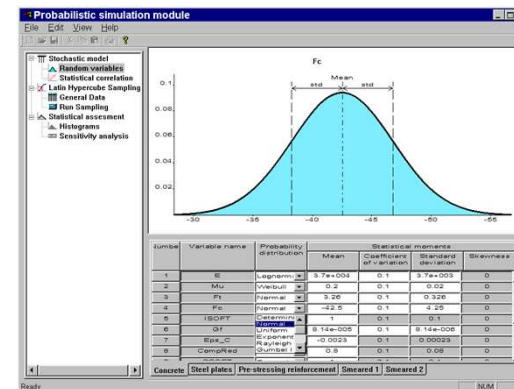


Uncertainty modelling using software FReET

D. Novak, M. Vorechovsky, R. Rusina

Brno University of Technology Brno, Czech Republic



Outline

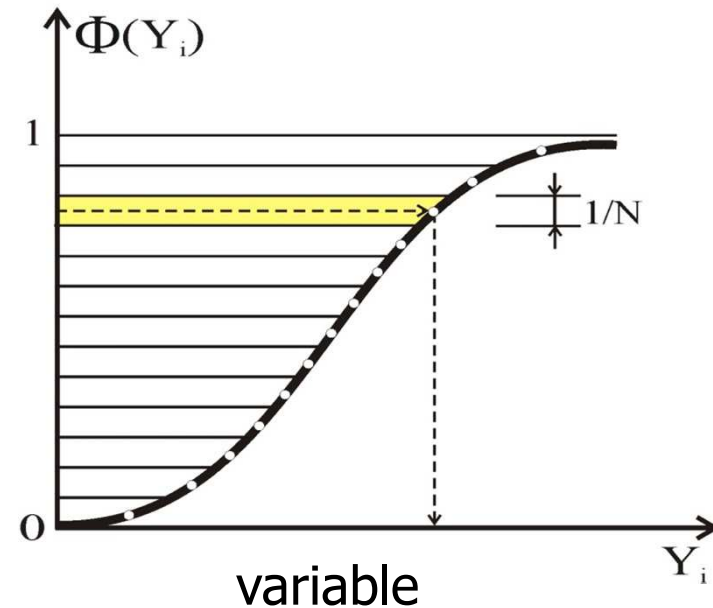
- Introduction
- Methods and main features
- Software FReET
- Selected applications
 - Probabilistic analyses of concrete structures
 - Statistical size effect studies
 - Identification of computational model parameters

Methods and main features

- Small-sample simulation of Monte Carlo type
- Imposing statistical correlation
- Sensitivity analysis
- Reliability analysis

Latin hypercube sampling

- The range (0; 1) of PDF $\Phi(Y_i)$ of each random variable Y_i is divided into N non-overlapping intervals of equal probability $1/N$ (McKay et al. 1979. Iman & Conover 1980, Iman & Shortencarier 1984).
- The centroids are selected randomly based on random permutations of integers.
- Every interval of each variable is used only once during the simulation process.

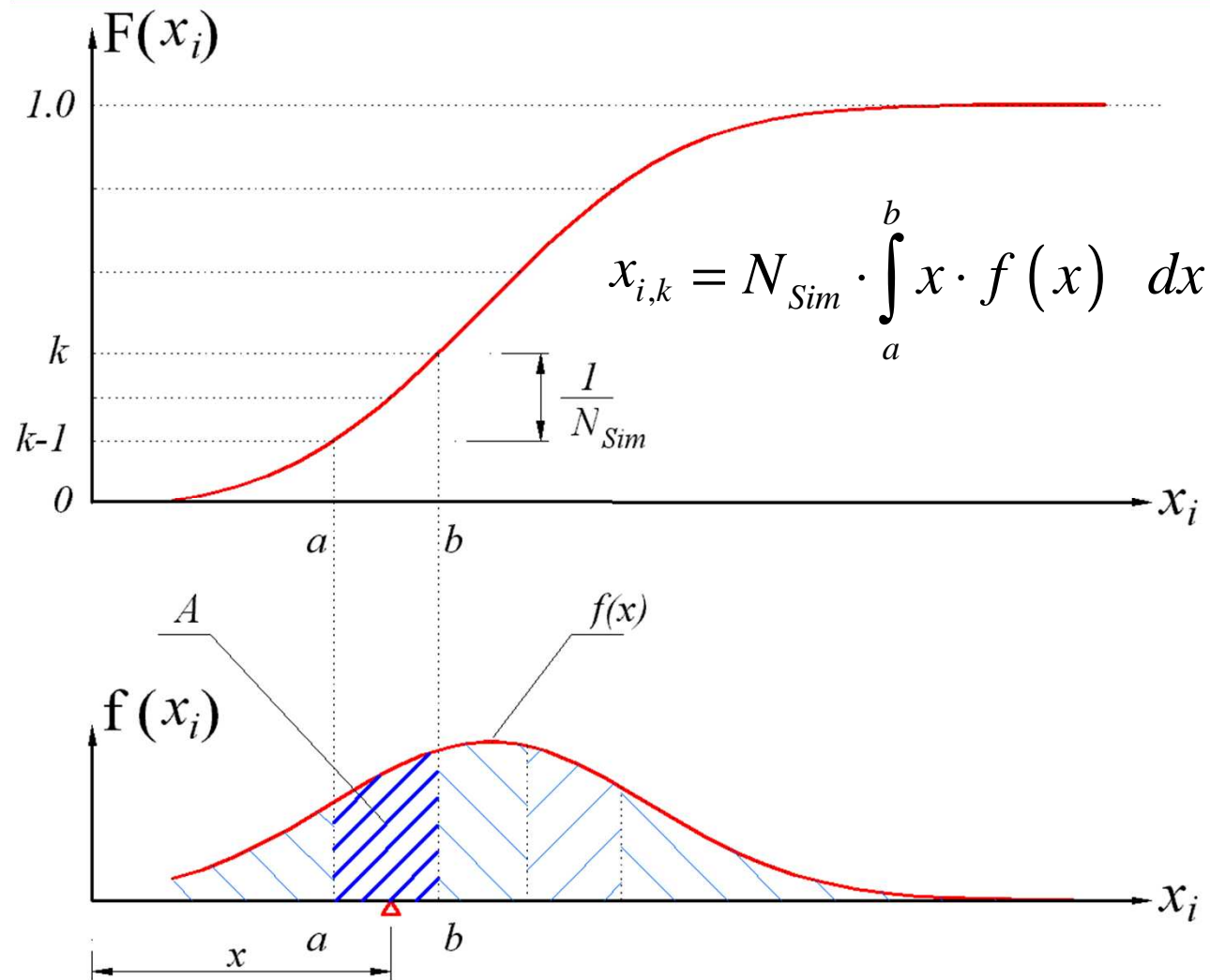


9	1	10	4	1
4	5	3	7	10
8	3	9	10	8
6	2	8	9	3
10	4	4	8	8
7	10	5	1	2
5	9	6	5	4
2	6	7	2	6
1	7	1	6	7
3	6	2	3	5

simulation

LHS: Step 1 - simulation

Huntington & Lyrintzis
(1998)



- Mean value: accurately
- Stand. deviation: significant improvement

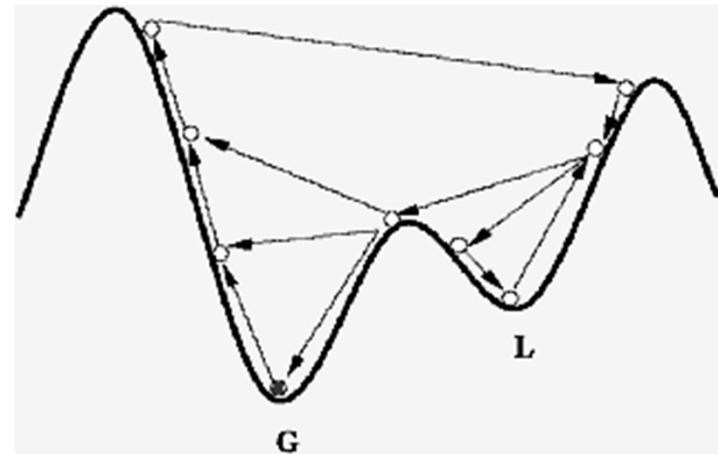
LHS: Step 2 – imposing statistical correlation

		variable	
simulation	x_1	y_1	z_1
	x_2	y_2	z_2
	x_3	y_3	z_3
	x_4	y_4	z_4
	x_5	y_5	z_5
	x_6	y_6	z_6
	x_7	y_7	z_7
	x_8	y_8	z_8

	x_{NSim}	y_{NSim}	z_{NSim}

- Simulated annealing: Probability to escape from local minima
- Cooling - decreasing of system excitation
- Boltzmann PDF, energetic analogy

$$P_r(E) \approx e^{\left(\frac{-\Delta E}{k_b \cdot T}\right)}$$

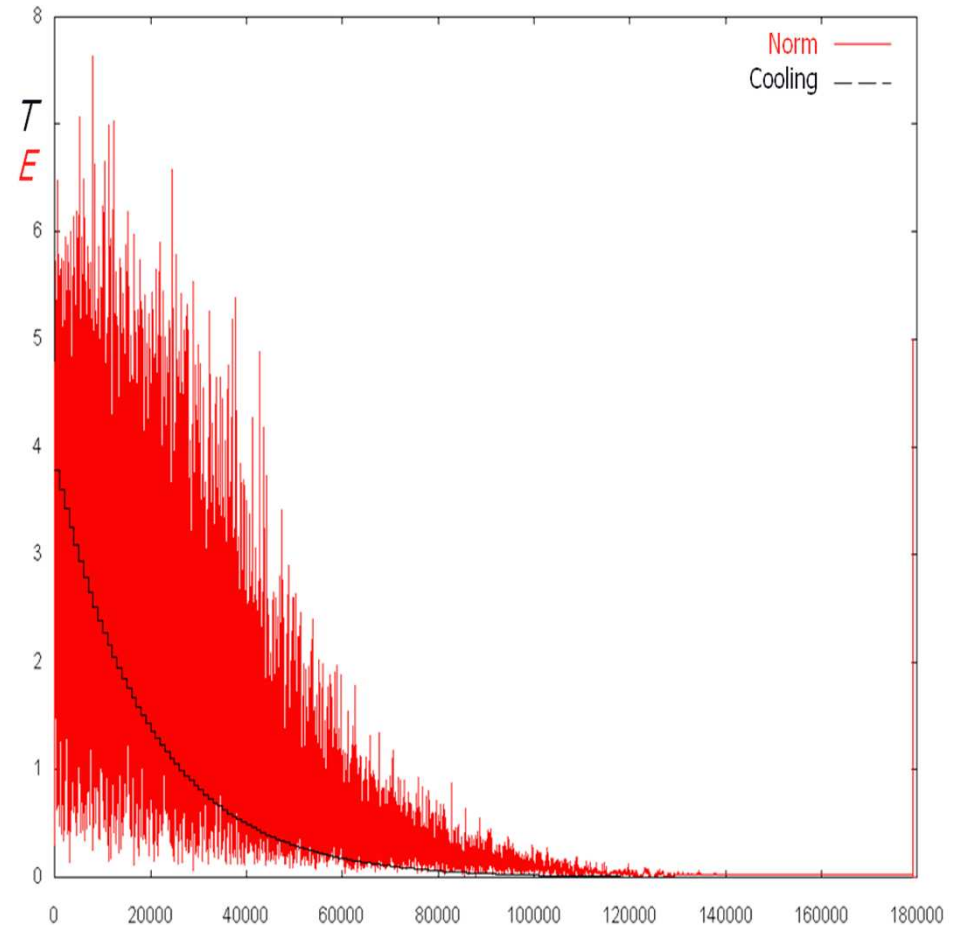


LHS: Step 2 – imposing statistical correlation

variable

	x_1	y_1	...	z_1
	x_2	y_2	...	z_2
	x_3	y_3	...	z_3
	x_4	y_4	...	z_4
	x_5	y_5	...	z_5
	x_6	y_6	...	z_6
	x_7	y_7	...	z_7
	x_8	y_8	...	z_8

	x_{NSim}	y_{NSim}	...	z_{NSim}
simulation				



Sensitivity analysis

Nonparametric rank-order correlation between input variables and output response variable

- Kendall tau $\tau_i = \tau(q_{ji}, p_j), \quad j = 1, 2, \dots, N$
- Spearman

$$r^s = 1 - \frac{6 \sum_{i=1}^n d_i^2}{n(n-1)(n+1)}$$

- Robust - uses only orders
- Additional result of LHS simulation, no extra effort
- Bigger correlation coefficient = high sensitivity
- Relative measure of sensitivity (-1, 1)

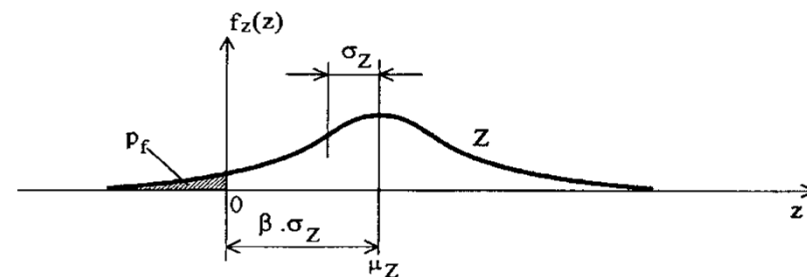
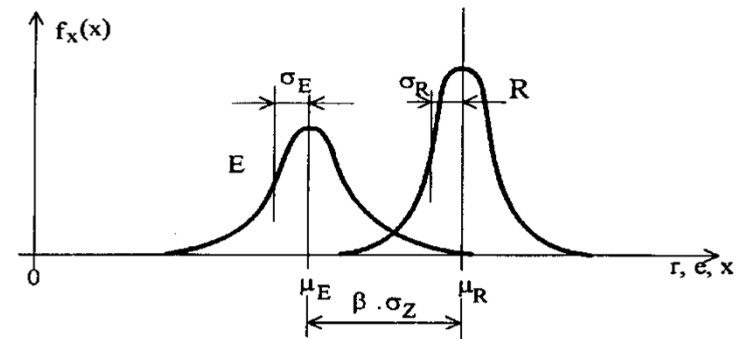
INPUT	OUTPUT
$x_{1,1}$	R_1
...	...
...	...
...	...
$x_{1,N}$	R_N



INPUT	OUTPUT
$q_{1,1}$	p_1
...	...
...	...
...	...
$q_{1,N}$	p_N

Reliability analysis

- Simplified – rough estimates, as constrained by extremally small number of simulations (10-100)!
- Cornell safety index
- Curve fitting
- FORM, importance sampling response surface...



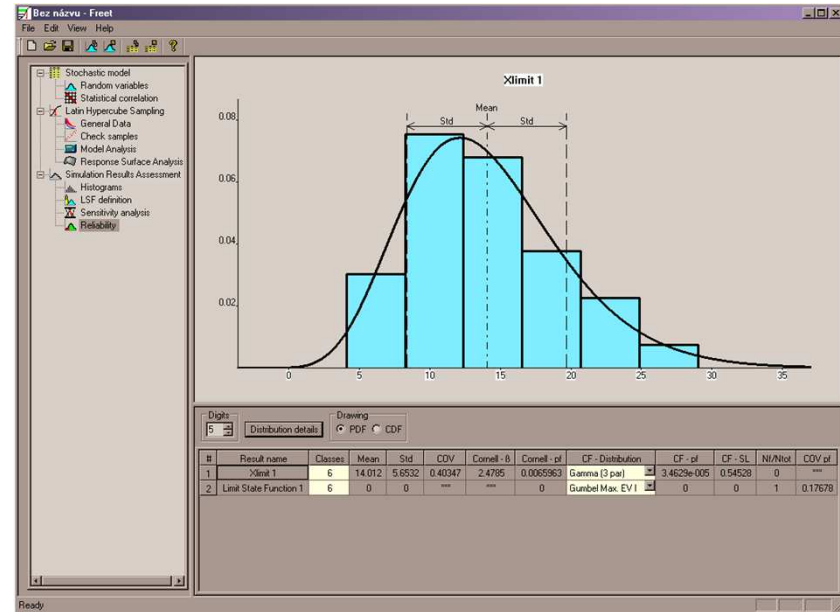
FREET

Probabilistic techniques

- Crude Monte Carlo simulation
- Latin Hypercube Sampling (3 types)
- First Order Reliability Method (FORM)
- Curve fitting
- Simulated Annealing
- Bayesian updating

Response/Limit state function

- Closed form (direct) using implemented Equation Editor (simple problems)
- Numerical (indirect) using user-defined DLL function prepared practically in any programming language (C++, Fortran, Delphi, etc.)
- General interface to third-parties software using user-defined *.BAT or *.EXE

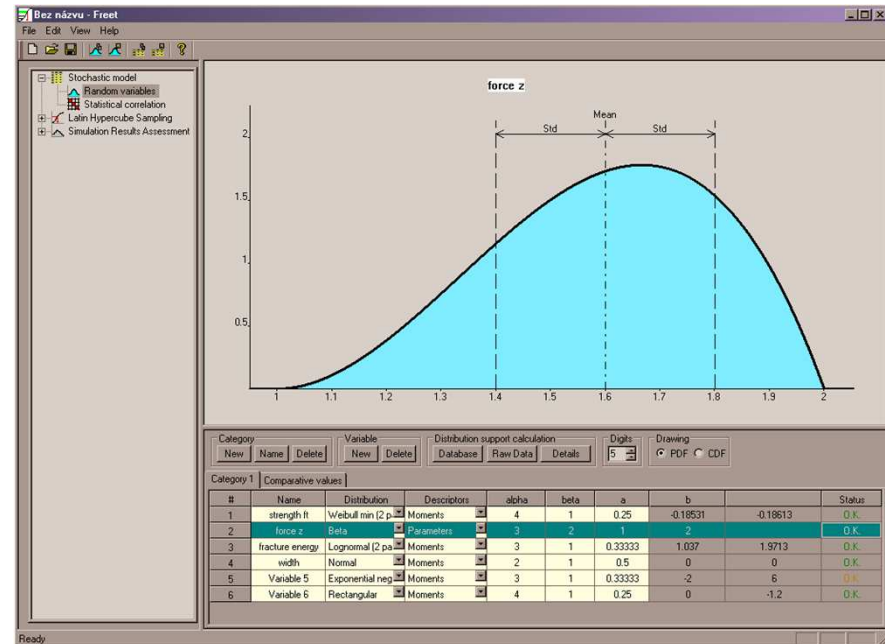


<http://www.freet.cz>

FREET

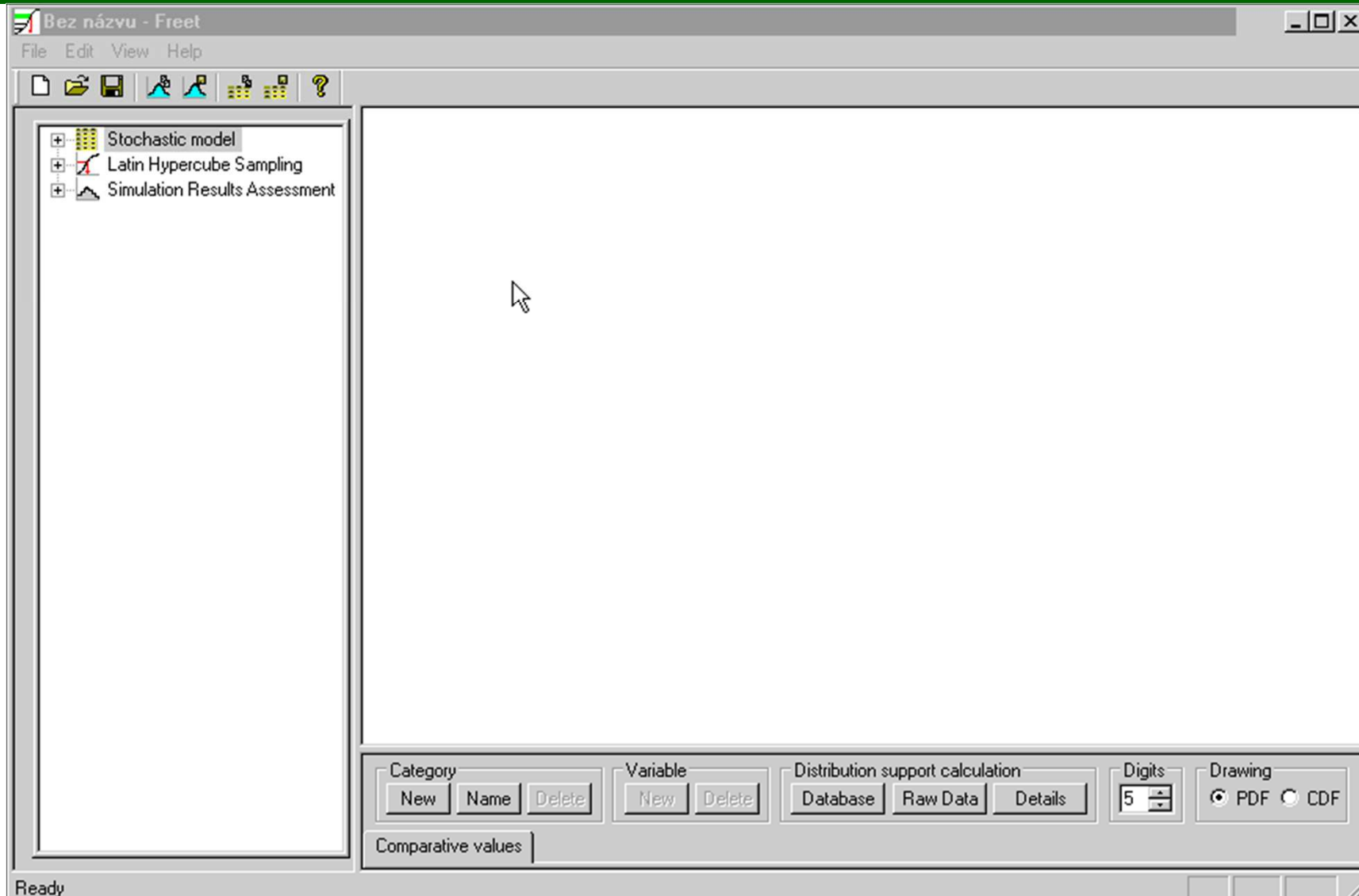
Stochastic model (inputs)

- Friendly Graphical User Environment (GUE)
- 30 probability distribution functions (PDF), mostly 2-parametric, some 3-parametric, two 4-parametric (Beta PDF and normal PDF with Weibullian left tail)
- Unified description of random variables optionally by statistical moments or parameters or a combination of moments and parameters
- PDF calculator
- Statistical correlation (also weighting option)
- Categories and comparative values for PDFs
- Basic random variables visualization, including statistical correlation in both Cartesian and parallel coordinates



<http://www.freet.cz>

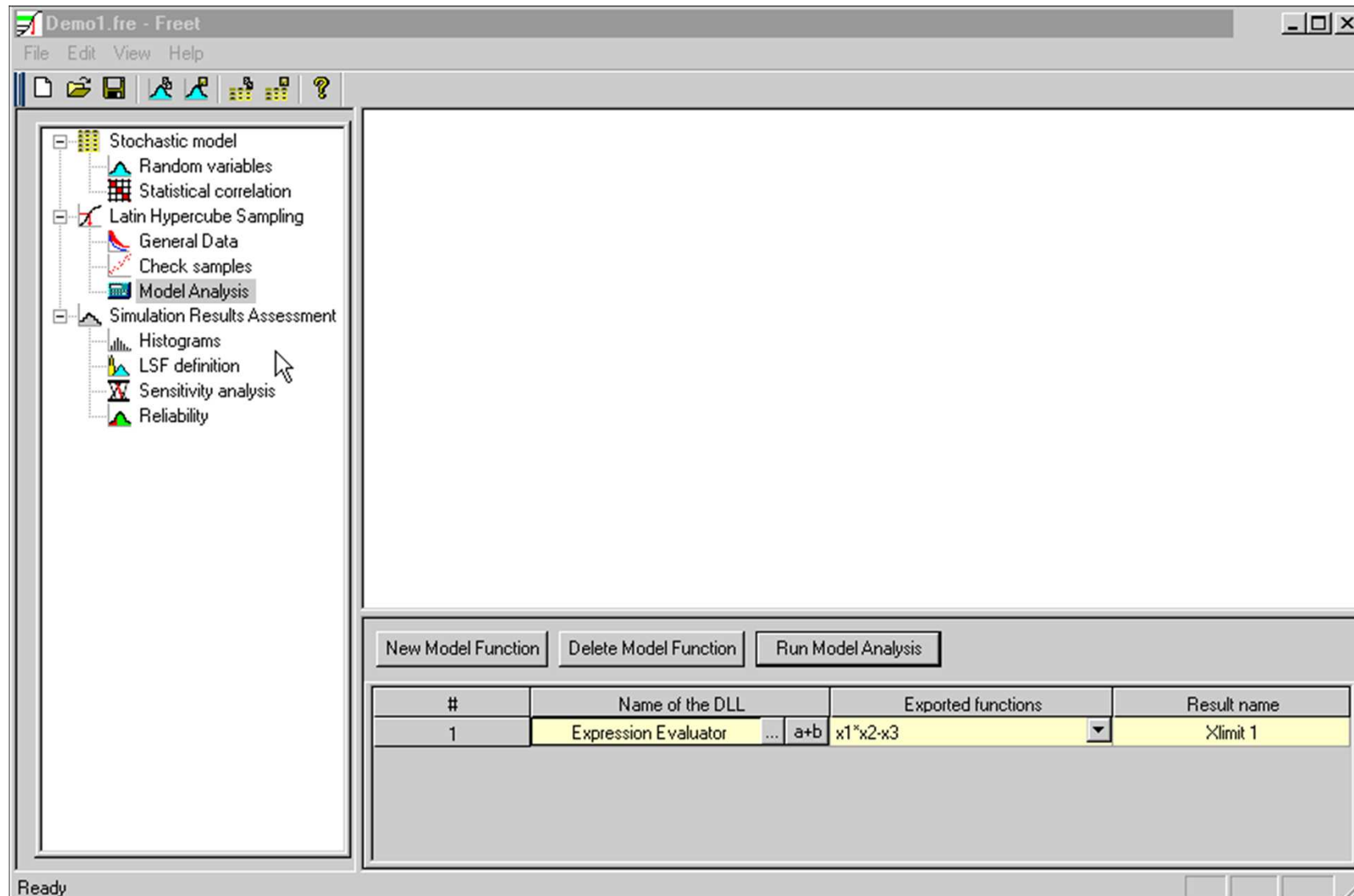
FREET – Stochastic model



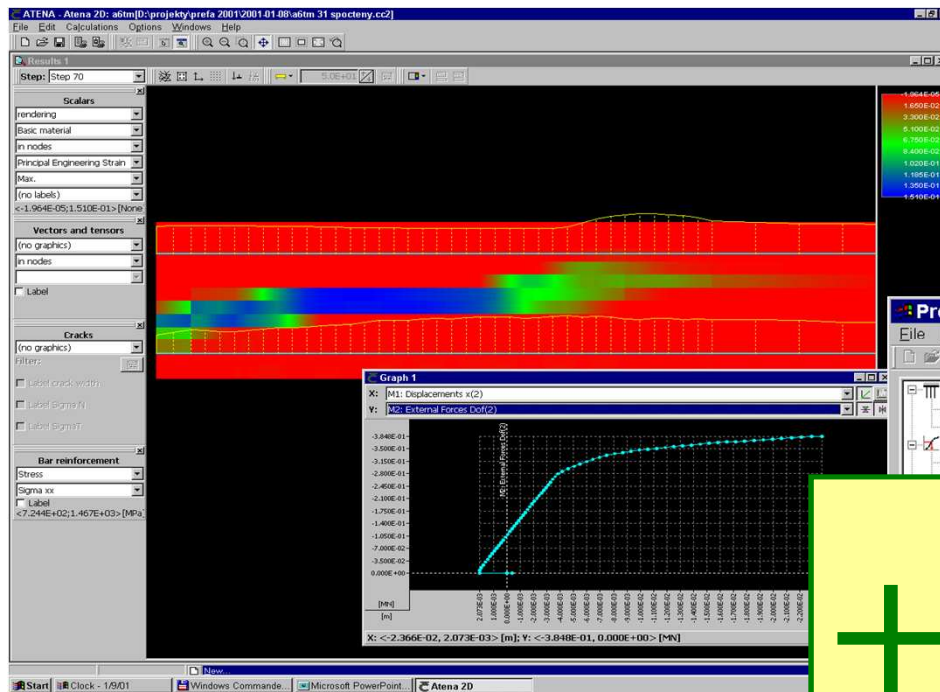
FREET – Correlation, simulation



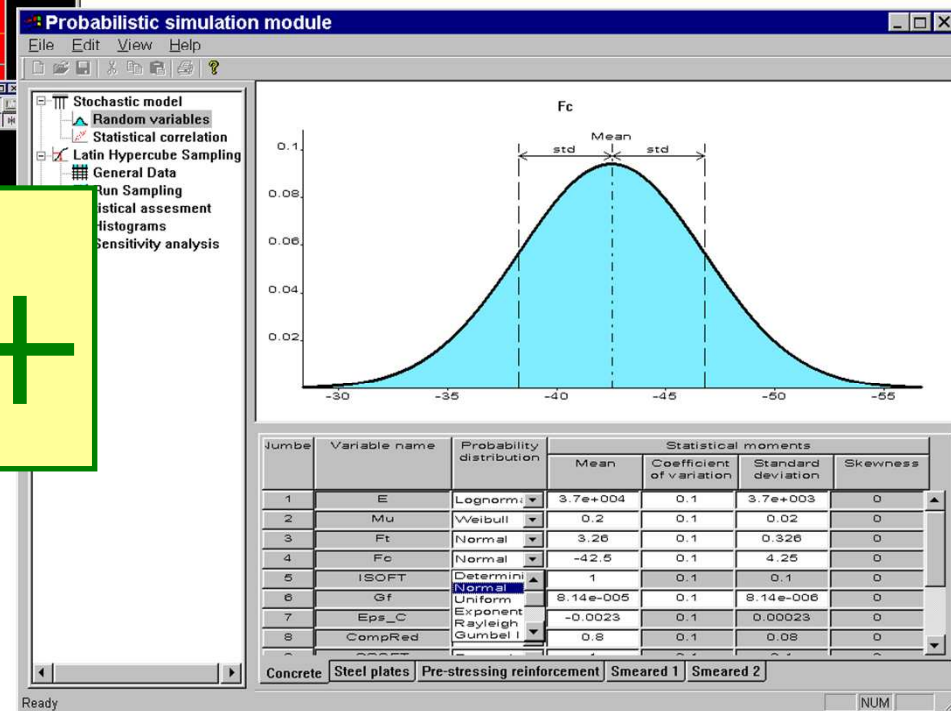
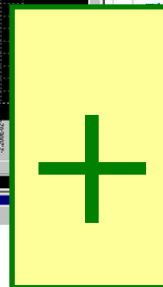
FREET – Assessment



Software tools: SARA Studio



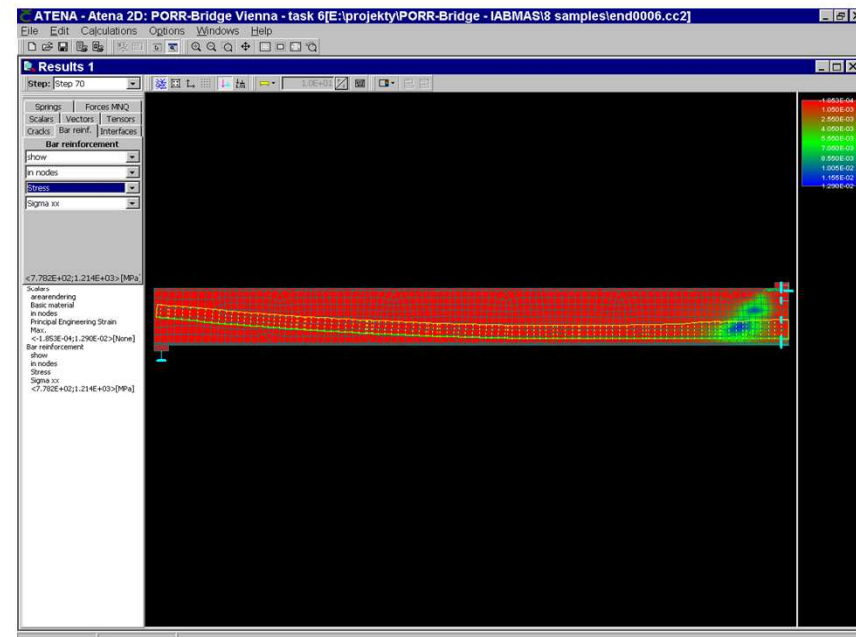
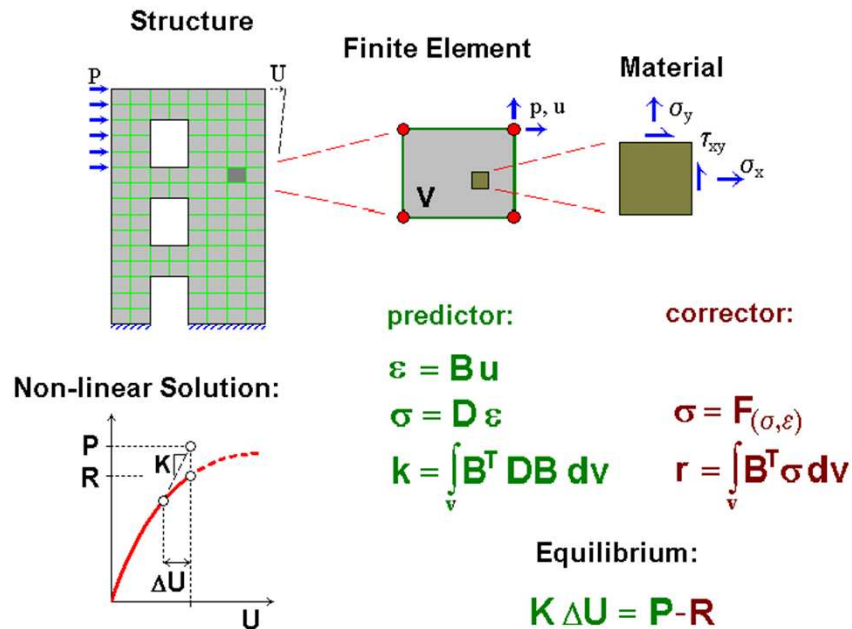
Probabilistic software FReET
<http://www.freet.cz>



Software for nonlinear fracture mechanics analysis ATENA

ATENA

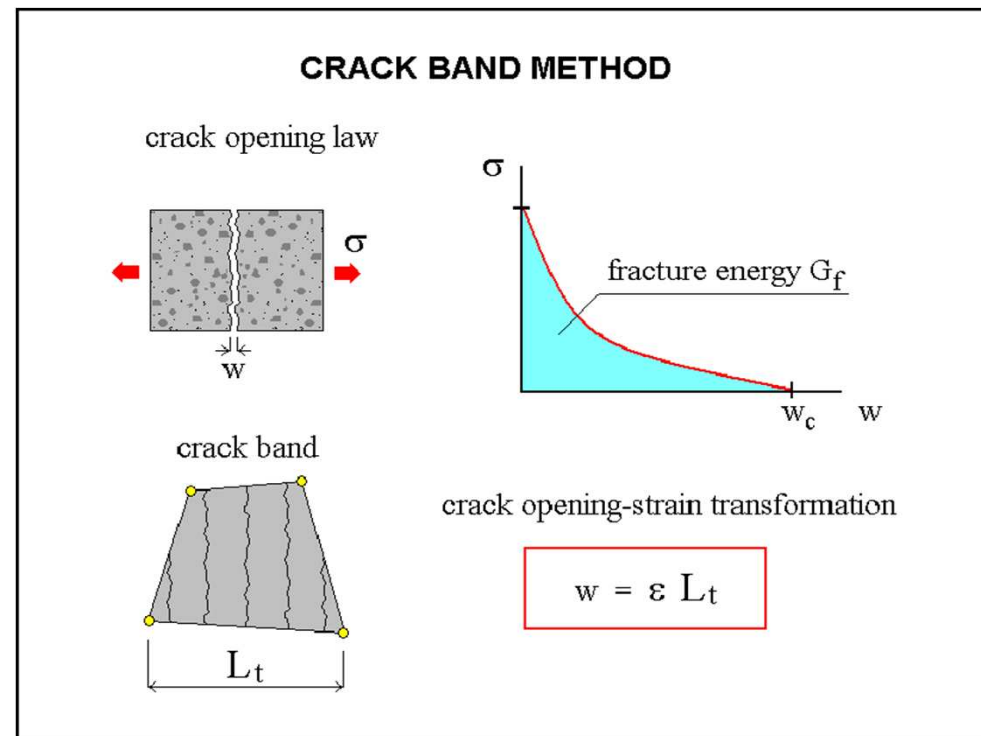
- **Well-balanced approach** for practical applications of advanced FEM in civil engineering
- **Numerical core – state-of-art background**
- User friendly **Graphical user environment**



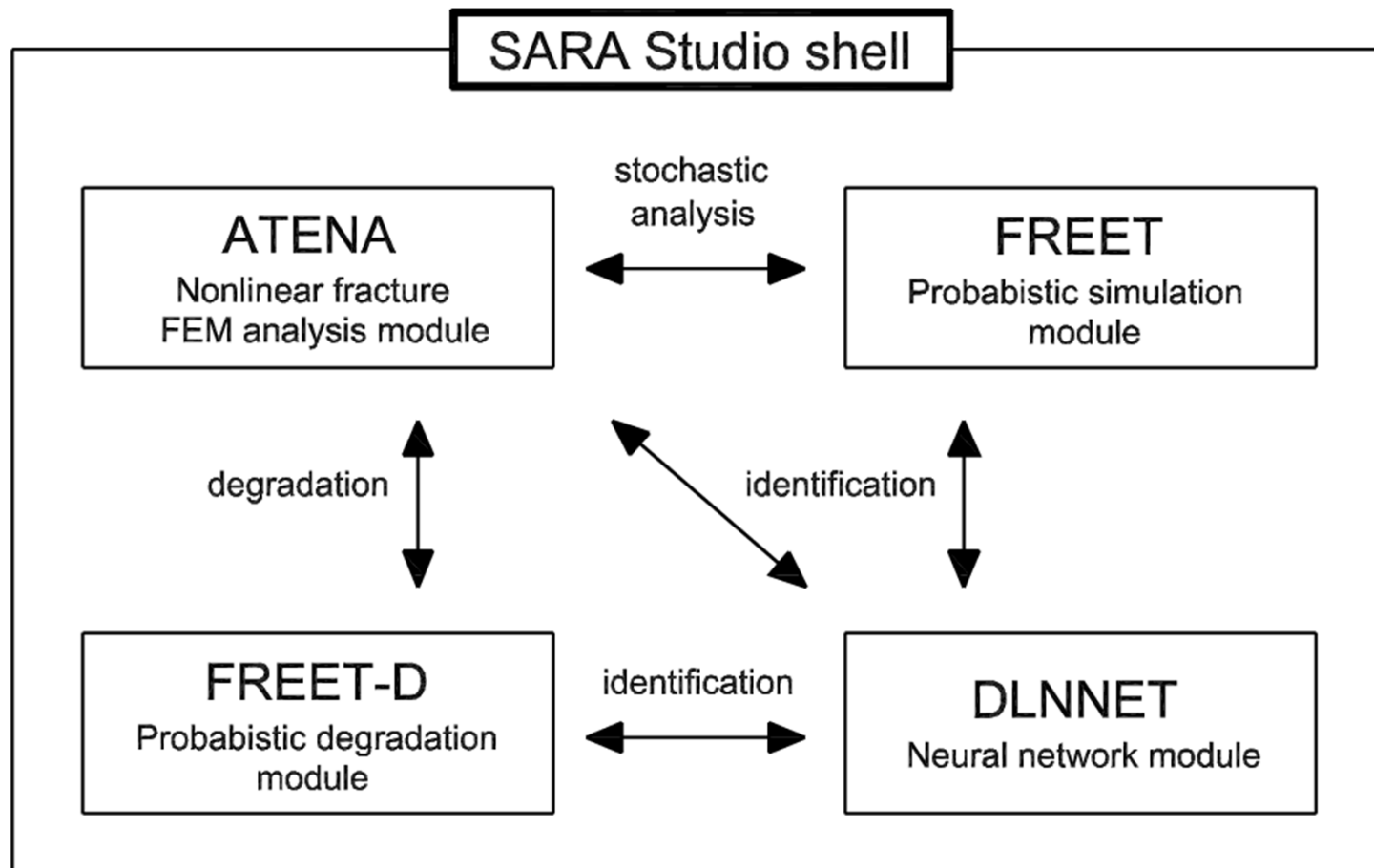
Material models for concrete: ATENA software

Numerical core – advanced nonlinear material models concrete in tension

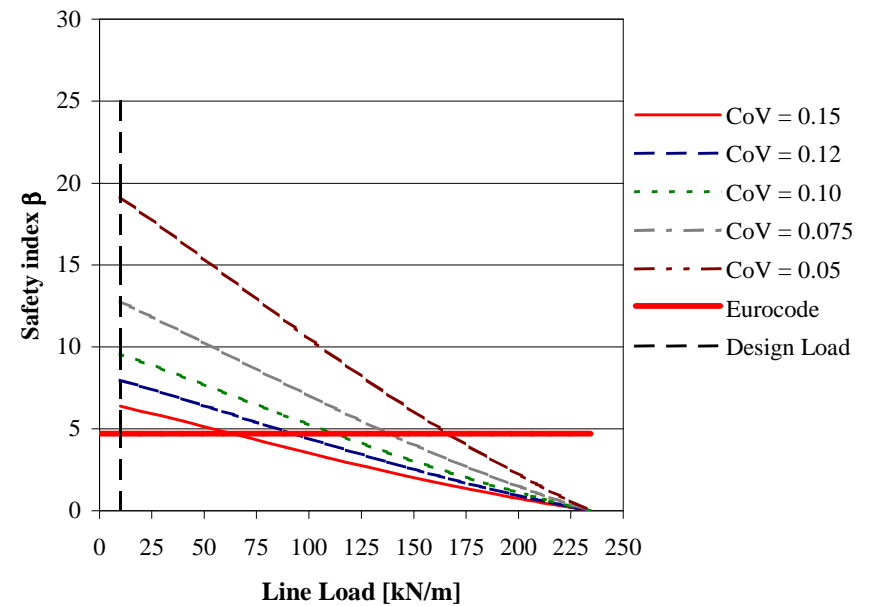
- tensile cracks
- post-peak behavior
- smeared crack approach
- crack band method
- fracture energy
- fixed or rotated cracks
- crack localization
- size-effect is captured



Software tools: SARA Studio



Probabilistic analyses of concrete structures – Coll de Isarco viaduct

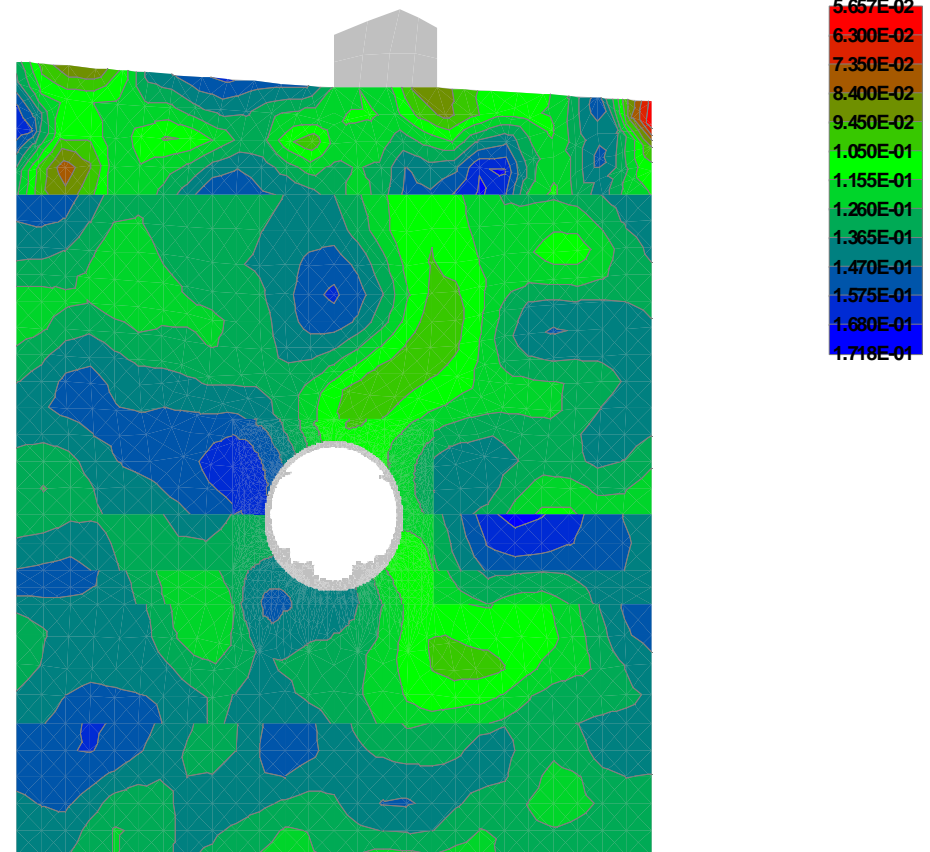


Probabilistic analyses of concrete structures – Coll de Isarco viaduct

Random variable description	Symbol	Units	Mean value	COV	Distribution type	Reference
<i>Concrete grade B500</i>						
Modulus of elasticity	E_c	GPa	36.95	0.15	Lognormal	⁶
Poisson's ratio	μ	-	0.2	0.05	Lognormal	Estimation
Tensile strength	f_t	MPa	3.257	0.18	Weibull	⁶
Compressive strength	f_c	MPa	42.5	0.10	Lognormal	^{6,7}
Specific fracture energy	G_f	N/m	81.43	0.20	Weibull	⁸
Uniaxial compressive strain	ε_c	-	0.0023	0.15	Lognormal	⁶
Reduction of strength	c_{Red}	-	0.8	0.06	Rectangular	Estimation
Critical comp displacement	w_d	m	0.0005	0.10	Lognormal	Estimation
Specific material weight	ρ	MN/m ³	0.023	0.10	Normal	⁹
<i>Prestressing strands</i>						
Modulus of elasticity	E_s	GPa	200.0	0.03	Lognormal	¹⁰
Yield stress	f_y	MPa	1600.0	0.07	Lognormal	¹⁰
Prestressing force	F	MN	21.85	0.04	Normal	⁹
Area of strands	A_s	m ²	0.0237	0.001	Normal	⁹

Probabilistic analyses of concrete structures – soil structure interaction

- Stability of concrete tunnel tube in complicated geological conditions
- Influence of spatial variability of Young modulus and material constants of Drucker-Prager criterion (based on cohesion and angle of internal friction)
- Analyzed part 50 x 60m, diameter of tunnel 11m, wall thickness 0.5m
- Plain strain state, 5000 finite elements

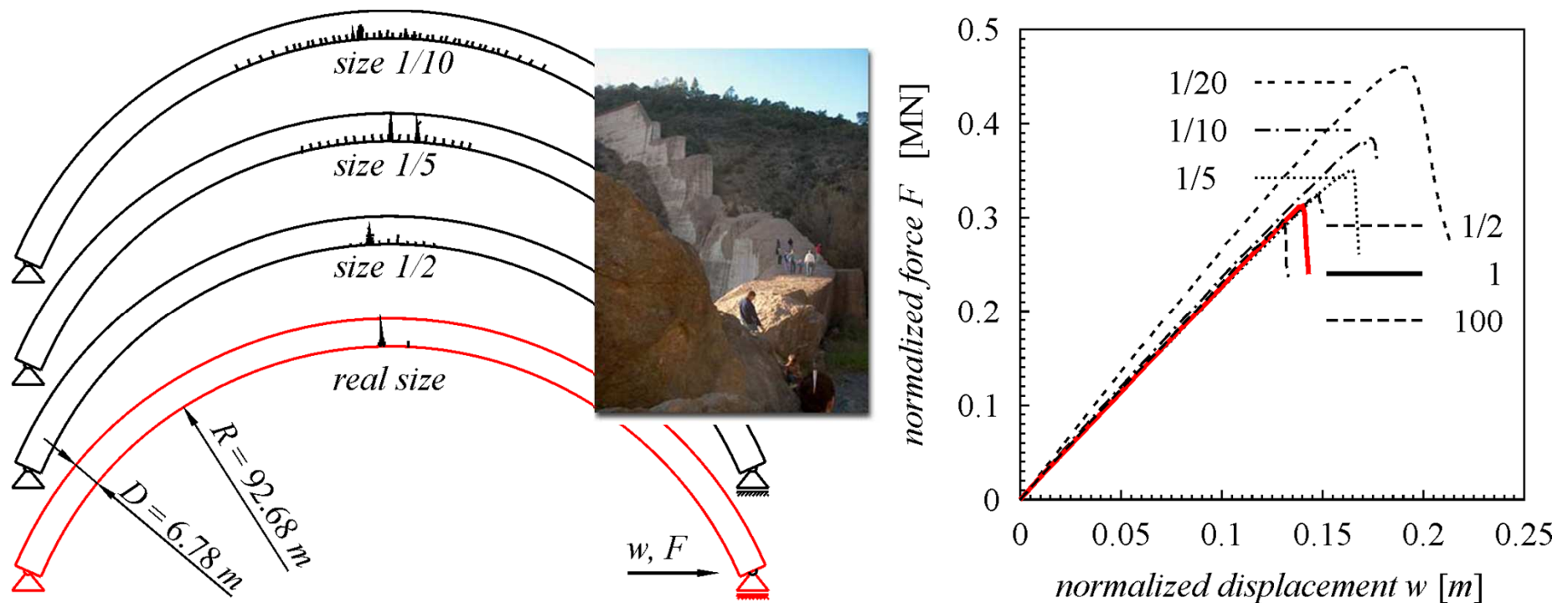


Statistical size effect studies

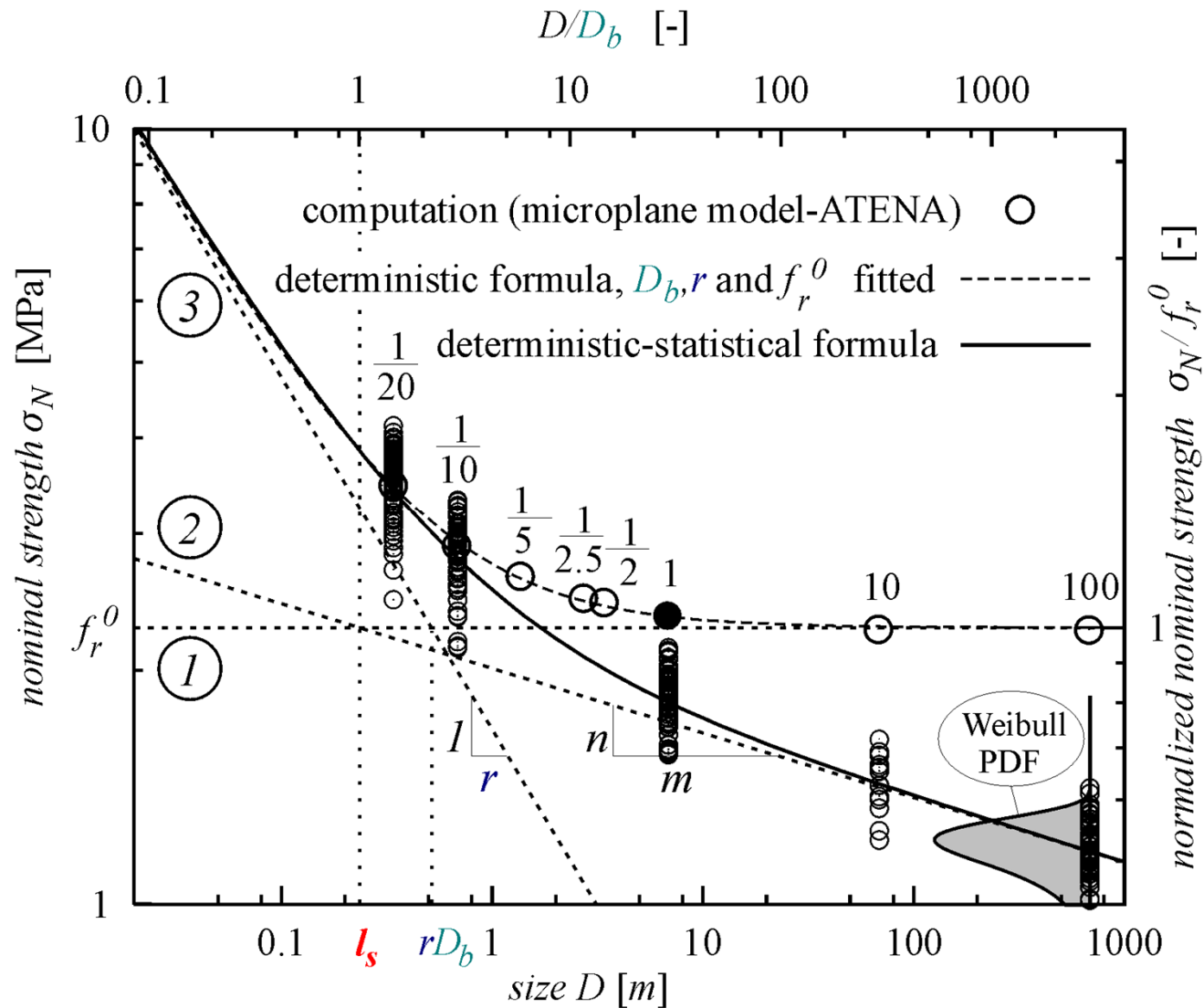
Malpasset dam (failed 1959)

Calculation for different sizes, microplane M4 model

Bažant, Vořechovský, Novák - Icosar 2005

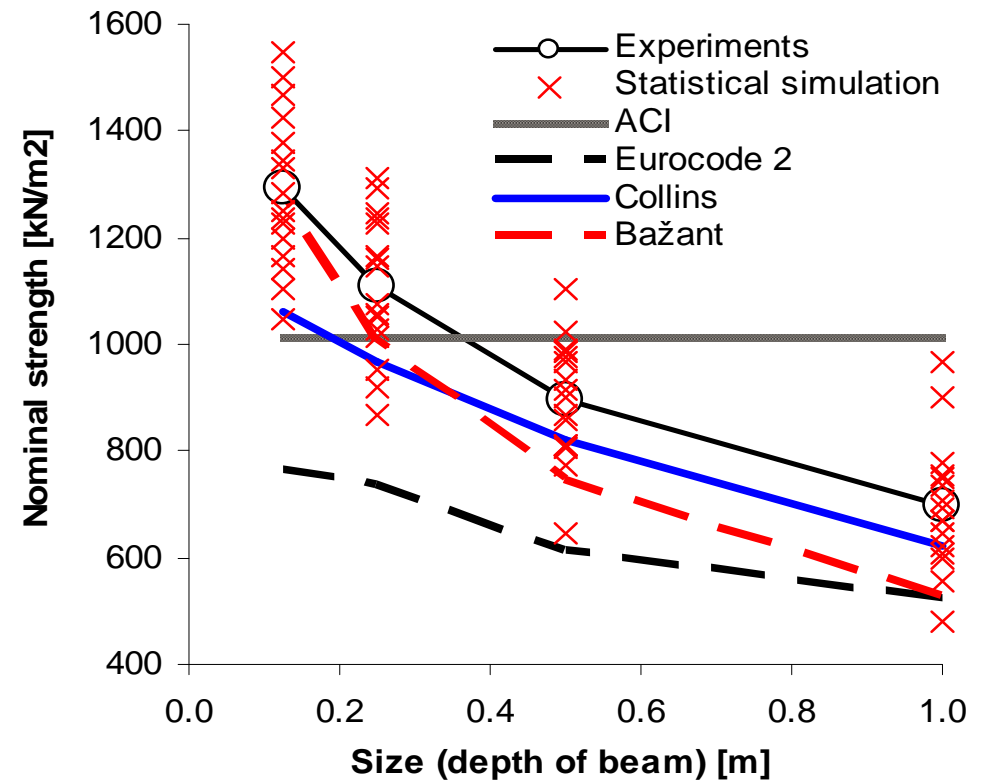
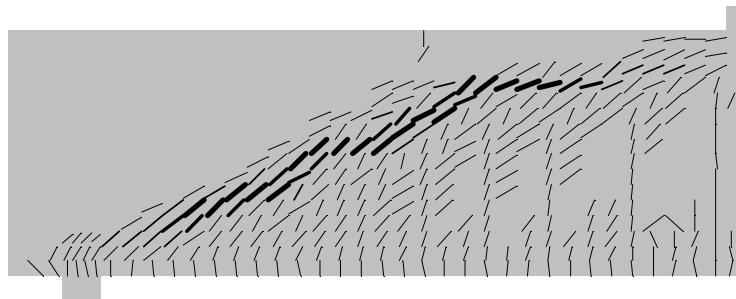
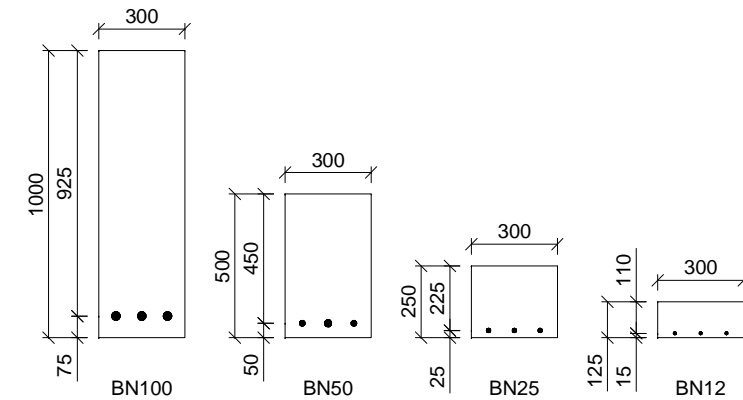


Statistical size effect studies

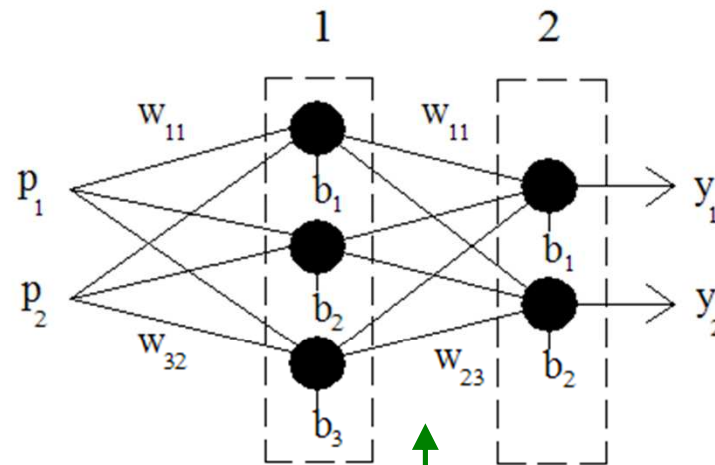
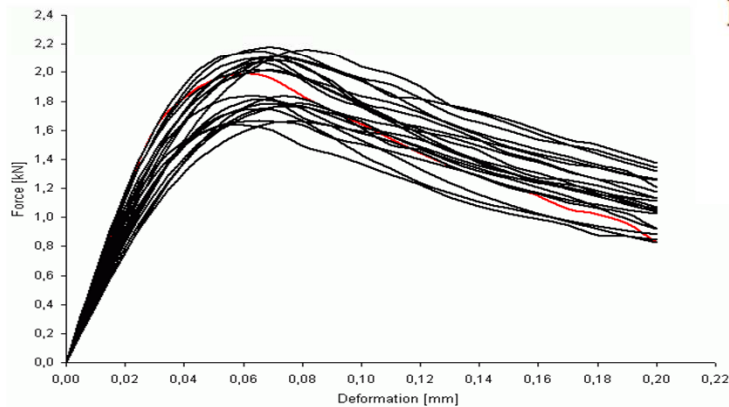
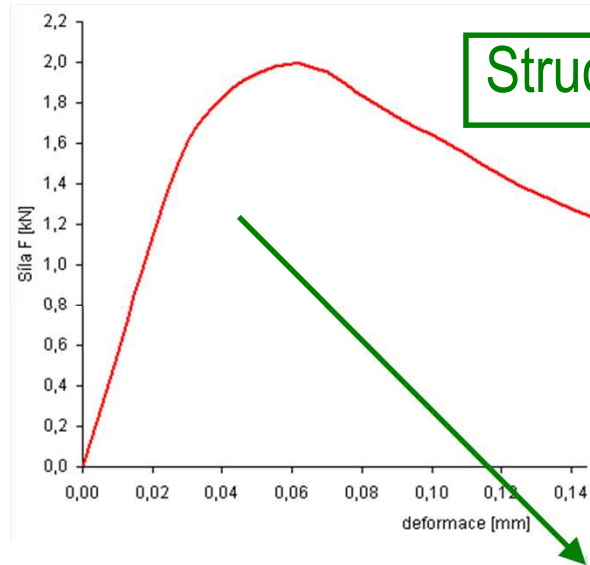


Statistical size effect studies

Shear failure, verification of design formulas



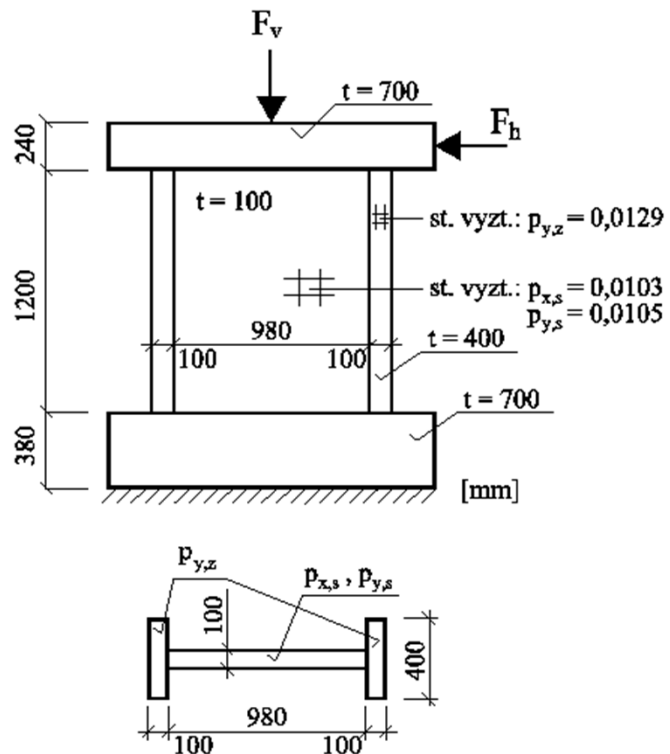
Identification of computational model parameters



Material model parameters

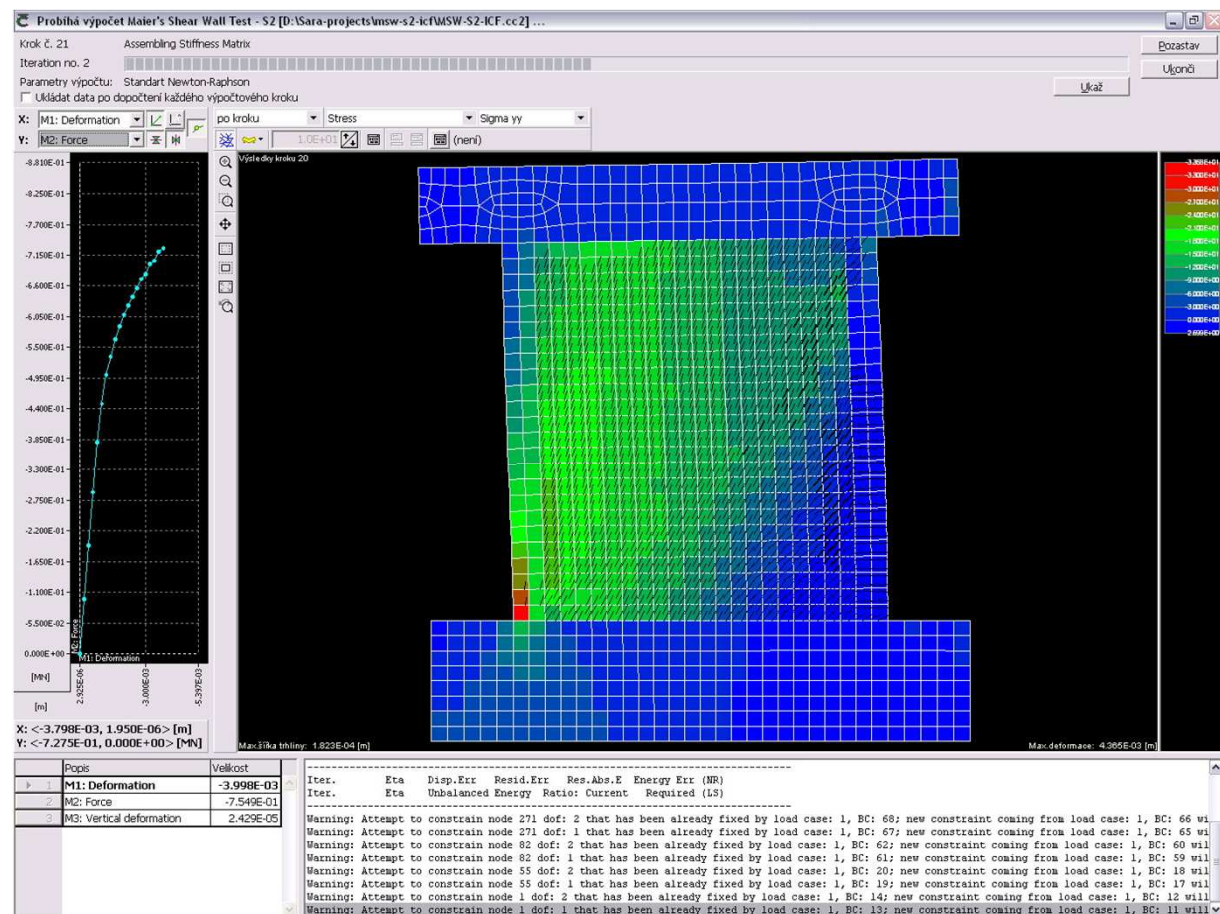
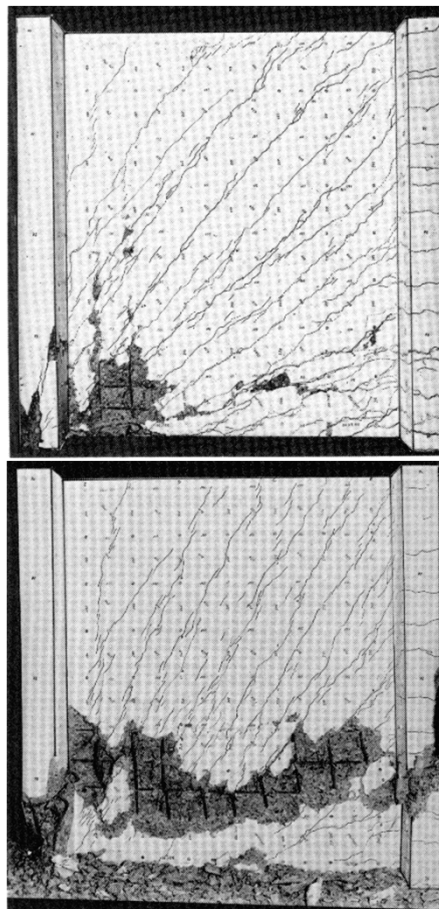
Stochastic calculation (LHS) – training set for calibration of synaptic weights and biases

Identification of computational model parameters – shear wall test

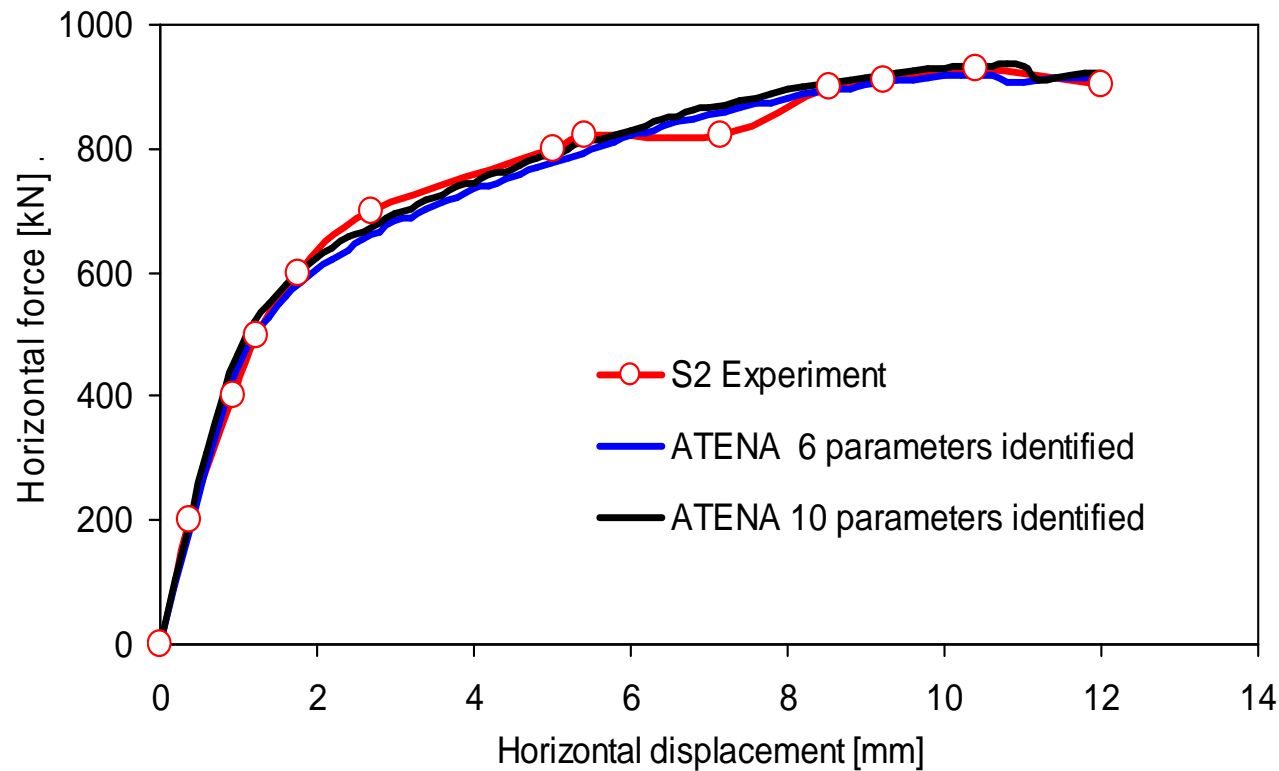


Variable	Symbol	Unit	Mean value	COV
Modulus of elasticity	E	GPa	30	0.10
Tensile strength	f_t	MPa	2.5	0.10
Compressive strength	f_c	MPa	30	0.10
Fracture energy	G_F	N/m	75	0.20
Compressive strain	ϵ_c	-	0.0025	0.20
Max. comp. displacement	w_d	m	0.003	0.30
Bilinear diagram of steel for smeared reinforcement	x_1	m	0.0027	0.10
	f_{x1}	kN	574	0.10
	x_2	m	0.015	0.10
	f_{x2}	kN	764	0.10

Identification of computational model parameters



Identification of computational model parameters



DLNNET	6 par.	10 par.
E [MPa]	29,9	33,0
f_t [MPa]	2,47	2,47
f_c [MPa]	34,51	35,3
G_f [MN/m]	75,0	77,85
e_c [-]	2,51E-03	2,57E-03
w_d [m]	3,00E-03	3,10E-03
x_1	2,72E-03	2,74E-03
fx_1	566,9	570,7
x_2	1,50E-02	1,47E-02
fx_2	764	768,8

Conclusions

- **FREET** - software for statistical, sensitivity and reliability analysis
- Suitable for analysis of computationally intensive problems (eg. continuum mechanics, FEM)
- Software development:
 - 1) Stand alone module - definition of reliability problem (user-defined response/limit state function)
 - 2) Integration with software **ATENA** - nonlinear fracture mechanics of reinforced concrete structures (Červenka Consulting)

Thank You for Your attention!

<http://www.freet.cz>

