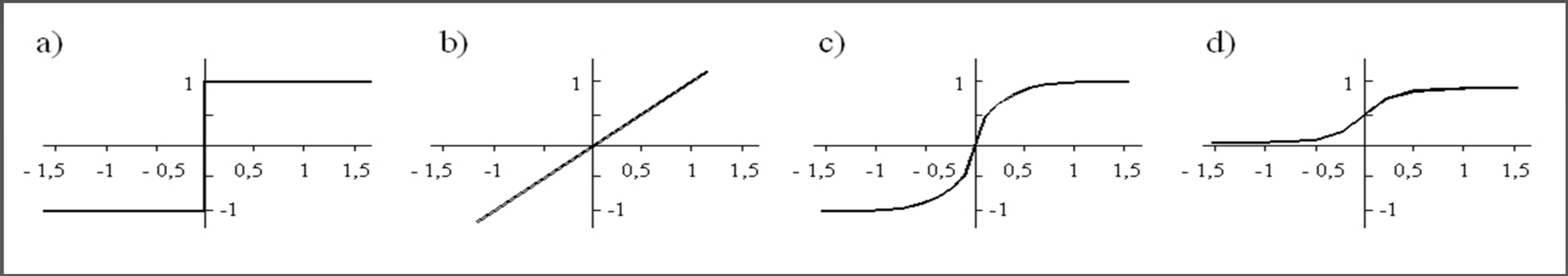
Artificial neural network

Behavior of ANN is determined by:

- number of hidden layers and neurons in them
- synaptic weights – conductivity of connecting paths
- biases
- transfer (activation) functions (binary, linear, nonlinear neurons)

Types of transfer functions:

a) two-valued function
b) linear transfer function
c) hyperbolic tangent (symmetric sigmoid function)
d) sigmoid function



Training of artificial neural network

ANN activities → active period (simulation of process)
ANN activities → adaptive period (training)

Training of neural network:

– training set, i.e. ordered pair $[p_i, y_i]$

(p_i – input vector, y_i – output vector)

Minimization of criterion:

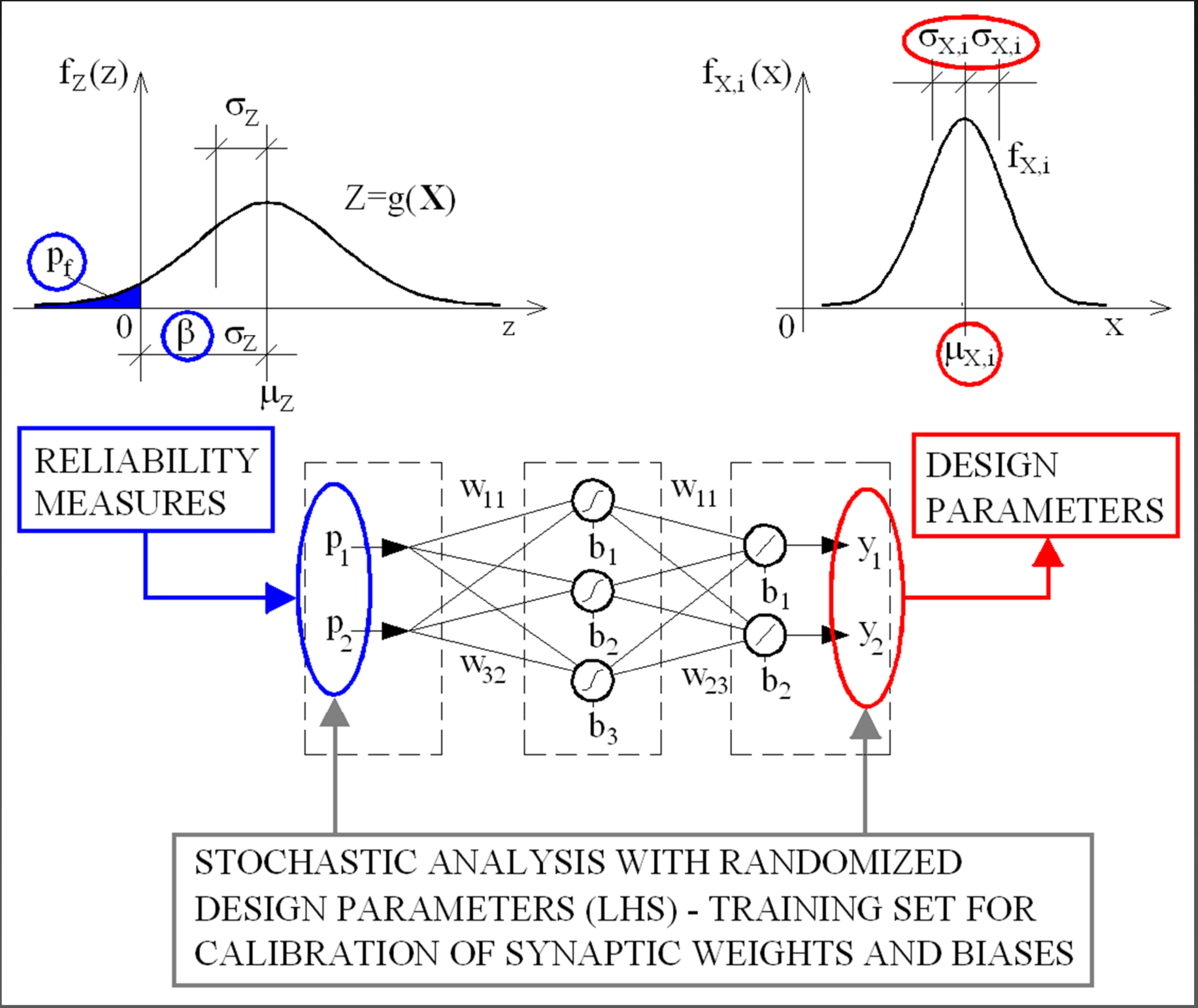
$$E = \frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{K}\left(y_{ik}^{v} - y_{ik}^{*}\right)^2$$

N – number of ordered pairs input - output in training set;

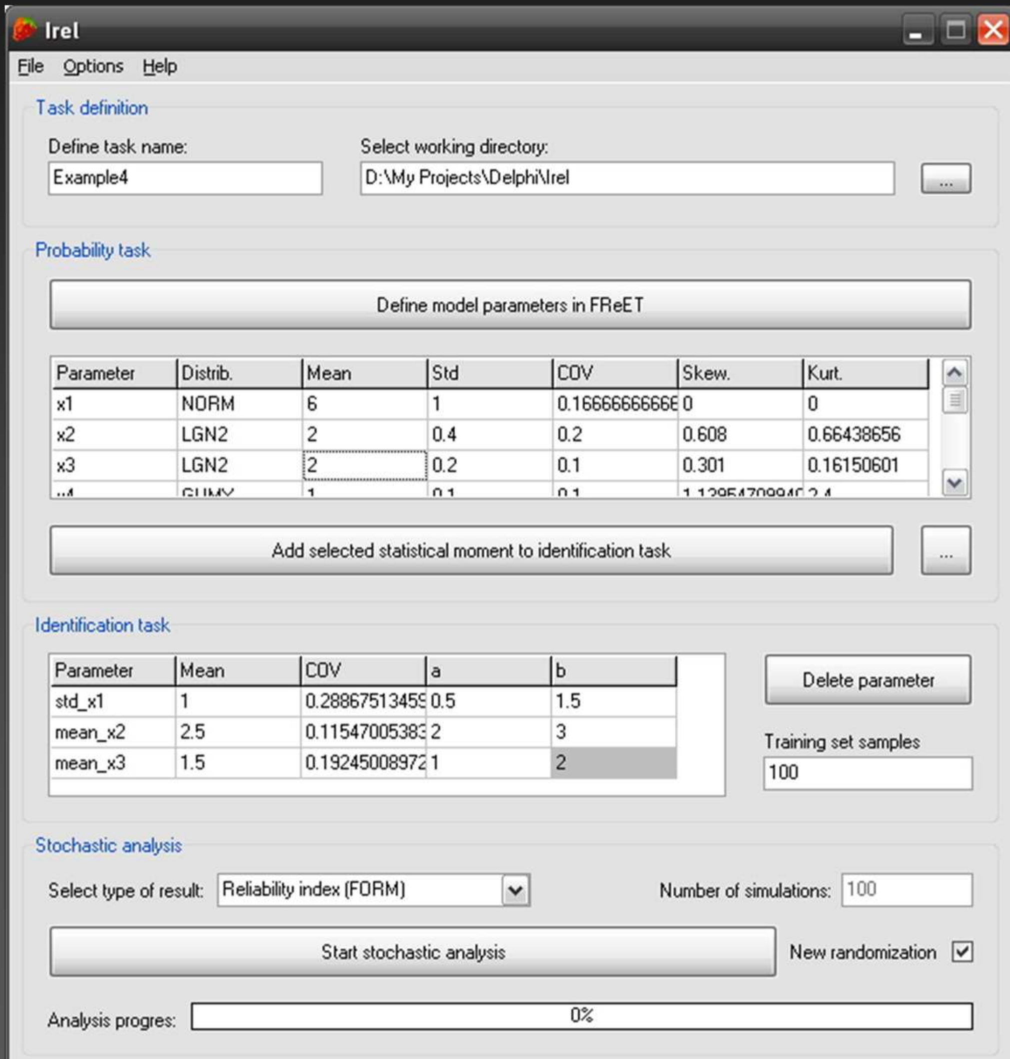
$y_{ik}{}^{*}$ – required output value of k-th output neuron at i-th input;

$y_{ik}{}^{v}$ – real output value (at same input).

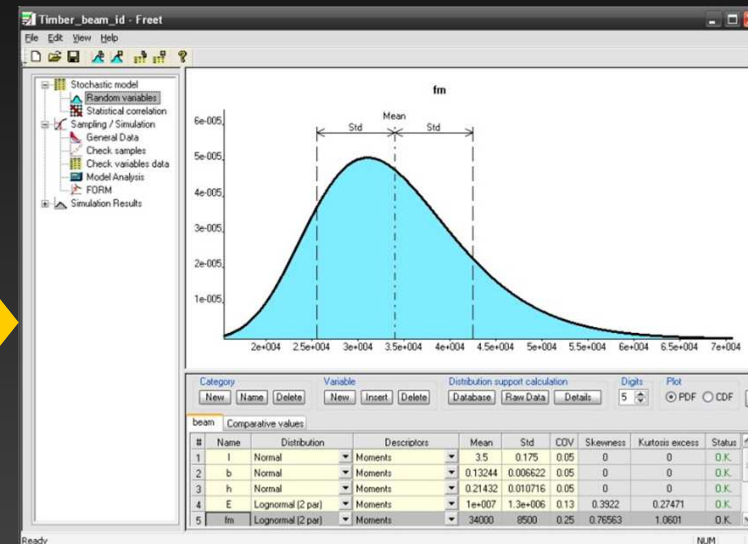
ANN based inverse reliability analysis



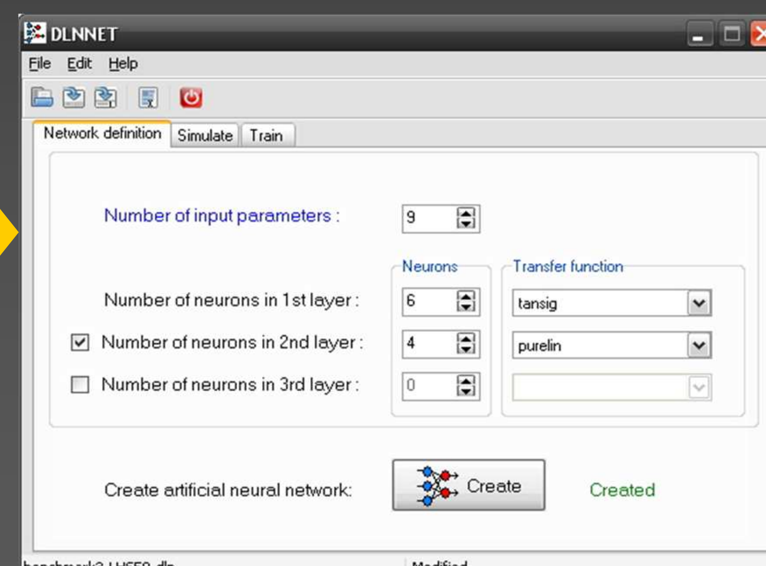
Software tools



IREL: Inverse reliability



FReET: Simulation and reliability



DLNNET: Neural networks

Example 1

A limit state function with single design parameter θ. Target reliability index $\beta = 2.0$.

$$g = \exp\left[-\theta\left(u_1 + 2u_2 + 3u_3\right)\right] - u_4 + 1.5$$

Random variables:

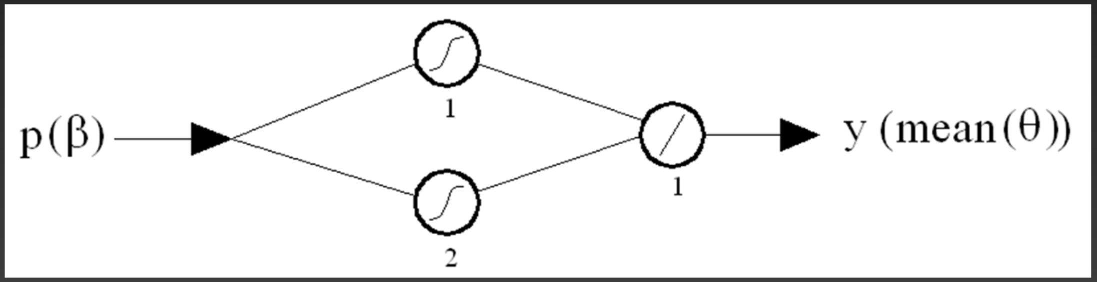
Variable	Distribution	Mean	Std	COV
u_1	Normal	0	1	--
u_2	Normal	0	1	--
u_3	Normal	0	1	--
u_4	Normal	0	1	--
θ	Lognormal (2 par)	?	--	0.30

Design parameter and its randomization (LHS):

Variable	Distribution	Mean	Std	*a*	*b*
mean(θ)	Rectangular	0.30	0.0577	0.20	0.40

Example 1

Artificial neural network:



Training set :

– 41 LHS simulations of *mean(θ)*

– multiple FORM analyses with each LHS simulation ⟶ β

Results:

mean(θ)	β	β_{target}
0.37245	2.0	2.0

Example 2

A set of three limit state functions g_1, g_2, g_3 with target reliability indexes $\beta_1 = 3.0$, $\beta_2 = 3.5$, $\beta_3 = 4.0$.

$$g_1 = x_1^2 - 4x_2 - 2x_3x_4$$
$$g_2 = 2x_1x_4 - x_2x_3$$
$$g_3 = x_1x_2x_4 - 2x_3$$

Random variables:

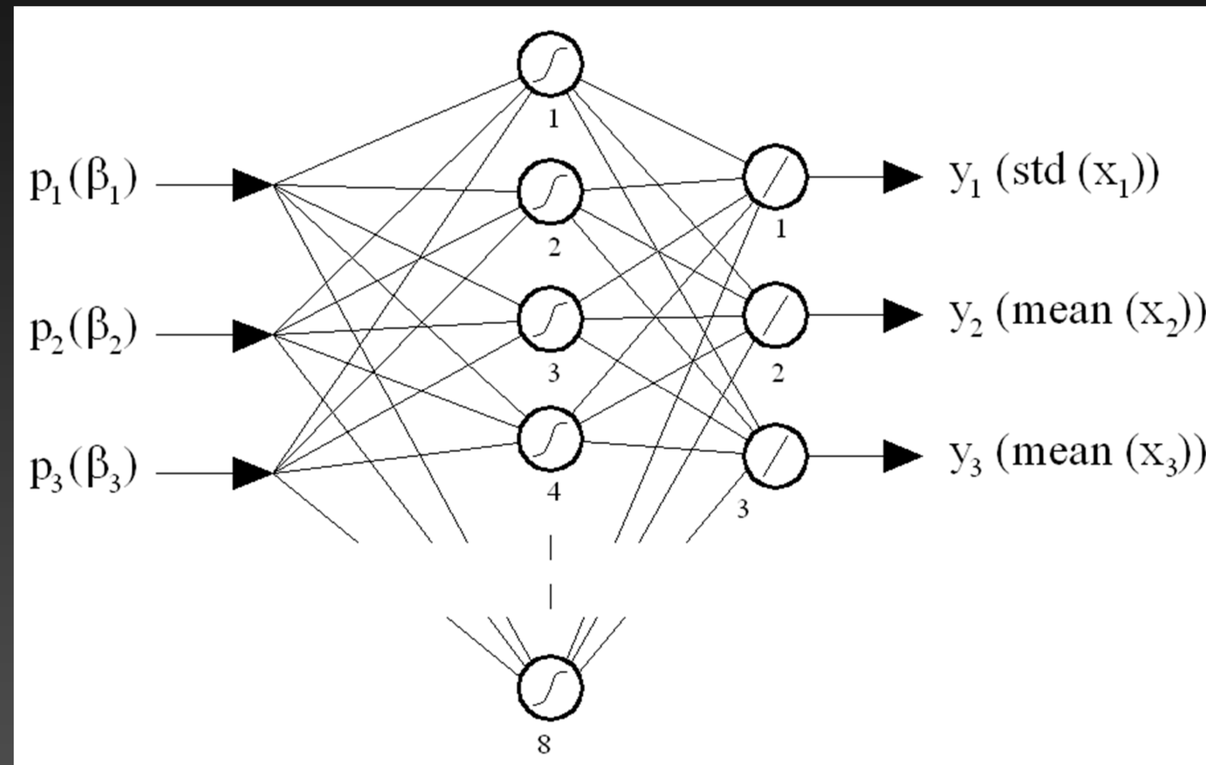
Variable	Distribution	Mean	Std	COV
x_1	Normal	6	?	--
x_2	Lognormal (2 par)	?	--	0.2
x_3	Lognormal (2 par)	?	--	0.1
x_4	Gumbel max EV 1	1.0	0.1	0.1

Design parameter and its randomization (LHS):

Variable	Distribution	Mean	Std	*a*	*b*
$std(x_1)$	Rectangular	1.0	0.2887	0.5	1.5
$mean(x_2)$	Rectangular	2.5	0.2887	2	3
$mean(x_3)$	Rectangular	1.5	0.2887	1	2

Example 2

Artificial neural network:



Training set :

– 100 LHS simulations

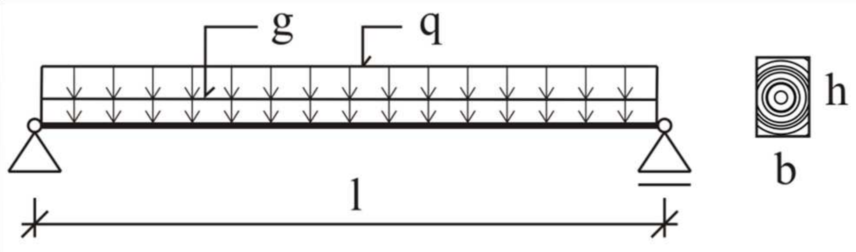
– multiple FORM analyses with each LHS simulation → β

Results:

std(x_3)	*mean(x_2)*	*mean(x_3)*	β_1 ($\beta_{1,target}$)	β_2 ($\beta_{2,target}$)	β_3 ($\beta_{3,target}$)
0.7688	2.1950	2.0779	2.9991 (3.0)	3.5019 (3.5)	3.9981 (4.0)

Example 3

The aim is to design proportions of rectangular cross-section (mean values of width *b* and height *h*) of timber beam to satisfy reliability level given in Eurocodes.



Two limit states are considered:

(1) ultimate limit state – reliability index $\beta = 3.8$.

(2) serviceability limit state – reliability index $\beta = 1.5$.

$$g_1 = M_R - M_E$$

$$g_2 = u_{\lim, fin} - u_{net, fin}$$

$$M_R = \theta_R \frac{1}{6} b h^2 k_{\mathrm{mod}} f_m$$

$$M_E = \theta_E \frac{1}{8} (g+q) l^2$$

$$u_{\lim, fin} = \frac{l}{200}$$

$$u_{net, fin} = \theta_E \left(u_{1, fin} + u_{2, fin} \right)$$

$$u_{1, fin} = \frac{5}{384} \frac{g l^4}{E \frac{1}{12} b h^3} \left(1 + k_{1,def} \right)$$

$$u_{2, fin} = \frac{5}{384} \frac{q l^4}{E \frac{1}{12} b h^3} \left(1 + k_{2,def} \right)$$

Example 3

Random variables:

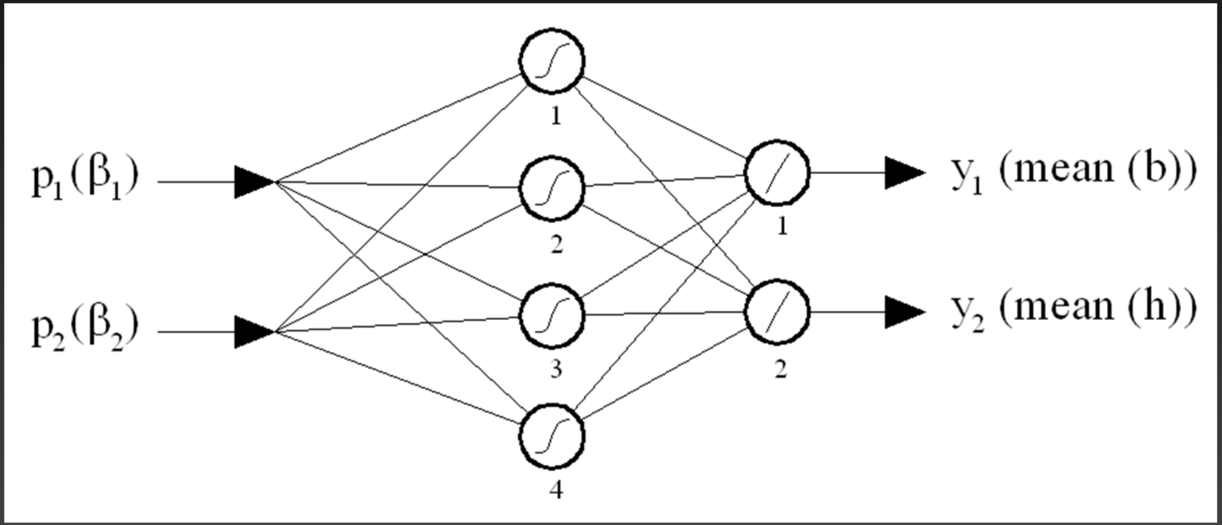
Variable	Distribution	Mean	Std	COV
l [m]	Normal	3.5	0.175	0.05
b [m]	Normal	?	--	0.05
h [m]	Normal	?	--	0.05
E [GPa]	Lognormal (2 par)	10	1.3	0.13
f_m [MPa]	Lognormal (2 par)	34	8.5	0.25
g [kN/m]	Gumbel max EV 1	1.686	0.169	0.10
q [kN/m]	Gumbel max EV 1	2.565	0.770	0.30
θ_R [-]	Lognormal (2 par)	1	0.1	0.10
θ_E [-]	Lognormal (2 par)	1	0.1	0.10

Design parameter and its randomization (LHS):

Variable	Distribution	Mean	Std	*a*	*b*
mean(b)	Rectangular	0.125	0.0144	0.10	0.15
mean(h)	Rectangular	0.225	0.0144	0.20	0.25

Example 3

Artificial neural network:



Training set :

– 100 LHS simulations

– multiple FORM analyses with each LHS simulation ⟹ β

Results:

mean(b)	*mean(h)*	β_1 ($\beta_{1,target}$)	β_2 ($\beta_{2,target}$)
0.13244	0.21432	3.8001 (3.8)	1.5001 (1.5)

⟹ Design:

b = 140 mm

h = 220 mm

β_1 = 4.068

β_2 = 1.912

Summary

+ Methodology for inverse reliability analysis:
  Artificial neural network + stochastic analysis (LHS simulation method)
+ Helpful in case of fully probabilistic design
+ Design parameters – deterministic as well as random ones
+ Multiple design parameters problem as well as multiple limit state function problem can be solved
+ Statistical correlation among variables can be imposed
+ Sotware tools ready for routine applications and we welcome interesting problems for collaboration

Thank you for paying attention!