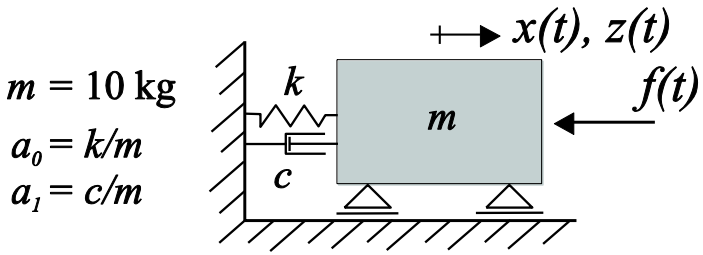
Parameter identification using the skewed Kalman Filter

Katrin Runtemund, Gerhard Müller
Chair for Structural Mechanics, TU München

Motivation and the investigated identification problem

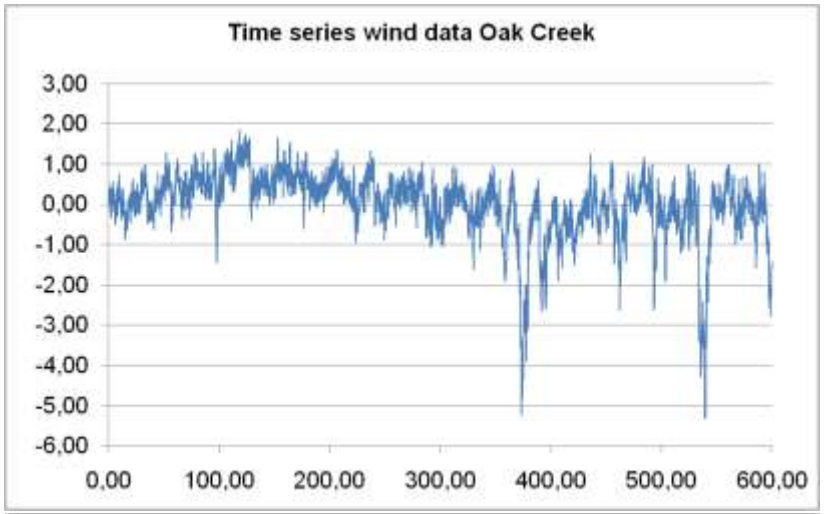
Problem: damped SDOF system excited by a **skewed process**

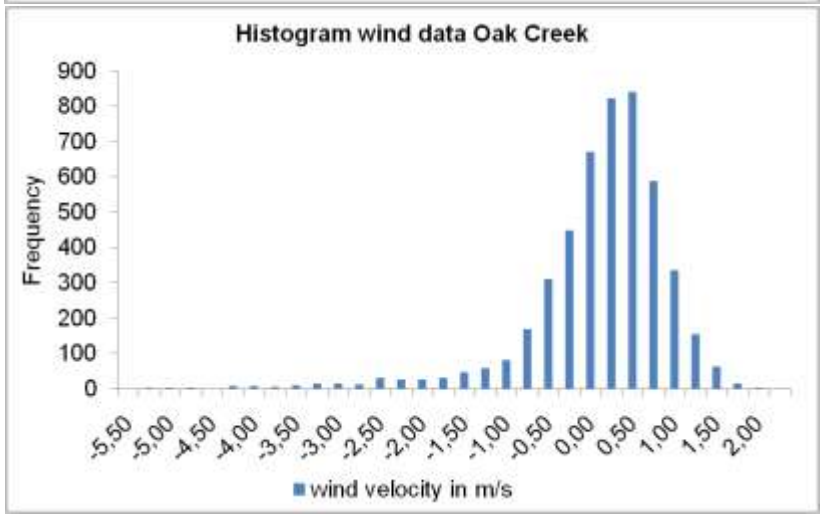
wanted: **estimation of the stiffness, damping paramter** a_0=k/m and a_1=c/m

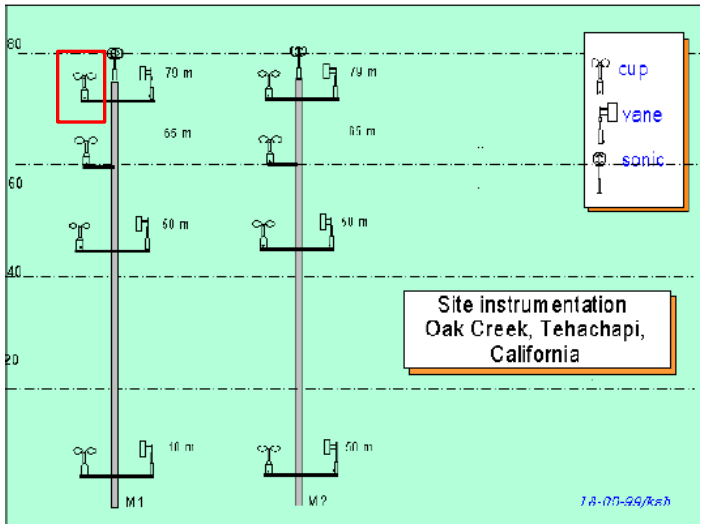


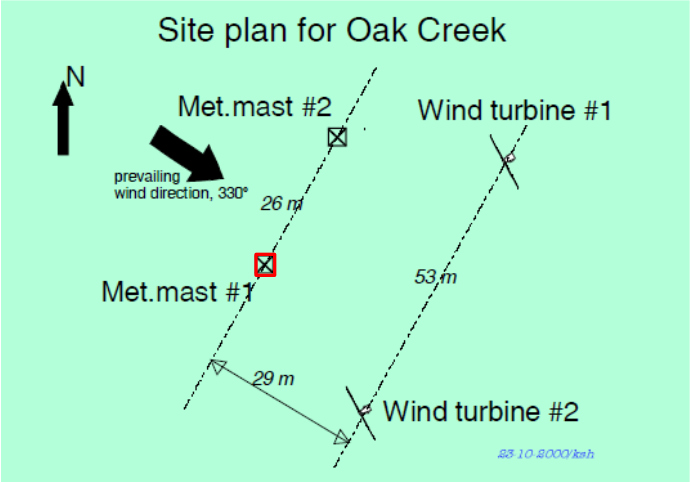
given:

- mathematical model of the system
- the statistical characteristics of the input force
  - ⇒ **wind data** was downloaded from the internet database: ”Database of Wind Characteristics” located at DTU, Denmark, www.winddata.com) from the Oak Creek wind farm site
- the „true“ system response w(t) was simulated using the wind data









website: www.winddata.com of DTU

Wind speed measurement	mast 1; heigth 79 m; frequency 8 Hz; duration 600s
Statistics	mean 0 m/s; variance 0,69 m/s; skewness -1,99

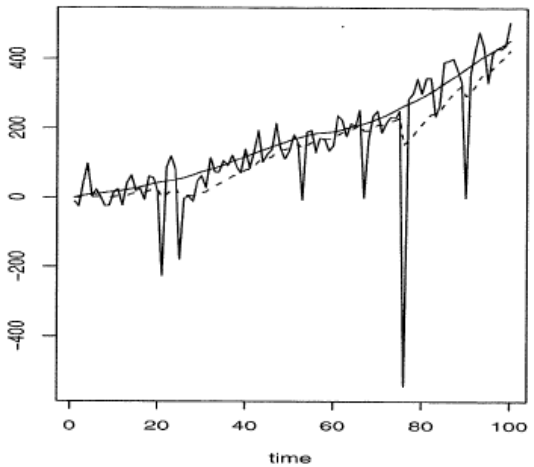
Overview

- Skewed Kalman Filter
- Extension of the skewed Kalman Filter for parameter identification problems
- Identification results of the application
- Conclusion

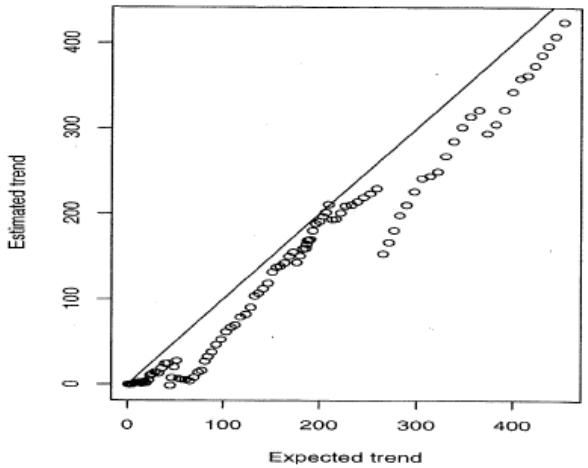
Skewed Kalman Filter

Skewed Kalman Filter (sKF)

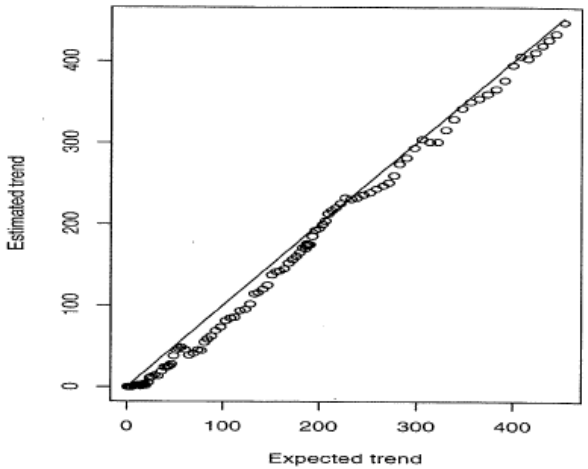
Trend & Noise



Kalman Filter



Skewed Kalman Filter



- introduced by P. Naveau, M. Genton and X. Shen

  Paper: “A skewed Kalman Filter”, *Journal of Multivariate Analysis, 2004*

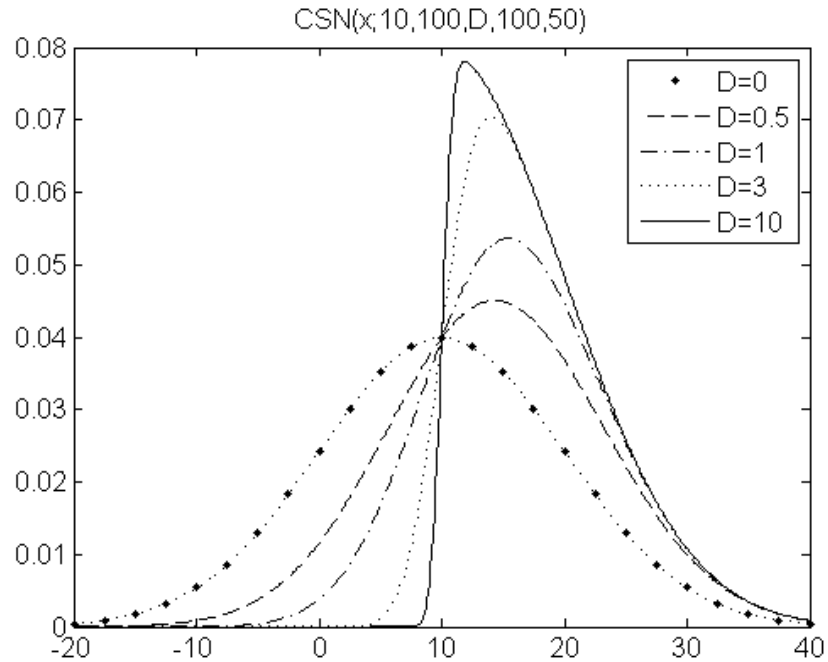
- based on the **closed skew-normal distribution** (CSN)

  $\Rightarrow$ introduced by G. González-Farías, J.A. Domínguez-Molina, A. K. Gupta (2004)

Figure: M. G. Genton: Skew-Elliptical Distributions and their Applications – A Journal Beyond Normality

Closed skew-normal distribution (CSN)

- family of distributions including the normal one, but with 3 additional parameters
- allows a continuous variation from a normal to a half-normal distribution
- closed under
  - ⇒ linear transformation
  - ⇒ marginalization
  - ⇒ conditioning
  - ⇒ summation*



Closed skew-normal distribution (CSN)

- CSN probability density function of a *n*-dimensional random vector **X**

$$X \sim CSN_{n,m}(\mu, \Sigma, D, \nu, \Delta) \qquad p_{n,m}(\mathbf{x}) = \frac{\phi_n(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})\boldsymbol{\Phi}_m(\mathbf{D}(\mathbf{x}-\boldsymbol{\mu});\ \boldsymbol{\nu}, \boldsymbol{\Delta})}{\boldsymbol{\Phi}_m(\mathbf{0};\ \boldsymbol{\nu}, \boldsymbol{\Delta} + \mathbf{D}\boldsymbol{\Sigma}\mathbf{D}^T)}$$

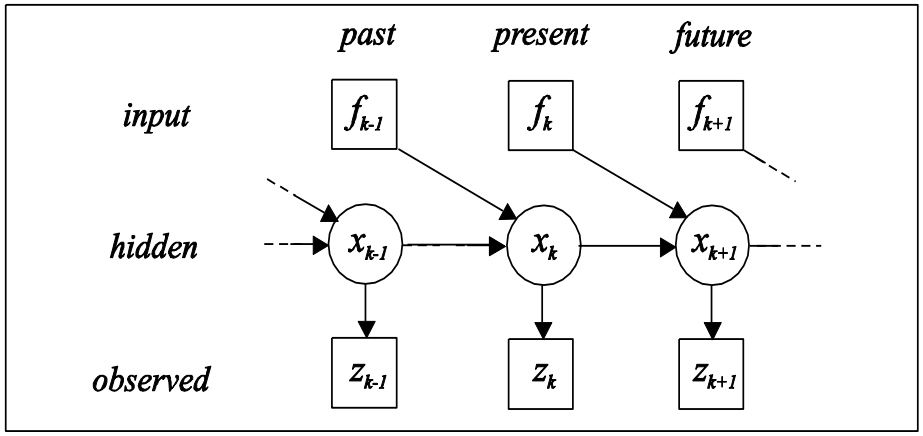
mean values:	$\mu \in \mathbb{R}^n$ and $\nu \in \mathbb{R}^m$	$\phi_n(*\,;\ \eta, \Omega) = N(\eta, \Omega)$
covariance matrices:	$\Sigma \in \mathbb{R}^{n\times n}$ and $\Delta \in \mathbb{R}^{m\times m}$	$\boldsymbol{\Phi}_n(*\,; \boldsymbol{\eta}, \boldsymbol{\Omega})$: cumulative distribution function
arbitrary matrix	$\mathrm{D} \in \mathbb{R}^{m\times n}$	$\mathrm{C} = \Phi_m(0; \nu, \Delta + D\Sigma D^T) = const.$

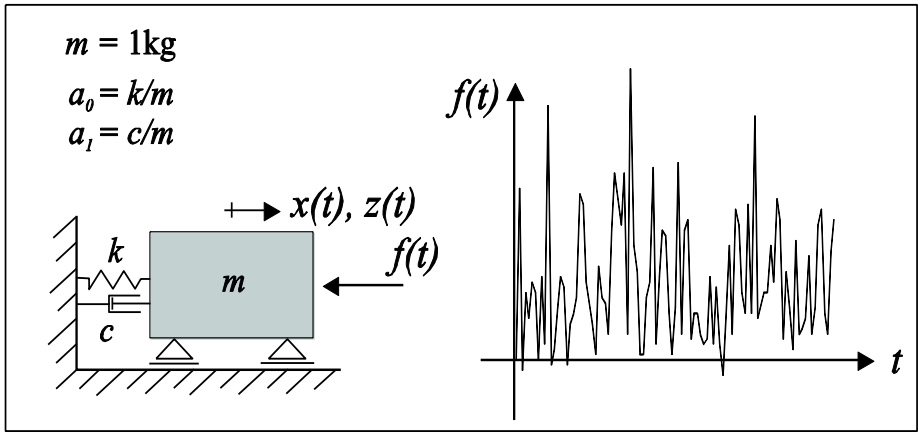
D	regulates skewness continuous variation from the normal ($\mathrm{D} = 0$) to a half normal PDF
μ, Σ	location and scale parameters
ν	closure under conditioning
Δ	closure under marginalization
C	closure under summation

Standard Kalman Filter

- The standard KF is based on a linear state space model
  - ⇒ linear system equation: $\mathbf{x}_k = \mathbf{T}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{f}_k + \mathbf{S}\mathbf{w}_k \qquad \mathbf{w}_k \sim \mathrm{N}(0, \Sigma_w)$
  - ⇒ observation equation: $\mathbf{z}_k = \mathbf{A}\mathbf{x}_k + \mathbf{v}_k \qquad \mathbf{v}_k \sim \mathrm{N}(0, \Sigma_v)$

- the state follows a first order hidden Markov process represented as **Bayesian Dynamic Network**





Implementation of skewness into the state space model

Alternative 1:

- Introducing the skewness in the sytem equation by modeling the input force as **additive CSN noise**:

$$\mathbf{f}_k \sim \mathrm{CSN}(\mu, \Sigma, \mathrm{D}, \nu, \Delta) \quad \Rightarrow \quad \mathbf{x}_k = \overbrace{\mathbf{T}\mathbf{x}_{k-1}}^{skewed} + \overbrace{\mathbf{B}\mathbf{f}_k}^{skewed} + \overbrace{\mathbf{S}\mathbf{w}_k}^{noise}$$

- problem: the sum of two CSN of dimension (n, m) is of dimension **(n, 2m)**

$\Rightarrow$ rapid increase of the dimension of the matrices $\mathbf{D}_k, \mathbf{\Delta}_k$ at each time step

Implementation of skewness into the state space model

Alternative 2:

- splitting up the state in a linear and a skewed part $\mathbf{x}_k = [\mathbf{u}_k^T, \mathbf{s}_k^T]^T$

$$\mathbf{x}_k = \mathbf{T}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{f}_k + \mathbf{S}\mathbf{w}_k$$
$$\mathbf{z}_k = \mathbf{A}\mathbf{x}_k + \mathbf{v}_k$$

$$\Rightarrow$$

$$\mathbf{u}_k = \mathbf{T}\mathbf{u}_{k-1} + \mathbf{S}\mathbf{w}_k$$
$$\mathbf{z}_k = \mathbf{G}\mathbf{x}_k = \underbrace{\mathbf{A}\mathbf{u}_k}_{linear} + \underbrace{\mathbf{S}\mathbf{s}_k}_{skewed} + \underbrace{\mathbf{v}_k}_{noise}$$

- generation of the skewed process

$$\begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} \mu \\ \nu \end{bmatrix}, \begin{bmatrix} \Sigma & -\Sigma \mathrm{D}^T \\ -\mathrm{D}\Sigma & \Delta + \mathrm{D}\Sigma\mathrm{D}^T \end{bmatrix} \right) \Rightarrow \mathrm{X} \mid \mathrm{Y} < \mathrm{D}\mu \sim \mathrm{CSN}(\mu, \Sigma, \mathrm{D}, \nu, \Delta)$$

Implementation of skewness into the state space model

Alternative 2:

- splitting up the state in a linear and a skewed part $\mathbf{x}_k = [\mathbf{u}_k^T, \mathbf{s}_k^T]^T$

$$\mathbf{x}_k = \mathbf{T}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{f}_k + \mathbf{S}\mathbf{w}_k \qquad \mathbf{u}_k = \mathbf{T}\mathbf{u}_{k-1} + \mathbf{S}\mathbf{w}_k$$

$$\mathbf{z}_k = \mathbf{A}\mathbf{x}_k + \mathbf{v}_k \qquad \mathbf{z}_k = \mathbf{G}\mathbf{x}_k = \underbrace{\mathbf{A}\mathbf{u}_k}_{linear} + \underbrace{\mathbf{S}\mathbf{s}_k}_{skewed} + \underbrace{\mathbf{v}_k}_{noise}$$

- generation of the skewed process

  step 1: time-update $\mathbf{y}_k = -\mathrm{L}_k \mathbf{y}_{k-1} + \eta_k$

  step 2: determine the joint distribution

$$\begin{bmatrix} \mathbf{y}_k \\ \mathbf{y}_{k-1} \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} -\mathrm{L}_k \mu_{k-1} + \mu_{\eta,k} \\ \mu_{k-1} \end{bmatrix}, \begin{bmatrix} \mathrm{L}_k \Sigma_{k-1} \mathrm{L}_k^T + \Sigma_{\eta,k} & -\Sigma_{k-1} \mathrm{L}_k^T \\ -\mathrm{L}_k \Sigma_{k-1} & \Sigma_{k-1} \end{bmatrix} \right)$$

  step 3: conditioning $\mathbf{s}_k = \mathbf{y}_k \,|\, \mathbf{y}_{k-1} < \mathrm{D}_k \mu_k \sim \mathrm{CSN}(\mu, \Sigma, \mathrm{D}, \nu, \Delta)$

Bayesian approach of the skewed Kalman filter

1. **Initialising** the filter using the prior PDF $p(\mathbf{u}_{k-1}, \mathbf{y}_{k-1} | \mathbf{z}_{1:k-1})$

2. **Prediction**: time-update conditional on the observations up to time k-1 using the system equations $p(\mathbf{u}_k, \mathbf{y}_k, \mathbf{y}_{k-1} | \mathbf{z}_{1:k-1})$

3. **Measurement update** after observing $\mathbf{z}_k$:
   from the prediction error $\mathbf{e}_k = \mathbf{z}_k - \mathbf{A}_k \bar{\boldsymbol{\mu}}_{u,k-1} - \mathbf{B}_k \mathrm{E}[\mathbf{s}_k | \mathbf{z}_{1:k-1}]$
   likelihood $p(\mathbf{z}_k, | \mathbf{u}_k, \mathbf{y}_k, \mathbf{y}_{k-1}, \mathbf{z}_{1:k-1})$ is obtained using the observation equation

4. Applying the **Bayes' Rule** leads to the posterior PDF

$$p(\mathbf{u}_k, \mathbf{y}_k, \mathbf{y}_{k-1} | \mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k | \mathbf{u}_k, \mathbf{y}_k, \mathbf{y}_{k-1}) p(\mathbf{u}_k, \mathbf{y}_k, \mathbf{y}_{k-1} | \mathbf{z}_{1:k-1})}{p(\mathbf{z}_k | \mathbf{z}_{1:k-1})}$$

Bayesian approach to the skewed Kalman filter

$$p(\mathbf{u}_k, \mathbf{y}_k, \mathbf{y}_{k-1} | \mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k | \mathbf{u}_k, \mathbf{y}_k, \mathbf{y}_{k-1}) p(\mathbf{u}_k, \mathbf{y}_k, \mathbf{y}_{k-1} | \mathbf{z}_{1:k-1})}{p(\mathbf{z}_k | \mathbf{z}_{1:k-1})}$$

5. **Marginalization** leads to the posterior PDF of the linear part $p(\mathbf{u}_k | \mathbf{z}_{1:k})$

6. **Marginalization and Conditioning** with respect to $\mathbf{y}_{k-1}$ leads to the posterior PDF of the skewed part $p(\mathbf{s}_k | \mathbf{z}_{1:k})$

Advantages of the method:

- computationally efficient: the closure properties of the CSN allow to reduce the filter algorithm to the recursive calculation of the parameters

- At each time step a different skewness can be implemented by changing the matrix L_k of the process $\mathbf{y}_k = -\mathrm{L}_k \mathbf{y}_{k-1} + \eta_k$

PREDICTION (TIME UPDATE)	CORRECTION (MEASUREMENT UPDATE)
Project the state ahead (***prior estimate***) **Linear part** $\bar{\boldsymbol{\mu}}_{u,k} = \mathbf{T}_k \boldsymbol{\mu}_{u,k-1}$ $\bar{\boldsymbol{\Sigma}}_{uu,k} = \mathbf{T}_k \boldsymbol{\Sigma}_{uu,k-1} \mathbf{T}_k^T + \mathbf{S}_k \boldsymbol{\Sigma}_{ww,k-1} \mathbf{S}_k^T$ **Skewed part** $\bar{\boldsymbol{\mu}}_{y,k} = -\mathbf{L}_k \mu_{y,k-1}$ $\bar{\boldsymbol{\Sigma}}_{yy,k} = \mathbf{L}_k \boldsymbol{\Sigma}_{yy,k-1} \mathbf{L}_\mathrm{k}^T + \boldsymbol{\Sigma}_{\eta\eta,k}$ $\bar{\mathbf{D}}_k = \boldsymbol{\Sigma}_{yy,k-1} \mathbf{L}_k^T \bar{\boldsymbol{\Sigma}}_{yy,k}^{-1}$ $\bar{\mathbf{v}}_k = \mu_{y,k-1} - \bar{\mathbf{D}}_k \bar{\boldsymbol{\mu}}_{y,k}$ $\bar{\boldsymbol{\Delta}}_k = \boldsymbol{\Sigma}_{yy,k-1} - \bar{\mathbf{D}}_k \bar{\boldsymbol{\Sigma}}_{yy,k} \bar{\mathbf{D}}_k^T$	Compute the **Kalman gain matrix** of the linear and skewed part $\mathbf{K}_k = \bar{\boldsymbol{\Sigma}}_{uu,k} \mathbf{A}_k^T \left(\boldsymbol{\Sigma}_{vv,k} + \mathbf{A}_k \bar{\boldsymbol{\Sigma}}_{uu,k} \mathbf{A}_k^T + \mathbf{S}_k \mathrm{Var}[\mathbf{s}_k \vert \mathbf{z}_{1:k-1}] \mathbf{S}_k^T \right)^{-1}$ $\mathbf{K}_{s,k} = \mathbf{C}_k \mathbf{S}_k^T \left(\boldsymbol{\Sigma}_{vv,k} + \mathbf{A}_k \bar{\boldsymbol{\Sigma}}_{uu,k} \mathbf{A}_k^T + \mathbf{S}_k \mathrm{Var}[\mathbf{s}_k \vert \mathbf{z}_{1:k-1}] \mathbf{S}_k^T \right)^{-1}$ Update estimate with measurement (***posterior estimate***) $\mathbf{e}_k = \left(\mathbf{z}_k - \mathbf{A}_k \bar{\boldsymbol{\mu}}_{u,k} - \mathbf{S}_k \mathrm{E}[\mathbf{s}_k \vert \mathbf{z}_{1:k-1}] \right)$ **Linear part** $\bar{\boldsymbol{\mu}}_{y,k} = \bar{\boldsymbol{\mu}}_{u,k} + \mathbf{K}_k \mathbf{e}_k$ $\boldsymbol{\Sigma}_{uu,k} = \bar{\boldsymbol{\Sigma}}_{uu,k} - \mathbf{K}_k \mathbf{A}_k \bar{\boldsymbol{\Sigma}}_{uu,k}$ **Skewed part** $\boldsymbol{\mu}_{y,k} = \bar{\boldsymbol{\mu}}_{y,k} + \mathbf{K}_{s,k} \mathbf{e}_k$ $\boldsymbol{\Sigma}_{yy,k} = \bar{\boldsymbol{\Sigma}}_{yy,k} - \mathbf{K}_{s,k} \mathbf{S}_k \mathbf{C}_k$ $\mathbf{D}_k = \boldsymbol{\Sigma}_{yy,k}^* \mathbf{L}_\mathrm{k}^{*\mathrm{T}} \boldsymbol{\Sigma}_{yy,k}^{-1}$ $\mathbf{v}_k = \boldsymbol{\mu}_{y,k-1} + \mathbf{C}_{k-1} \mathbf{C}_k^{-1} \mathbf{K}_{s,k} \mathbf{e}_k - \mathbf{D}_k \boldsymbol{\mu}_{y,k}$ $\boldsymbol{\Delta}_k = \boldsymbol{\Sigma}_{yy,k}^* - \mathbf{D}_k \boldsymbol{\Sigma}_{yy,k} \mathbf{D}_k^T$

PREDICTION (TIME UPDATE)	CORRECTION (MEASUREMENT UPDATE)
Project the state ahead (***prior estimate***) **Linear part** $\bar{\boldsymbol{\mu}}_{u,k} = \mathbf{T}_k \boldsymbol{\mu}_{u,k-1}$ $\boxed{\boldsymbol{\Sigma}_{uu,k} = \mathbf{T}_k \boldsymbol{\Sigma}_{uu,k-1} \mathbf{T}_k^T + \mathbf{S}_k \boldsymbol{\Sigma}_{ww,k-1} \mathbf{S}_k^T}$ **Skewed part** $\bar{\boldsymbol{\mu}}_{y,k} = -\mathbf{L}_k \mu_{y,k-1}$ $\bar{\boldsymbol{\Sigma}}_{yy,k} = \mathbf{L}_k \boldsymbol{\Sigma}_{yy,k-1} \mathbf{L}_k^T + \boldsymbol{\Sigma}_{\eta\eta,k}$ $\bar{\mathbf{D}}_k = \boldsymbol{\Sigma}_{yy,k-1} \mathbf{L}_k^T \bar{\boldsymbol{\Sigma}}_{yy,k}^{-1}$ $\bar{\mathbf{v}}_k = \mu_{y,k-1} - \bar{\mathbf{D}}_k \bar{\boldsymbol{\mu}}_{y,k}$ $\bar{\boldsymbol{\Delta}}_k = \boldsymbol{\Sigma}_{yy,k-1} - \bar{\mathbf{D}}_k \bar{\boldsymbol{\Sigma}}_{yy,k} \bar{\mathbf{D}}_k^T$	Compute the **Kalman gain matrix** of the linear and skewed part $\boxed{\mathbf{K}_k = \bar{\boldsymbol{\Sigma}}_{uu,k} \mathbf{A}_k^T \left(\boldsymbol{\Sigma}_{vv,k} + \mathbf{A}_k \bar{\boldsymbol{\Sigma}}_{uu,k} \mathbf{A}_k^T\right.} + \boxed{\left.\mathbf{S}_k \mathrm{Var}[\mathbf{s}_k \mid \mathbf{z}_{1:k-1}] \mathbf{S}_k^T\right)}^{-1}$ $\mathbf{K}_{s,k} = \mathbf{C}_k \mathbf{S}_k^T \left(\boldsymbol{\Sigma}_{vv,k} + \mathbf{A}_k \bar{\boldsymbol{\Sigma}}_{uu,k} \mathbf{A}_k^T + \mathbf{S}_k \mathrm{Var}[\mathbf{s}_k \mid \mathbf{z}_{1:k-1}] \mathbf{S}_k^T\right)^{-1}$ Update estimate with measurement (***posterior estimate***) $\mathbf{e}_k = \left(\mathbf{z}_k - \mathbf{A}_k \bar{\boldsymbol{\mu}}_{u,k} - \boxed{\mathbf{S}_k \mathrm{E}[\mathbf{s}_k \mid \mathbf{z}_{1:k-1}]}\right)$ **Linear part** $\boxed{\begin{aligned} \bar{\boldsymbol{\mu}}_{y,k} &= \bar{\boldsymbol{\mu}}_{u,k} + \mathbf{K}_k \mathbf{e}_k \\ \boldsymbol{\Sigma}_{uu,k} &= \bar{\boldsymbol{\Sigma}}_{uu,k} - \mathbf{K}_k \mathbf{A}_k \bar{\boldsymbol{\Sigma}}_{uu,k} \end{aligned}}$ **Skewed part** $\boldsymbol{\mu}_{y,k} = \bar{\boldsymbol{\mu}}_{y,k} + \mathbf{K}_{s,k} \mathbf{e}_k$ $\boldsymbol{\Sigma}_{yy,k} = \bar{\boldsymbol{\Sigma}}_{yy,k} - \mathbf{K}_{s,k} \mathbf{S}_k \mathbf{C}_k$ $\mathbf{D}_k = \boldsymbol{\Sigma}_{yy,k}^* \mathbf{L}_k^{*T} \boldsymbol{\Sigma}_{yy,k}^{-1}$ $\mathbf{v}_k = \boldsymbol{\mu}_{y,k-1} + \mathbf{C}_{k-1} \mathbf{C}_k^{-1} \mathbf{K}_{s,k} \mathbf{e}_k - \mathbf{D}_k \boldsymbol{\mu}_{y,k}$ $\boldsymbol{\Delta}_k = \boldsymbol{\Sigma}_{yy,k}^* - \mathbf{D}_k \boldsymbol{\Sigma}_{yy,k} \mathbf{D}_k^T$

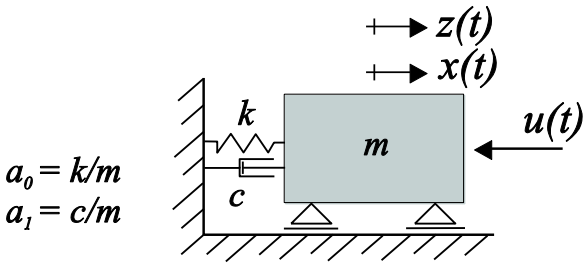
Skewed Kalman Filter

Kalman Filter

Extended skewed Kalman Filter (EsKF)

Parameter identification using the Extended Kalman Filter (EKF)

Example: damped single degree of freedom system



wanted: **estimation of the stiffness, damping paramter** a_0=k/m and a_1=c/m

$$\mathbf{u}_k = \mathbf{T}\mathbf{u}_{k-1} + \mathbf{S}\mathbf{w}_k$$

Problem: The transfer matrices $\mathbf{T}_{k,k-1}$, $\mathbf{B}_{k,k-1}$, $\mathbf{S}_{k,k-1}$ depend on the unknown parameters:

$$T_k = e^{-\delta\Delta t}\begin{bmatrix} \frac{\delta}{\omega_d}\sin(\omega_d\Delta t) + \cos(\omega_d\Delta t) & \frac{1}{\omega_d}\sin(\omega_d\Delta t) \\ -\frac{\omega_0^2}{\omega_d}\sin(\omega_d\Delta t) & \cos(\omega_d\Delta t) - \frac{\delta}{\omega_d}\sin(\omega_d\Delta t) \end{bmatrix}$$

$$\boxed{\begin{aligned} a_0 &= \frac{k}{m} = \omega^2 \\ a_1 &= \frac{c}{m} = 2\delta \end{aligned}}$$

$$\omega_\mathrm{d} = \sqrt{\omega^2 - \delta^2}$$

Parameter identification using the Extended Kalman Filter (EKF)

step 1: Extending the state by the **parameter vector $\mathbf{p}_k$** to include the model parameters to be identified

$$x_{ext,k} = \begin{bmatrix} \mathbf{x}_k \\ \mathbf{p}_k \end{bmatrix} = \begin{bmatrix} w_k \\ v_k \\ \boxed{\begin{matrix} a_{0,k} \\ a_{1,k} \end{matrix}} \end{bmatrix}$$

extended state space model

$$x_{ext,k} = f\left(\mathbf{x}_{k-1}, \mathbf{p}_{k-1}, u_{k-1}, \mathbf{w}_{k-1}, \mathbf{w}_{p,k-1}\right)$$

$$z_{ext,k} = \mathbf{A}_{ext,k}\mathbf{x}_{ext,k} + \mathbf{v}_k$$

step 2: Approximation of the nonlinear function $f(\cdot)$ by a first order Taylor series near the current state estimate $\hat{x}_{k-1}$

$$x_{ext,k} \approx f\left(\hat{x}_{0,k-1}\right) + \left.\frac{\partial f\left(x_{0,k-1}\right)}{\partial x_{0i,k-1}}\right|_{\hat{x}_{0i,k-1}} \left(x_{0i,k-1} - \hat{x}_{0i,k-1}\right)$$

$$x_{0,k-1} = \begin{bmatrix} \mathbf{x}_{k-1} \\ \mathbf{p}_{k-1} \\ u_{k-1} \\ \mathbf{w}_{k-1} \\ \mathbf{w}_{p,k-1} \end{bmatrix} \approx \begin{bmatrix} \hat{\mathbf{x}}_{k-1} \\ \hat{\mathbf{p}}_{k-1} \\ \hat{u}_{k-1} \\ \mathbf{0}_{2\times1} \\ \mathbf{0}_{2\times1} \end{bmatrix} = \hat{x}_{0,k-1}$$

Parameter identification using the Extended Kalman Filter (EKF)

step 2: Approximation of the nonlinear function $f(\cdot)$ by a first order Taylor series near the current state estimate $\hat{x}_{k-1}$

$$\mathrm{x}_{ext,k} \approx f\left(\hat{\mathbf{x}}_{0,k-1}\right) + \mathbf{T}_{ext,k} \underbrace{\begin{bmatrix} \mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1} \\ \mathbf{p}_{k-1} - \hat{\mathbf{p}}_{p,k-1} \end{bmatrix}}_{\varepsilon_{ext,\hat{x},k-1}} + \mathbf{B}_{ext,k} \underbrace{\left(\mathbf{u}_{k-1} - \hat{\mathbf{u}}_{k-1}\right)}_{\varepsilon_{ext,u,k-1}} + \mathbf{S}_{ext,k} \underbrace{\begin{bmatrix} \mathbf{w}_{k-1} \\ \mathbf{w}_{p,k-1} \end{bmatrix}}_{\varepsilon_{ext,w,k-1}}$$

$$\mathbf{T}_{ext,k} = \begin{bmatrix} \mathbf{T}_k\left(\hat{\mathbf{p}}_{k-1}\right) & \boxed{\mathcal{J}_{f,k}\left(\hat{\mathbf{p}}_{k-1}\right)} \\ 0_{2\times 2} & \mathbf{I}_{2\times 2} \end{bmatrix} \quad \text{with} \quad \mathcal{J}_{f,k}\left(\hat{\mathbf{p}}_{k-1}\right) = \left.\frac{\partial f\left(\mathbf{x}_{0,k-1}\right)}{\partial \mathbf{p}_{k-1}}\right|_{\hat{\mathbf{x}}_{0i,k-1}}$$

$$\mathbf{B}_{ext,k} = \begin{bmatrix} \mathbf{B}_k\left(\hat{\mathbf{p}}_{k-1}\right) \\ 0_{2\times 1} \end{bmatrix}$$

$$\mathbf{S}_{ext,k} = \begin{bmatrix} \mathbf{S}_k\left(\hat{\mathbf{p}}_{k-1}\right) & \mathcal{J}_{f,k}\left(\mathbf{w}_{p,k-1}\right) \\ 0_{2\times 1} & \mathbf{I}_{2\times 2} \end{bmatrix} \quad \text{with} \quad \mathcal{J}_{f,k}\left(\mathbf{w}_{p,k-1}\right) = \left.\frac{\partial f\left(\mathbf{x}_{0,k-1}\right)}{\partial \mathbf{w}_{p,k-1}}\right|_{\hat{\mathbf{x}}_{0i,k-1}}$$

Jacobian Matrix

$$\mathcal{J}_f\left(\mathbf{p}_k\right) = \begin{bmatrix} \frac{\partial w}{\partial a_0} & \frac{\partial v}{\partial a_0} \\ \frac{\partial w}{\partial a_1} & \frac{\partial v}{\partial a_1} \end{bmatrix}$$

Parameter identification using the Extended skewed Kalman Filter (EsKF)

- in case of the skewed state space model, the linearization is done around the current state of the linear part

$$\text{linear part:} \quad \mathbf{u}_{ext,k} = \mathbf{T}_{ext,k\text{-}1}\mathbf{u}_{ext,k-1} + \mathbf{S}_{ext,k\text{-}1}\mathbf{w}_{ext,k}$$

$$\text{skewed part:} \quad \mathbf{y}_k = -\mathrm{L}_k\mathbf{y}_{k-1} + \eta_k \quad \Rightarrow \quad \mathbf{s}_k = \mathbf{y}_k \,|\, \mathbf{y}_{k-1} < \mathrm{D}_k\mu_k$$

$$\mathbf{z}_k = \mathbf{G}\mathbf{x}_k = \mathbf{A}_{ext}\mathbf{u}_k + \mathbf{A}_{ext}\mathbf{S}_{ext,k\text{-}1}\mathbf{s}_k + \mathbf{v}_k \qquad \mathbf{A}_{ext} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

- the linearized extended state space model can now be investigated using the introduced skewed Kalman filter algorithm where

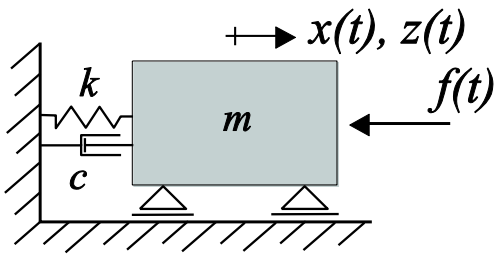
  $\Rightarrow$ the time-variante transfer matrices $\mathbf{T}_k$, $\mathbf{S}_k$ are replaced by the extended matrices $\mathbf{T}_{ext,k}$, $\mathbf{S}_{ext,k}$

Identification results

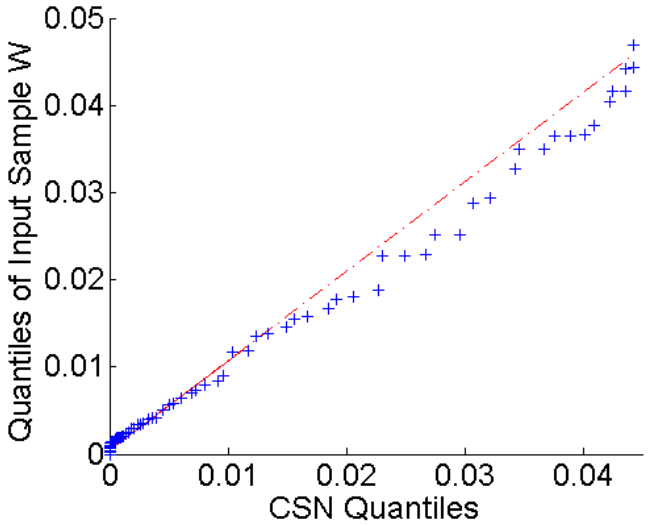
$m = 10\ \mathrm{kg}$

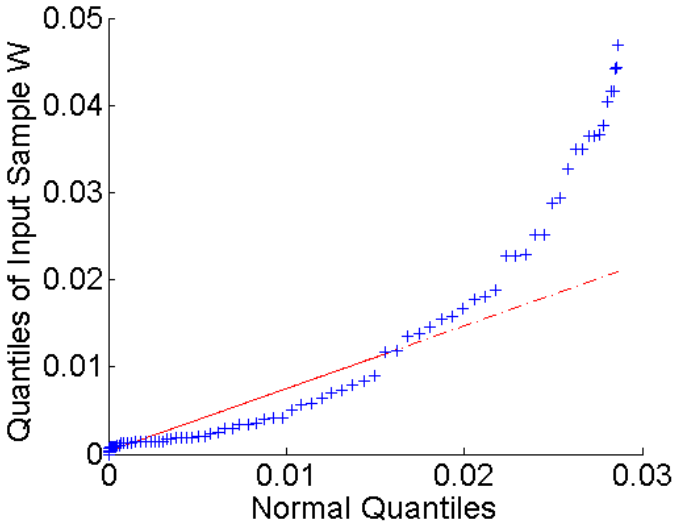
$a_0 = k/m$

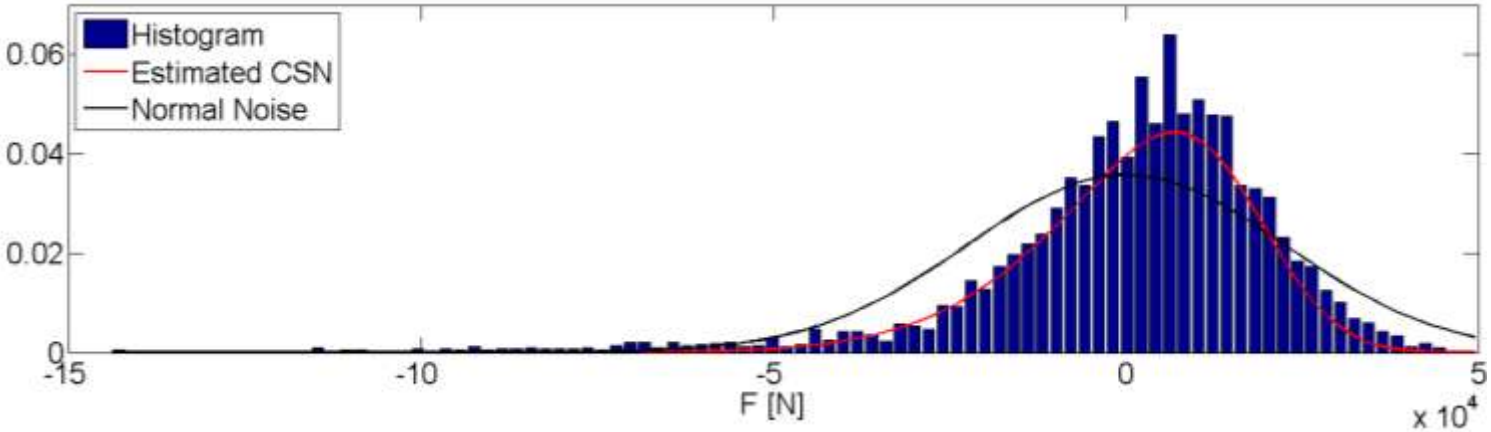
$a_1 = c/m$



Approximation of the PDF of the measurement wind data

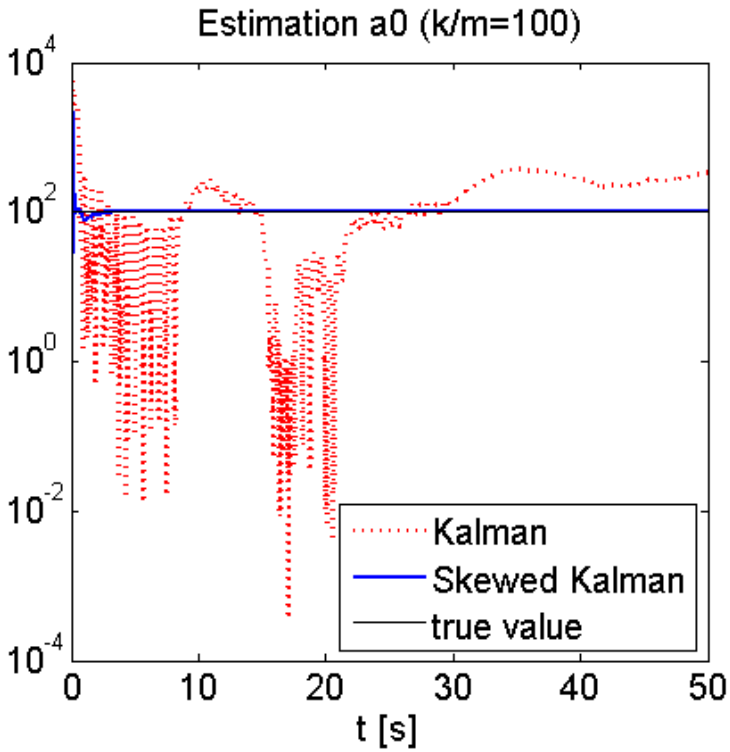


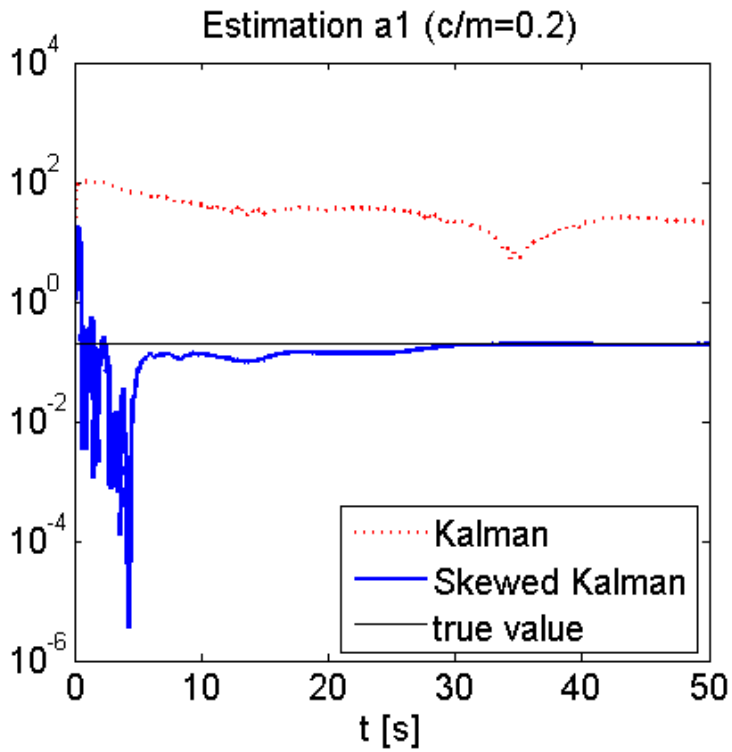




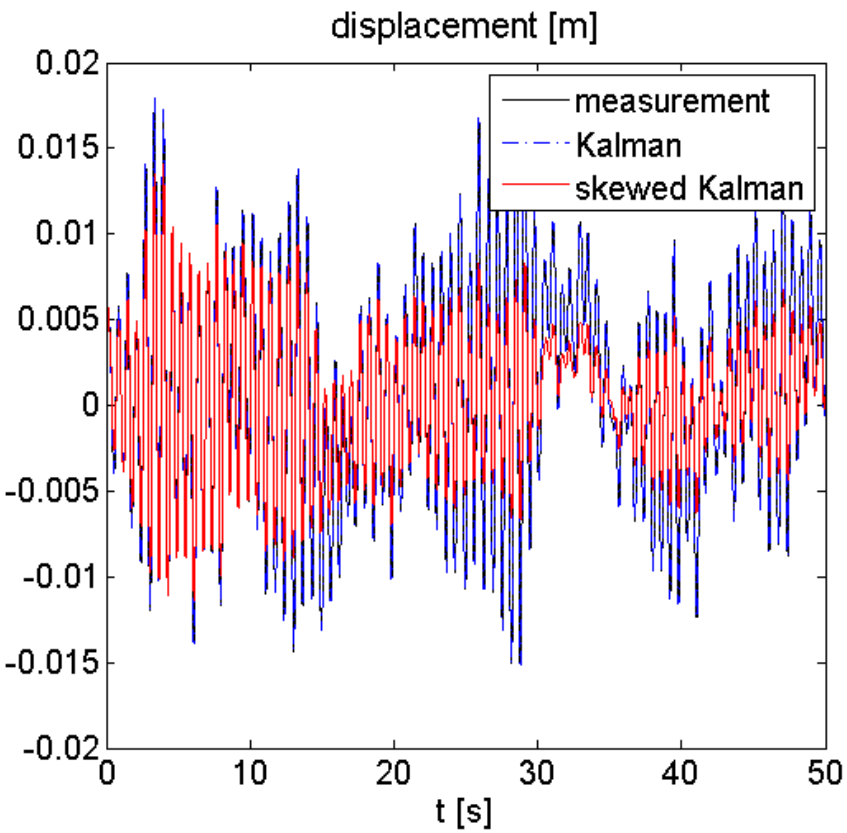
Identification results

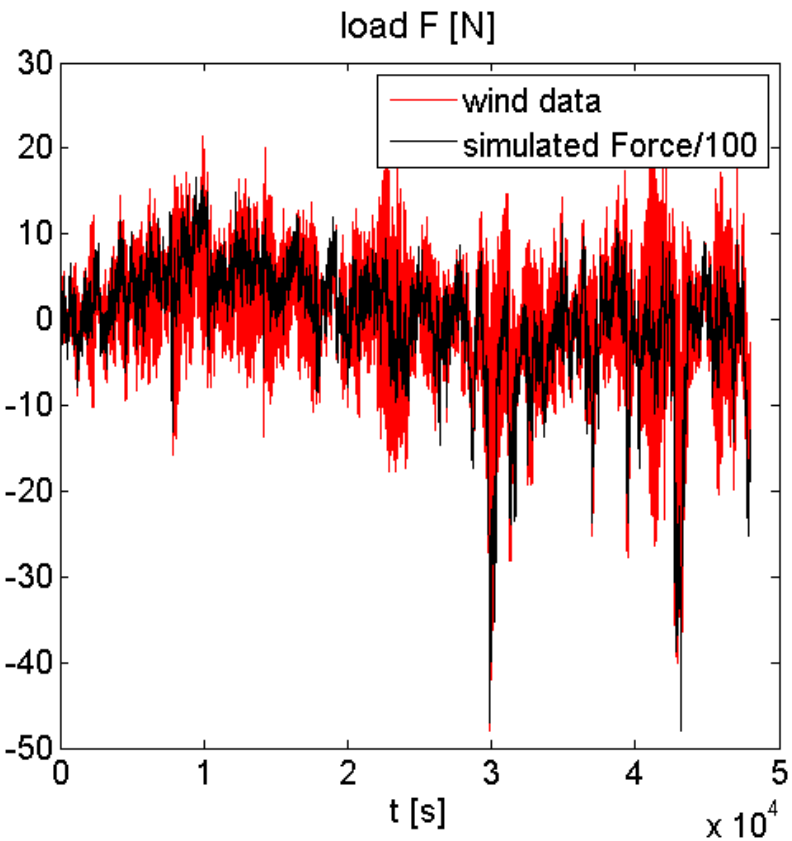
	true values	initial values	initial error	identified parameters		identification error	
				EKF	EsKF	EKF	EsKF
$a_0\,[\mathrm{s}^{-2}]$	100	150	50 %	20,40	101,84	79,6 %	2 %
$a_1\,[\mathrm{s}^{-1}]$	0,2	0,3	50 %	1,18	0,19	490 %	5%





Predicted displacement and generated load





Conclusion

- ✚ The method is computational efficient
- ✚ The method worked well for the introduced identification problem
- ✚ The CSN allows to describe the characteristics of the regarded skewed wind data
- ▬ The „form“ of the temperal evaluation of the wind data was approximated well, but the amplitude was wrong by a factor 100

Further work

- Find the error….any comments?
- Investigate the sensitivity of the parameter identification results depending on the initial conditions
- Extending the method to a multi-degree of freedom system

Thanks!

Any questions or comments?

PREDICTION (TIME UPDATE)	CORRECTION (MEASUREMENT UPDATE)
Project the state ahead (***prior estimate***) **Linear part** $\bar{\boldsymbol{\mu}}_{u,k} = \mathbf{T}_k \boldsymbol{\mu}_{u,k-1}$ $\bar{\boldsymbol{\Sigma}}_{uu,k} = \mathbf{T}_k \boldsymbol{\Sigma}_{uu,k-1} \mathbf{T}_k^T + \mathbf{S}_k \boldsymbol{\Sigma}_{ww,k-1} \mathbf{S}_k^T$ **Skewed part** $\bar{\boldsymbol{\mu}}_{y,k} = -\mathbf{L}_k \mu_{y,k-1}$ $\bar{\boldsymbol{\Sigma}}_{yy,k} = \mathbf{L}_k \boldsymbol{\Sigma}_{yy,k-1} \mathbf{L}_\mathrm{k}^T + \boldsymbol{\Sigma}_{\eta\eta,k}$ $\bar{\mathbf{D}}_k = \boldsymbol{\Sigma}_{yy,k-1} \mathbf{L}_k^T \bar{\boldsymbol{\Sigma}}_{yy,k}^{-1}$ $\bar{\mathbf{v}}_k = \mu_{y,k-1} - \bar{\mathbf{D}}_k \bar{\boldsymbol{\mu}}_{y,k}$ $\bar{\boldsymbol{\Delta}}_k = \boldsymbol{\Sigma}_{yy,k-1} - \bar{\mathbf{D}}_k \bar{\boldsymbol{\Sigma}}_{yy,k} \bar{\mathbf{D}}_k^T$	Compute the **Kalman gain matrix** of the linear and skewed part $\mathbf{K}_k = \bar{\boldsymbol{\Sigma}}_{uu,k} \mathbf{A}_k^T \left(\boldsymbol{\Sigma}_{vv,k} + \mathbf{A}_k \bar{\boldsymbol{\Sigma}}_{uu,k} \mathbf{A}_k^T + \mathbf{S}_k \mathrm{Var}[\mathbf{s}_k \vert \mathbf{z}_{1:k-1}] \mathbf{S}_k^T \right)^{-1}$ $\mathbf{K}_{s,k} = \mathbf{C}_k \mathbf{S}_k^T \left(\boldsymbol{\Sigma}_{vv,k} + \mathbf{A}_k \bar{\boldsymbol{\Sigma}}_{uu,k} \mathbf{A}_k^T + \mathbf{S}_k \mathrm{Var}[\mathbf{s}_k \vert \mathbf{z}_{1:k-1}] \mathbf{S}_k^T \right)^{-1}$ Update estimate with measurement (***posterior estimate***) $\mathbf{e}_k = \left(\mathbf{z}_k - \mathbf{A}_k \bar{\boldsymbol{\mu}}_{u,k} - \mathbf{S}_k \mathrm{E}[\mathbf{s}_k \vert \mathbf{z}_{1:k-1}] \right)$ **Linear part** $\bar{\boldsymbol{\mu}}_{y,k} = \bar{\boldsymbol{\mu}}_{u,k} + \mathbf{K}_k \mathbf{e}_k$ $\boldsymbol{\Sigma}_{uu,k} = \bar{\boldsymbol{\Sigma}}_{uu,k} - \mathbf{K}_k \mathbf{A}_k \bar{\boldsymbol{\Sigma}}_{uu,k}$ **Skewed part** $\boldsymbol{\mu}_{y,k} = \bar{\boldsymbol{\mu}}_{y,k} + \mathbf{K}_{s,k} \mathbf{e}_k$ $\boldsymbol{\Sigma}_{yy,k} = \bar{\boldsymbol{\Sigma}}_{yy,k} - \mathbf{K}_{s,k} \mathbf{S}_k \mathbf{C}_k$ $\mathbf{D}_k = \boldsymbol{\Sigma}_{yy,k}^* \mathbf{L}_\mathrm{k}^{*\mathrm{T}} \boldsymbol{\Sigma}_{yy,k}^{-1}$ $\mathbf{v}_k = \boldsymbol{\mu}_{y,k-1} + \mathbf{C}_{k-1} \mathbf{C}_k^{-1} \mathbf{K}_{s,k} \mathbf{e}_k - \mathbf{D}_k \boldsymbol{\mu}_{y,k}$ $\boldsymbol{\Delta}_k = \boldsymbol{\Sigma}_{yy,k}^* - \mathbf{D}_k \boldsymbol{\Sigma}_{yy,k} \mathbf{D}_k^T$