A Risk Based Approach for the Robustness Assessment of Timber Roofs

Simona Miraglia\textsuperscript{1}, Philipp Dietsch\textsuperscript{2}, Daniel Straub\textsuperscript{3}

\textsuperscript{1} Università degli Studi di Napoli ‘Federico II’
\textsuperscript{2} Chair for timber structures and building construction, TU München
\textsuperscript{3} Engineering Risk Analysis Group, TU München
Collapse of wide span roofs

- Exibition Hall Finland 2003
  - Frühwald et al.

- Siemens Arena Denmark 2003
  - Munch-Andersen

- Bad Reichenhall arena Germany 2006
  - Winter et al.

- Denmark Club Hall, Denmark 2010
  - Pedersen et al.
## Causes of failure


<table>
<thead>
<tr>
<th>Reference</th>
<th>Planning &amp; design %</th>
<th>Construction %</th>
<th>Use/maintenance %</th>
<th>Other(^a) %</th>
<th>Total %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matousek [1]</td>
<td>37</td>
<td>35</td>
<td>5</td>
<td>23</td>
<td>98</td>
</tr>
<tr>
<td>Brand &amp; Glatz [2]</td>
<td>40</td>
<td>40</td>
<td>-</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Yamamoto &amp; Ang [18]</td>
<td>36</td>
<td>43</td>
<td>21</td>
<td>-</td>
<td>100</td>
</tr>
<tr>
<td>Grunau [19]</td>
<td>40</td>
<td>29</td>
<td>31(^b)</td>
<td>-</td>
<td>100</td>
</tr>
<tr>
<td>Reygaertz [20]</td>
<td>49</td>
<td>22</td>
<td>29(^b)</td>
<td>-</td>
<td>100</td>
</tr>
<tr>
<td>Melchers, et al. [21]</td>
<td>55</td>
<td>24</td>
<td>21</td>
<td>-</td>
<td>100</td>
</tr>
<tr>
<td>Fraczek [22]</td>
<td>55</td>
<td>53</td>
<td>-</td>
<td>-</td>
<td>108(^c)</td>
</tr>
<tr>
<td>Allen [23]</td>
<td>55</td>
<td>49</td>
<td>-</td>
<td>-</td>
<td>104c</td>
</tr>
<tr>
<td>Hadipriono [24]</td>
<td>19</td>
<td>27</td>
<td>33</td>
<td>20</td>
<td>99</td>
</tr>
</tbody>
</table>

\(^a\) Includes cases where failure can not be associated with only one factor and may be due to several of them

\(^b\) Building materials, environmental influences, service conditions

\(^c\) Multiple errors for single failure case
Causes of failure

The errors occur more likely in the design phase, followed by the construction phase.

Material deficiency or maintenance

<table>
<thead>
<tr>
<th>Study</th>
<th>Planning &amp; design %</th>
<th>Construction %</th>
<th>Use/maintenance %</th>
<th>Other %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matousek [1]</td>
<td>37</td>
<td>35</td>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td>Brand &amp; Glatz [2]</td>
<td>40</td>
<td>40</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Yamamoto &amp; Ang [18]</td>
<td>36</td>
<td>43</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Grunau [19]</td>
<td>40</td>
<td>29</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>Reygaertz [20]</td>
<td>49</td>
<td>22</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>Melchers, et al. [21]</td>
<td>55</td>
<td>24</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Fraczek [22]</td>
<td>55</td>
<td>53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Allen [23]</td>
<td>55</td>
<td>49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hadipriono [24]</td>
<td>19</td>
<td>27</td>
<td>33</td>
<td>20</td>
</tr>
</tbody>
</table>

a Includes cases where failure cannot be associated with only one factor and may be due to several of them.
b Building materials, environmental influences, service conditions.
c Multiple errors for single failure case.
Robustness

= insensitivity to local failure and to progressive collapse

.....different measures
Redundancy factor, Robustness index, Reliability-Robustness index, Stiffness-Robustness index etc.

.....several code references
• Danish Code of Practice for the Safety of Structures
• EUROCODE
• Joint Committee for Structural Safety
A Robustness Measure

Damage Limit Requirement in EN 1991-1-7:

A failure should not lead to an area failed that exceeds the minimum between
- 15% of the floor area
- 100m²
Reliability & Risk

**Reliability / Probability of failure**
Probability of exceeding ultimate limit states for the structural system at any stage during its life

\[
Pr(F) = \int_{\Omega_F} f(x) \, dx = Pr\left( g(X) \leq 0 \right)
\]

**Risk**
Defined as the **“expected adverse consequences”**

\[
Risk = E[A_F] = \int_{0}^{A_{roof}} a \, f_{A_F}(a) \, da
\]
Case study

Dietsch-Winter 2010
Span: $L = 20.0$ m
Distance between the beams: $e = 6.0$ m
Width: $b = 180$ mm;
Height at Support: $h_a = 600$ mm
Angle upper Edge: $\delta = 10^\circ$
Angle lower edge: $\beta = 6^\circ$;
Inner Radius: $r = 20$ m
Lamella thickness: $t = 32$ mm
Height in Apex: $h_{ap} = 1163$ mm
Beam Failure Mechanism

Bending
- Purlins: Loss of the support
- Other beams: Redistribution of the load (30-40%)

Tension Orthogonal to the grain
- Purlins: Displacement of the support
- Other beams: None
- Beam ‘failed’: Stiffness reduction

Shear
- Purlins: Displacement of the support
- Other beams: None
- Beam ‘failed’: Stiffness reduction
Beam Failure Mechanism

Trigger for progressive collapse

- **Bending**
  - Purlins: Loss of the support
  - Other beams: Redistribution of the load (30-40%)

- **Tension Orthogonal to the grain**
  - Purlins: Displacement of the support
  - Other beams: None

- **Shear**
  - Purlins: Displacement of the support
  - Other beams: None
Timber Secondary Structure

SOLID TIMBER C24

- same utilization factor $0.9 < \eta < 1$
- same reliability of critical sections

(a) Simply supported
(b) Continuous
(c) Lap-Jointed
Secondary Structure Failure Scenario
Stochastic model of the snow load

Poisson spike process with rate $\lambda = 1.175$

\[
F_{Q_{\text{max}}}(q) = \sum_{i=0}^{\infty} \left[ F_{Q_{m}}(q) \right]^n \cdot p_N(n)
\]

\[
F_{Q_{\text{max}}}(q) = \exp \left\{ \lambda T \left[ \exp \left( -\exp \left( -\frac{q - b}{a} \right) \right) - 1 \right] \right\}
\]
Strength of timber (Solid, Glulam)

- Anisotropic
- Strength depends on direction of the grain
- Strength depends on size
- Knots
- Bark pockets
- Resin pocket
- Decay
- Slope of grain
- Lamelle di classe superiore
Bending Resistance: Isaksson’s model

\[ R_{ij} = \exp[v + \omega_i + \chi_{ij}] \]

- Short **weak zones** (knots or clusters) connected by sections of clear wood (series system)
- Strength is a correlated r.v.
- Bending Resistance is Lognormal r.v.
Weakened sections occur as Bernoulli process with \( p=0.30 \)

Bending strength of the weak-element \( R_D \) is Lognormal distributed with 20% lower mean value

Bending strengths of weak-elements \( R_D \) are strongly correlated (\( \rho=0.95 \))
## Random Variables of the model

<table>
<thead>
<tr>
<th>r.v.</th>
<th>Distribution</th>
<th>$\mu$</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow load on the ground [kN/m$^2$]</td>
<td>$Q$</td>
<td>0.384</td>
<td>0.40</td>
</tr>
<tr>
<td>Occurrence [1/y]</td>
<td>$T$</td>
<td>1.175</td>
<td>0.92</td>
</tr>
<tr>
<td>Shape Factor []</td>
<td>$C$</td>
<td>0.78</td>
<td>0.35</td>
</tr>
<tr>
<td>Density [kN/m$^3$]</td>
<td>$G$</td>
<td>4.20</td>
<td>0.10</td>
</tr>
<tr>
<td>Permanent load [kN/m$^2$]</td>
<td>$P$</td>
<td>0.4</td>
<td>0.10</td>
</tr>
<tr>
<td>Bending strength [MPa]</td>
<td>$R_{ij}$</td>
<td>36.97</td>
<td>0.25</td>
</tr>
<tr>
<td>Bending strength [MPa]</td>
<td>$R_{Dij}$</td>
<td>29.57</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Methods of Analysis

- Robustness
  - MCS (Pr(A_F >15%))

- Risk
  - MCS (E[A_F])

- Reliability
  - MCS (Pr(F))
  - FORM (Pr(F), reliability index $\beta$)
Monte Carlo simulations

<table>
<thead>
<tr>
<th>MCS (confidence interval 95%)</th>
<th>$\Pr\left( F(50,yr)\mid D \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Simply supp.</td>
<td>$4.51 \div 4.76 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>(b) Continuous</td>
<td>$1.75 \div 1.92 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>(c) Lap-Jointed</td>
<td>$1.39 \div 1.54 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

\[ \beta \text{ value 2.3-2.7} \]

<table>
<thead>
<tr>
<th>MCS (confidence interval 95%)</th>
<th>$\Pr\left( F(50,yr)\mid D \right)$ $(p=0.30)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Simply supp.</td>
<td>$9.38 \div 9.57 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>(b) Continuous</td>
<td>$5.21 \div 5.50 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>(c) Lap-Jointed</td>
<td>$2.94 \div 3.15 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

\[ \beta \text{ value 1.3-2.3} \]
Monte Carlo simulations

| MCS                  | $E[ A_f | F, \bar{D}]$ |
|----------------------|-------------------------|
| (a) Simply supp.     | 2.87                    |
| (b) Continuous       | 4.04                    |
| (c) Lap-Jointed      | 5.39                    |

- (a) Simply supp.
- (b) Continuous
- (c) Lap–jointed
Monte Carlo simulations

| MCS                      | $E[A_F | F, D]$ |
|--------------------------|----------------|
| (a) Simply supp.         | 2.52           |
| (b) Continuous           | 3.89           |
| (c) Lap-Jointed          | 5.30           |

Graph showing the cumulative distribution of $F_A | F_D(a)$ against % area, with the following line styles:
- (a) Simply supp.
- (b) Continuous
- (c) Lap–jointed
Monte Carlo simulations

The limit of $A_F$ as robustness requirement

$$F_{AF|F}(a) = F_{AF|F,D}(a) \cdot Pr(D|F) + F_{AF|F,D}(a) \cdot Pr(D|F)$$
Monte Carlo simulations

The limit of $A_F$ as robustness requirement

$$F_{A_F|F}(a) = F_{A_F|F,D}(a) \cdot Pr(D|F) + F_{A_F|F,D}(a) \cdot Pr(D|F)$$

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - F(A_F = 15%</td>
<td>F)$</td>
</tr>
<tr>
<td>(a) Simply supp.</td>
<td>0.027</td>
</tr>
<tr>
<td>(b) Continuous</td>
<td>0.035</td>
</tr>
<tr>
<td>(c) Lap-Jointed</td>
<td>0.032</td>
</tr>
</tbody>
</table>
**Risk**

\[ Risk = E[A_F] = \int_0^{A_{\text{roof}}} a \cdot f_{A_F}(a) \, da \]

<table>
<thead>
<tr>
<th>MCS</th>
<th>(E[A_F], \Pr(D) = 0.01)</th>
<th>(E[A_F], \Pr(D) = 0.10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Simply supp.</td>
<td>1.34 (\times 10^{-3})</td>
<td>1.43 (\times 10^{-3})</td>
</tr>
<tr>
<td>(b) Continuous</td>
<td>0.75 (\times 10^{-3})</td>
<td>0.88 (\times 10^{-3})</td>
</tr>
<tr>
<td>(c) Lap-Jointed</td>
<td>0.79 (\times 10^{-3})</td>
<td>0.87 (\times 10^{-3})</td>
</tr>
<tr>
<td>Results</td>
<td>Reliability</td>
<td>Robustness</td>
</tr>
<tr>
<td>-----------------</td>
<td>----------------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>(a) Simply supp.</td>
<td>$\Pr(F_{50y}) = 4.51 \div 4.76 \cdot 10^{-2}$</td>
<td>$\Pr(A_F &gt; 15%</td>
</tr>
<tr>
<td>(b) Continuous</td>
<td>$\Pr(F_{50y}) = 1.75 \div 1.92 \cdot 10^{-2}$</td>
<td>$\Pr(A_F &gt; 15%</td>
</tr>
<tr>
<td>(c) Lap-Jointed</td>
<td>$\Pr(F_{50y}) = 1.39 \div 1.54 \cdot 10^{-2}$</td>
<td>$\Pr(A_F &gt; 15%</td>
</tr>
</tbody>
</table>
Conclusions Purlins Assessment

- Statically Determined (Simply supp.) secondary system is more robust

- Statically undetermined (Continuous and Lap-Jointed) secondary system have the lowest Pr(F) and Risk

The more robust configuration might be not the optimal one
Conclusions Purlins Assessment

- Statically Determined (Simply supp.) secondary system is more robust

- Statically undetermined (Continuous and Lap-Jointed) secondary system have the lowest Pr(F) and Risk

