

# PRAGMATIC PROBABILISTIC MODELS FOR QUANTIFICATION OF TUNNEL EXCAVATION RISK

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# Presentation Outline

A Motivation

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C2 Failure probability of surface structure

C3 Failure due to random cave-in collapse

C4 Systems with varying geotechnical parameters

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# A Motivation

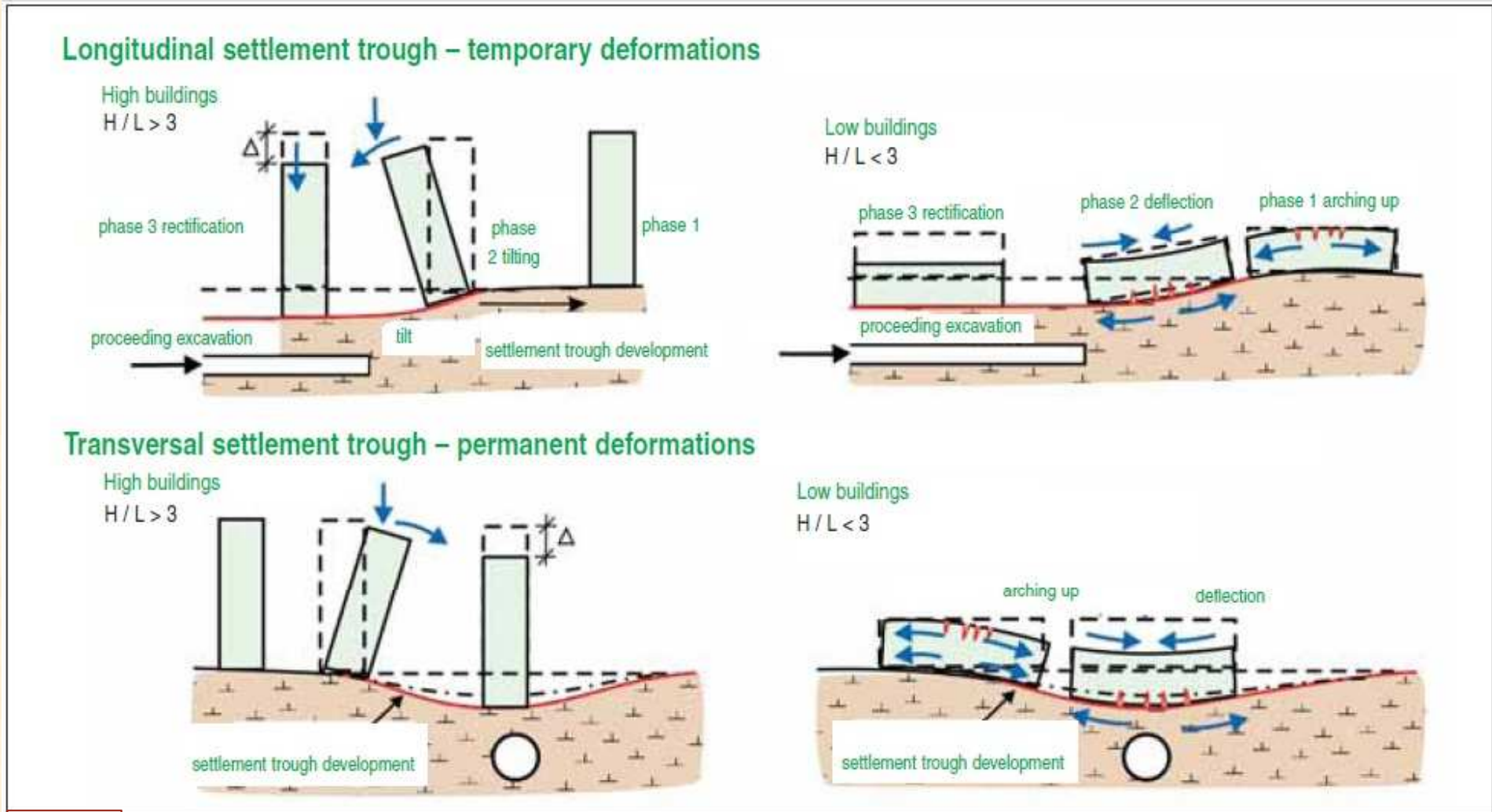


Fig.1

Courtesy: T. Ebermann et al, Tunnel 4/2010

# A Motivation



Fig.2

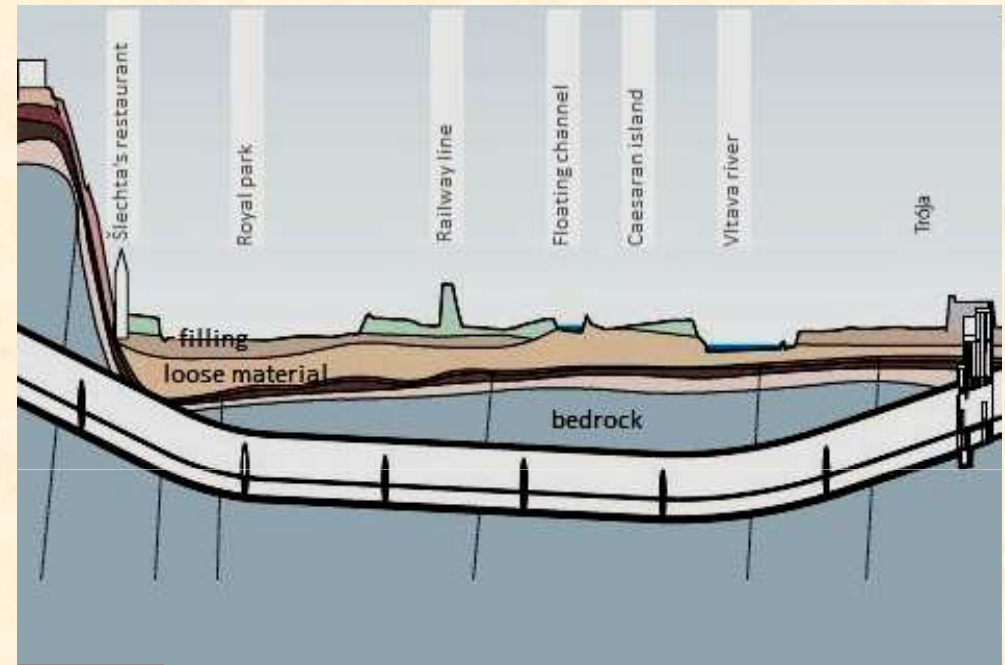


Fig.3

# B Modeling of successive excavation

## The convergence confinement method vs quasi-3D FEM

Loading with excavation forces

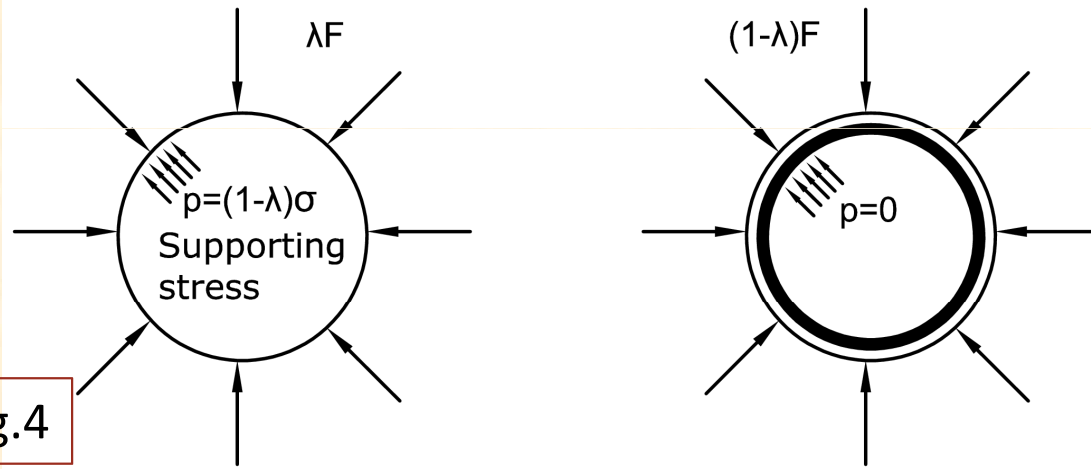
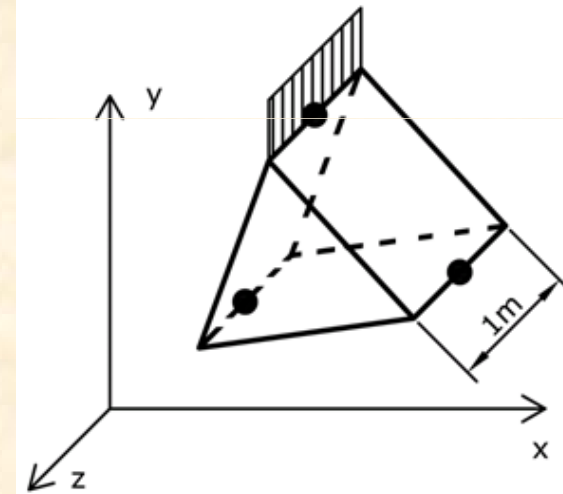


Fig.4

2D element - plane strain conditions



Direction of excavation

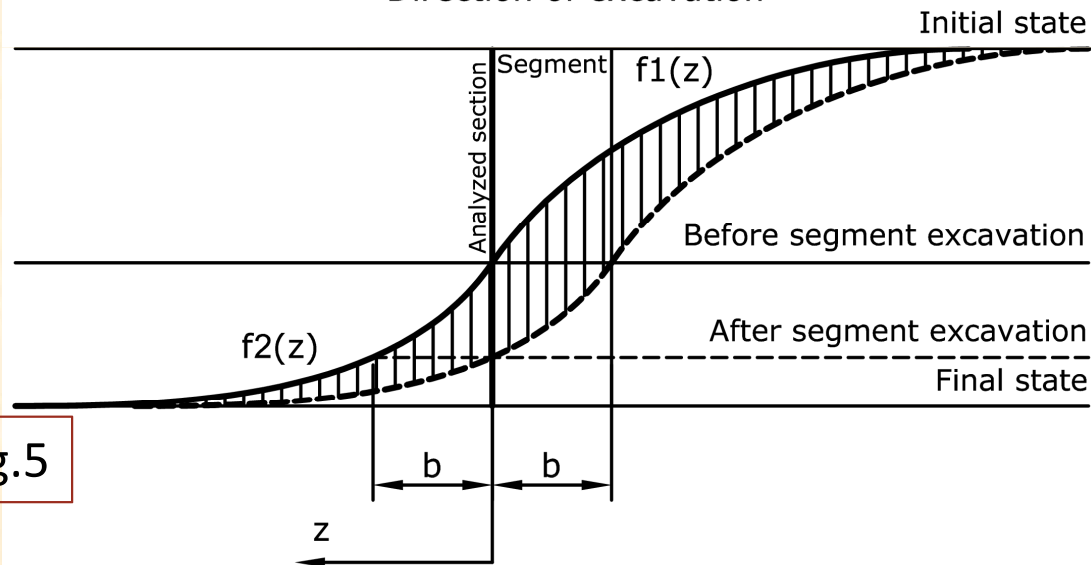
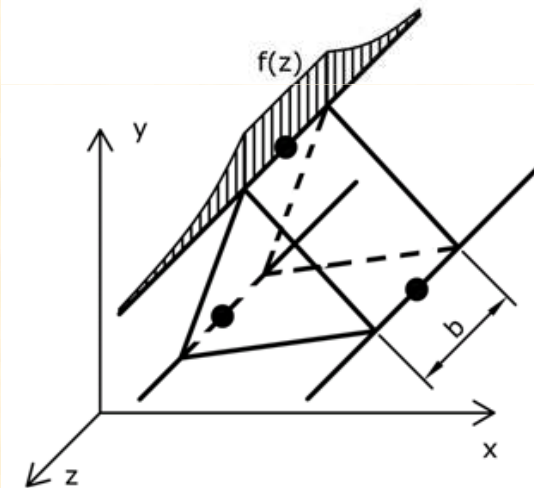


Fig.5

2D3D element - shape function  $f(z)$



# B Modeling of successive excavation

## Extensometric measurements as a source of the model data

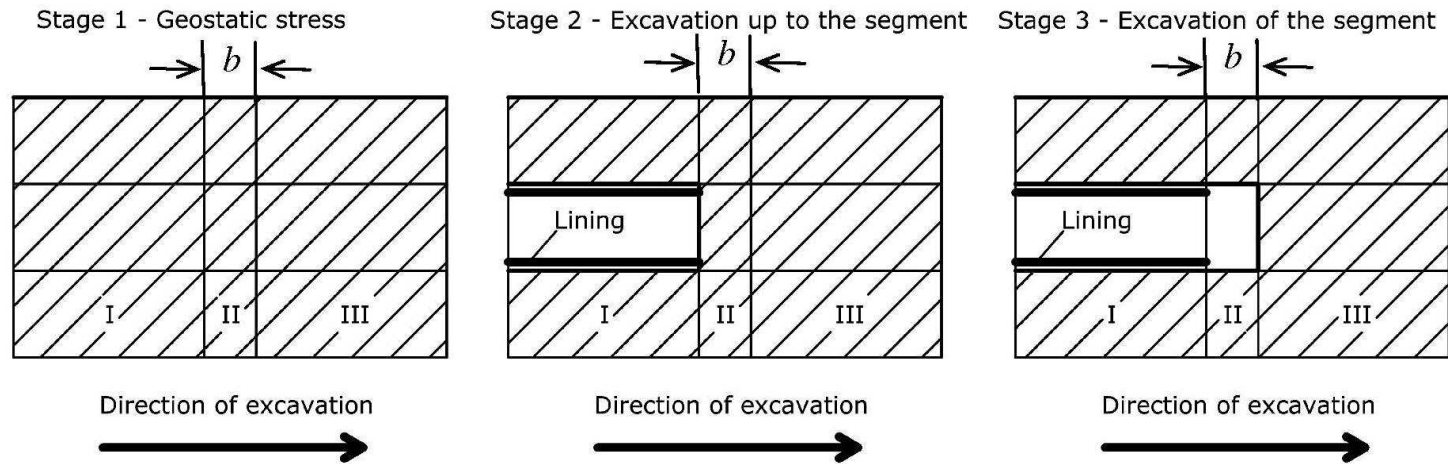


Fig.6

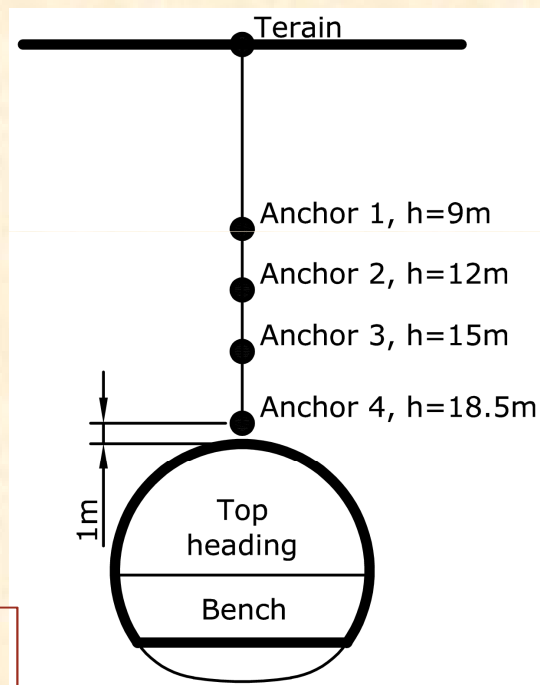


Fig.7

# B Modeling of successive excavation

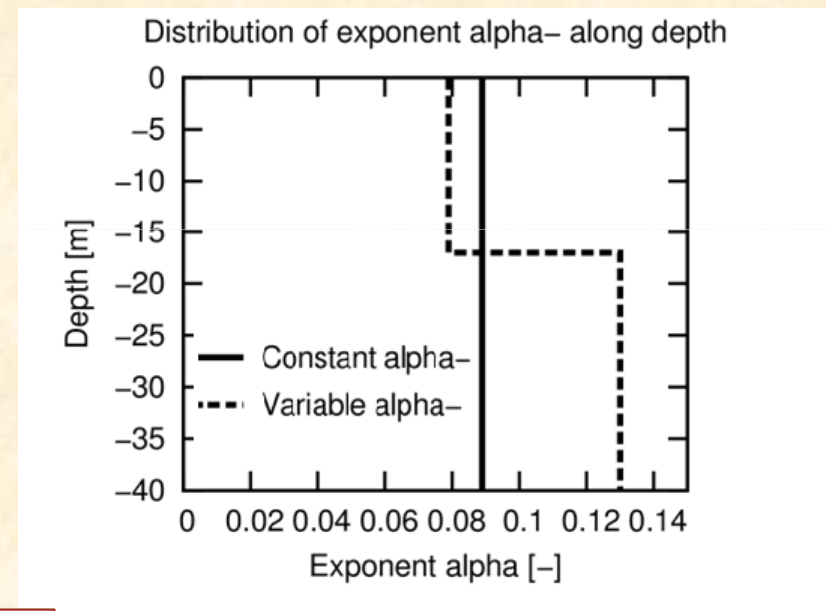
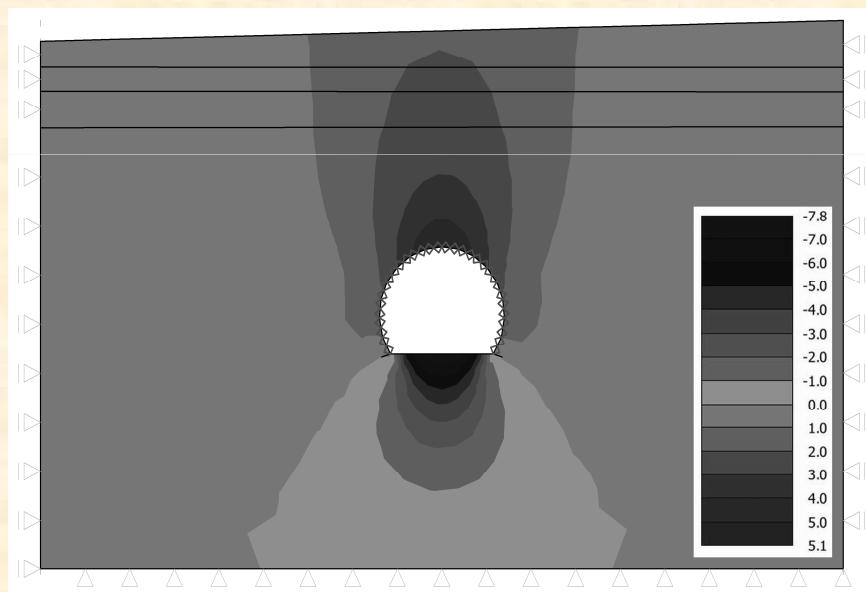
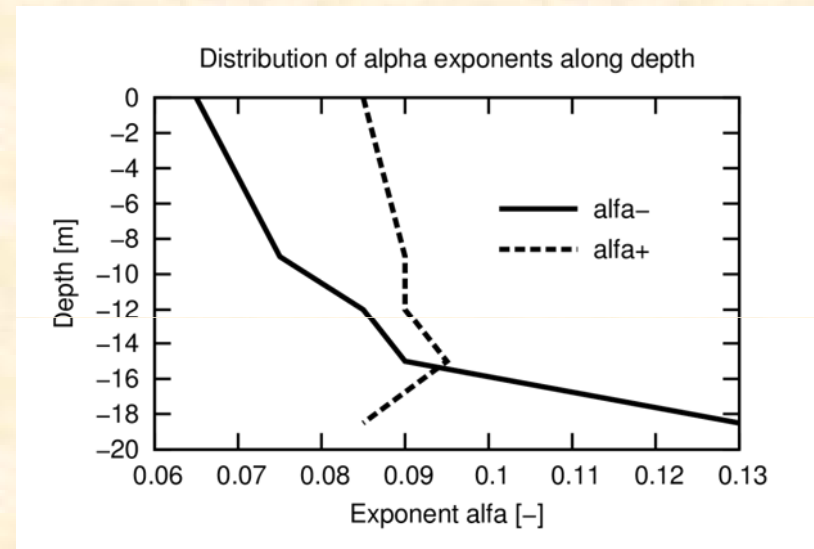
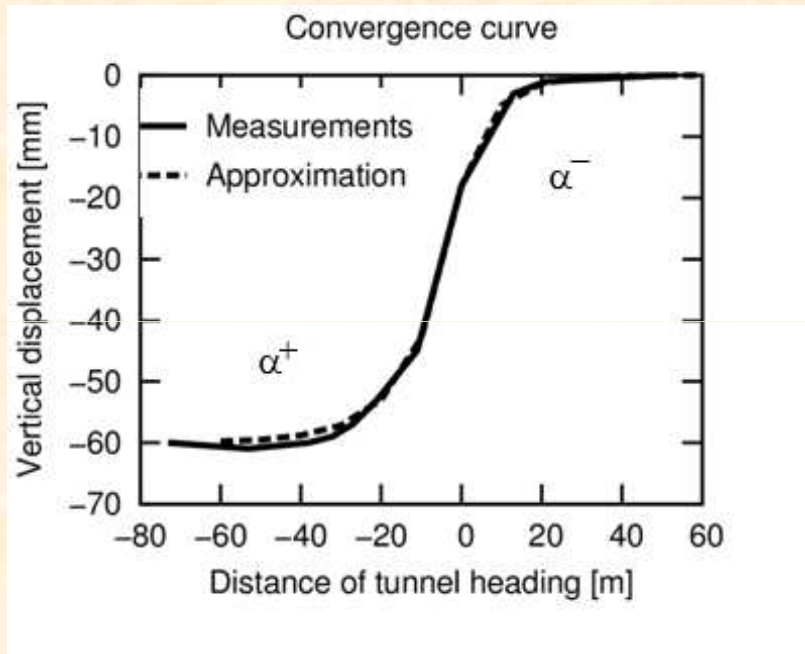


Fig.8

# C1 Categorization of failures and concept of risk analysis

## Most typical failures:

- Extensive deformations of the tunnel tube
- Exceeding of acceptable progress of the subsidence trough
- Cave-in collapse
- Occurrence of a tunnel segment surrounded by a suddenly weakened rock
- Occurrence of a resistant overburden the thickness of which randomly tends to diminish

## Risk assessment:

The simplest formula

$$R = P[Event] \times D. \quad (1)$$

can be generalized as

$$R = \sum_i P[Event] \times P[Consequence(i)|Event] \times D(i), \quad (2)$$

where  $D(i)$  is the expected financial loss caused by the Consequence.



# C2 Failure probability of surface structure

## The subsidence trough description

$$W(x, y) = \bar{W}(x, y) + w^*(x, y) \quad (3)$$

## A simple approximation reads

$$\begin{aligned} W(x; y) &= \bar{W}(0; 0)[g_1(x) + g^*][g_2(y) + g^*] \\ &\cong \bar{W}(x; y) + w^*[g_1(x) + g_2(y)], \end{aligned} \quad (4)$$

$$\text{where } \bar{W}(x, y) = \bar{W}_0 g_1(x) g_2(y), \quad w^* = \bar{W}_0 g^*. \quad (5)$$

## The loading effect

In longitudinal direction

$$\Delta_{2x} W(x; y) \cong \frac{\bar{W}_0 d_x^2}{2} g_1''(x) g_2(y), \quad \Delta_{2x} w^* \cong w^* \frac{d_x^2}{2} g_1''(x) \quad (6)$$

In transverse direction

$$\Delta_{2y} W(x; y) \cong \frac{\bar{W}_0 d_y^2}{2} g_1(x) g_2''(y), \quad \Delta_{2y} w^* \cong w^* \frac{d_y^2}{2} g_2''(y). \quad (7)$$

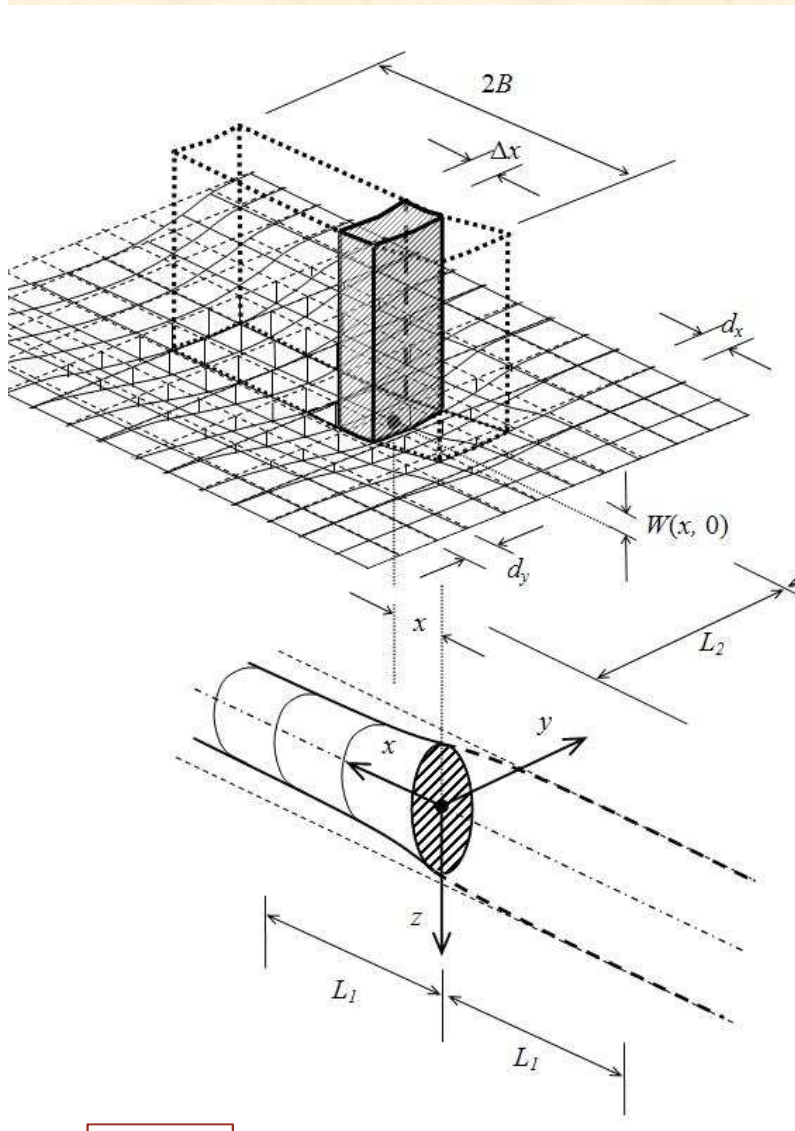


Fig.9

## C2 Failure probability of surface structure

- Let the structure resistance,  $R_x$  and/or  $R_y$ , be a maximum curvature the structure is able to sustain.
- The failure probability of a segment located at a distance  $x$

$$p_f(x) = P[R_x < \Delta_{2x} W(x;0) \cup R_y < \Delta_{2y} W(x;0)] \quad (8)$$

$$= p_{fx}(x) + p_{fy}(x) - P[R_x < \Delta_{2x} W(x;0) \cap R_y < \Delta_{2y} W(x;0)],$$

where

$$p_{fx}(x) = P[R_x < \Delta_{2x} W(x;0)] = \int_{\text{range of } w^*} F_{Rx} \left[ (\bar{W}_0 + w^*) g_1''(x) \frac{d_x^2}{2} \right] \cdot f_{W^*}(w^*) dw^* \quad (9)$$

$$p_{fy}(x) = P[R_y < \Delta_{2y} W(x;0)] = \int_{\text{range of } w^*} F_{Ry} \left[ (\bar{W}_0 g_1(x) + w^*) g_2''(0) \frac{d_y^2}{2} \right] \cdot f_{W^*}(w^*) dw^*.$$

- The unconditional probability of failure is

$$p_f = \frac{1}{2(L_1 + B)} \int_{-L_1-B}^{L_1+B} p_f(x) dx. \quad (10)$$

# C3 Failure due to random cave-in collapse

The sequence of collapses is mostly described by means of the Poisson model (Fig. 10). Hence, a random variable distance,  $U$ , is exponentially distributed, i.e.

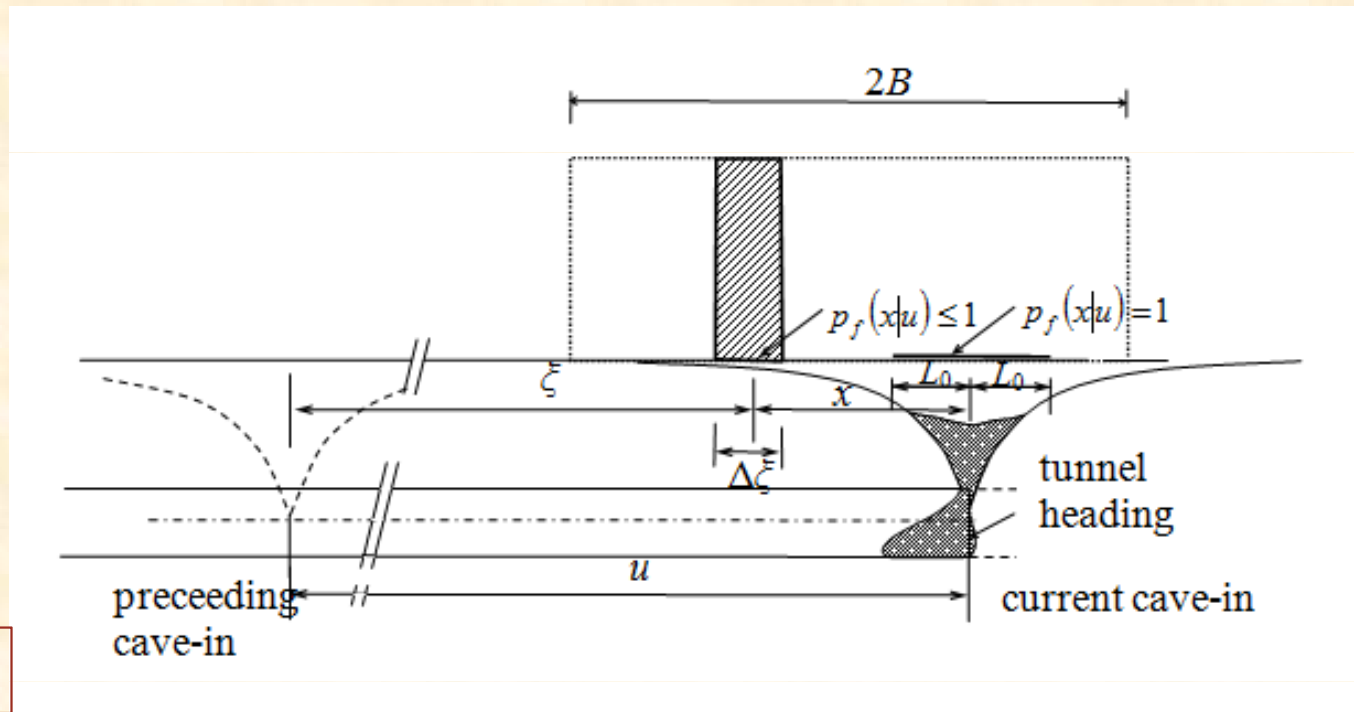


Fig.10

$$F_U(u) = P(U \leq u) = 1 - e^{-\lambda u},$$

$$f_U(u)du = P(u \leq U \leq u + du) = \lambda e^{-\lambda u} du, \quad (11)$$

where  $\lambda$  is the intensity of the process.

# C3 Failure due to random cave-in collapse

Modification of Eq. (11) based on an expert's judgment (Fig. 11)

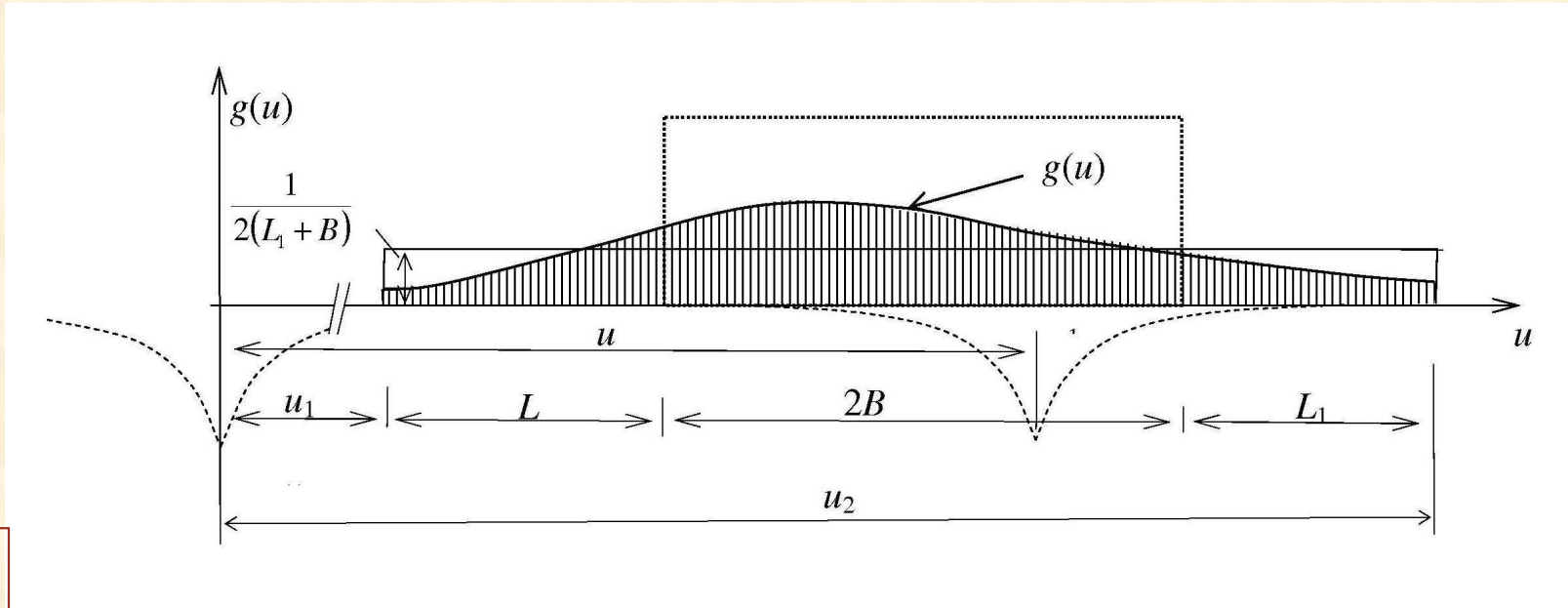


Fig.11

The expert's surrogate for Eq. (11) may be written as

$$f_U(u)du = P_1 g(u)du, \quad u_1 \leq U \leq u_2 \quad (12)$$

Evidently, the normalization condition must be fulfilled

$$\int_{u_1}^{u_2} f_U(u)du = P_1 \underbrace{\int_{u_1}^{u_2} g(u)du}_1 = P_1$$

# C3 Failure due to random cave-in collapse

## Two scenarios with regard to the cave-in location

(Fig.10)

a) The cave-in position is sufficiently far away from the selected segment

$$p_f(\xi|u) \leq 1$$

b) The cave-in occurs in the segment's close vicinity ( $|x| \leq L_0$ )

$$p_f(\xi|u) \approx 1$$

The unconditional probability of failure in a selected segment  $\xi$

$$p_f(\xi) = \int_{u_1}^{u_2} p_f(\xi|u) f_U(u) du + 2 L_0 f_U(\xi). \quad (13)$$

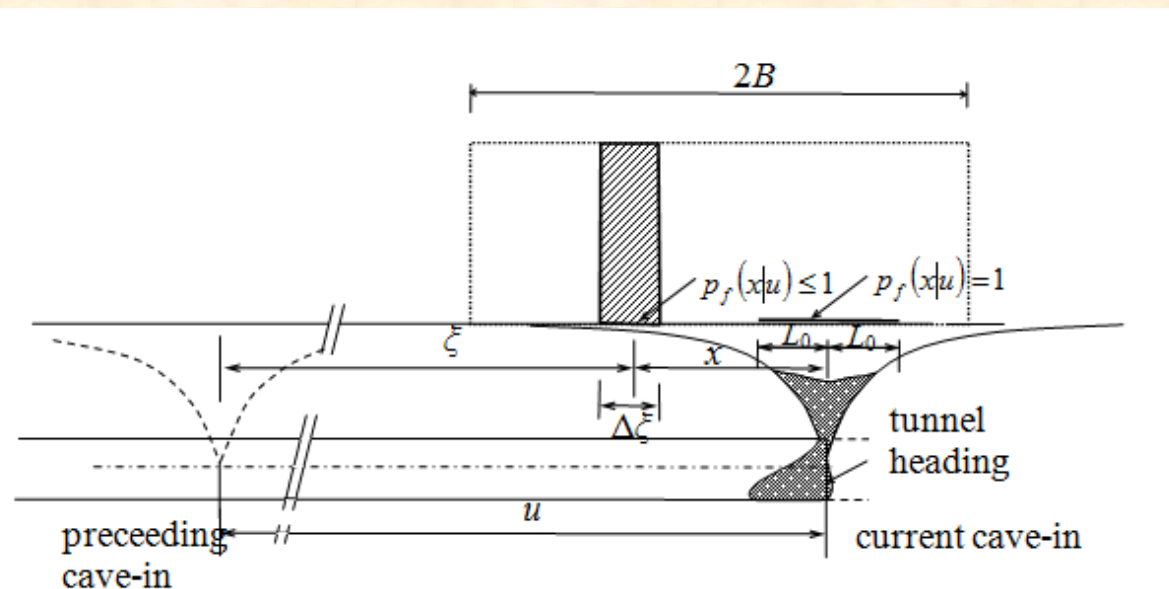


Fig.10

# C4 Random system with continuously varying geotechnical parameters

A loose material randomly separated from the rock overburden (Fig. 12)

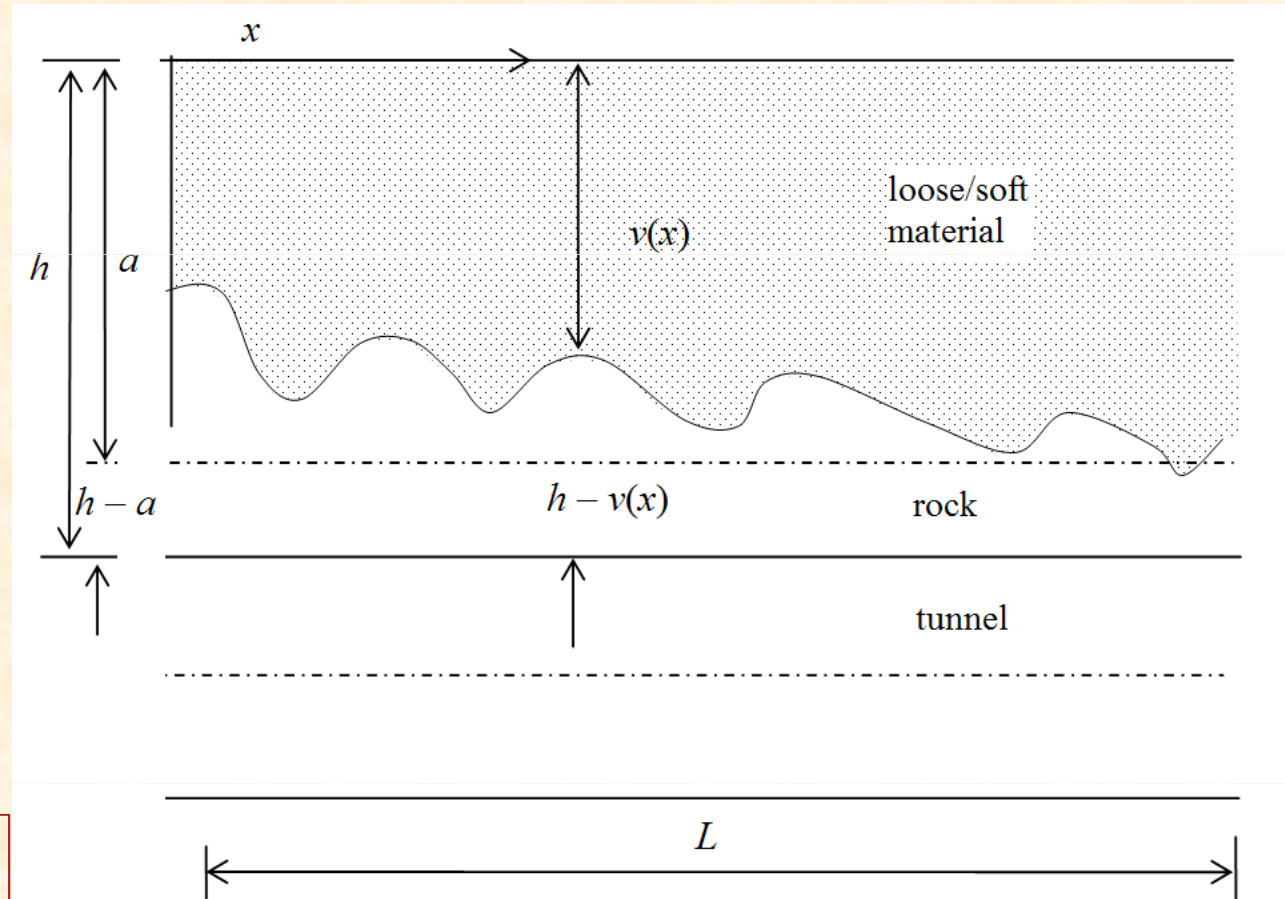


Fig.12

For a constant level  $a$ , the barrier up-crossing rate,  $\nu_a^+$ , is given by Rice's formula

$$\nu_a^+ = \int_0^\infty \dot{v} f_{v\dot{v}}(a, \dot{v}) d\dot{v} = \frac{1}{2\pi} \frac{\sigma_{\dot{v}}}{\sigma_v} \exp\left[-\frac{(a - \mu_v(x))^2}{2\sigma_v^2}\right] \quad (14)$$

# C4 Random system with continuously varying geotechnical parameters

Introducing the spectral density function  $S_v(\omega)$  along with corresponding relations

$$\sigma_v^2 = \int_{-\infty}^{\infty} S_v(\omega) d\omega, \quad \sigma_{\dot{v}}^2 = \int_{-\infty}^{\infty} \omega^2 S_v(\omega) d\omega \quad (15)$$

and considering a narrow band process characterized by frequency  $\omega_o$  we arrive at

$$\frac{\sigma_{\dot{v}}}{\sigma_v} \cong \omega_o \quad (16)$$

Evidently,  $\nu_a^+$  could also be regarded as the intensity of a Poisson process.

Hence, the first-passage probability is assessed as

$$P[N_f > 1] = 1 - \exp(-\tilde{\nu}_a^+ L), \quad (17)$$

where

$$N_f(\nu_a^+(x), L) = \int_0^L \nu_a^+(x) dx = \nu_a^+ L \quad (18)$$

is the number of up-crossings on a given length  $L$ .

# C4 Random system with continuously varying geotechnical parameters

Example - A segment of the Blanka tunnel (Fig. 3)

Interface between the rock overburden and soft material detected by in situ measurements is displayed in Fig. 13. A solid line obtained by the linear regression demonstrates a variable mean function of Gauss' process.

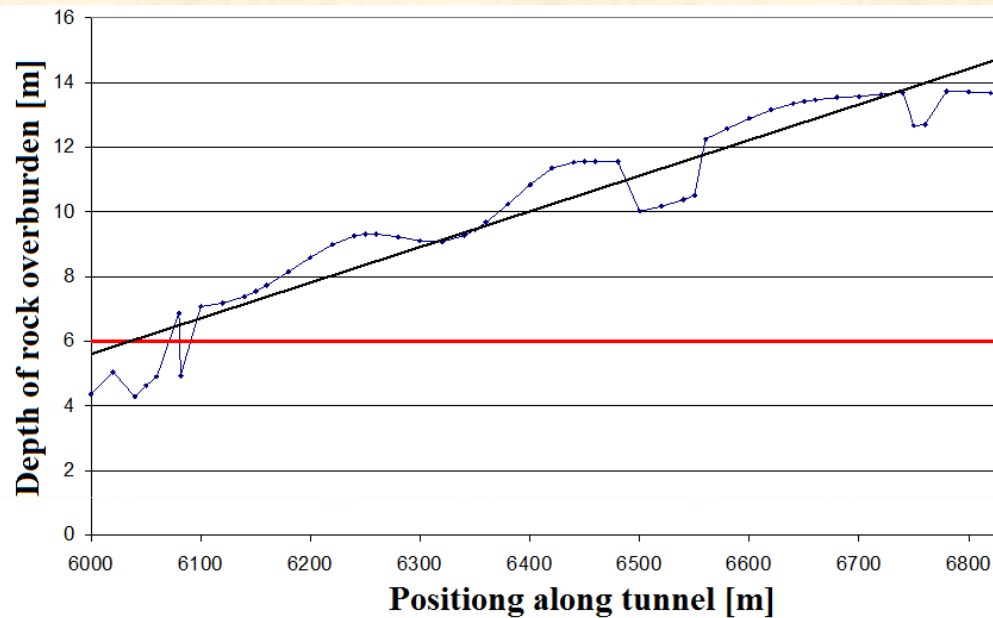
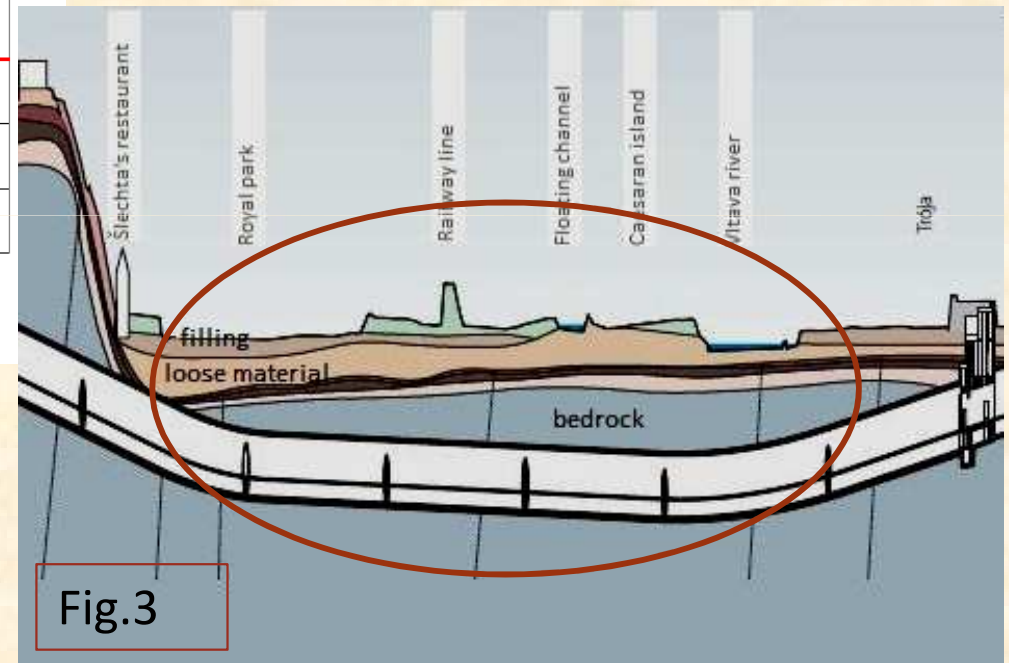


Fig.13





# C4 Random system with continuously varying geotechnical parameters

The results of a case study depicted in Fig. 14 show sensibility of the model to the wave length  $L_0$ . The standard deviation of Gauss' process  $\sigma = 0.841\text{m}$  was evaluated numerically and the depth of the rock layer was drawn as  $h - a = 6\text{m}$ .

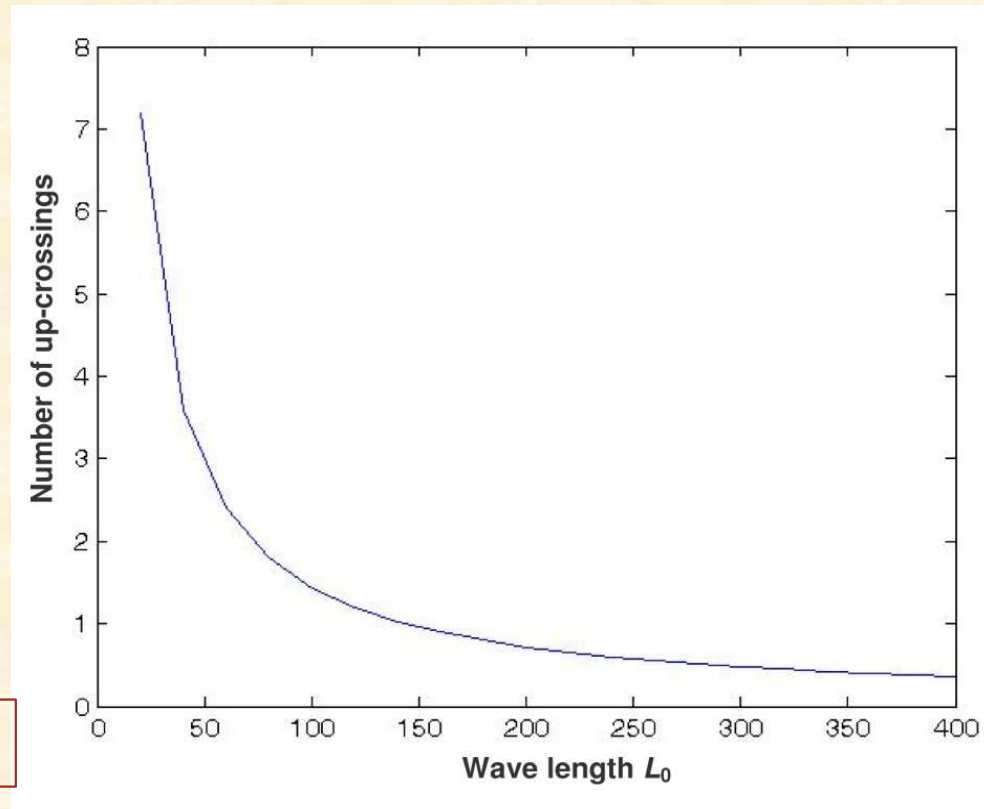


Fig.14

# D Conclusions

- Probability-based approaches are an efficient alternative to expert methods such as FTA and ETA. They operate either separately to estimate risks in a direct way or as an auxiliary tool for FTA and ETA.
- The proposed methodology suggests theoretical instruments making it possible to analyze most serious problems tunnel engineering has to face.
- All the phenomena discussed within the scope of this paper have been recently met during the excavation of the Blanka tunnel in Prague.
- Of course, there are certain drawbacks that could not be overlooked. The main point is material data which has to be properly predicted both by in situ measurements and laboratory tests. If reliable data is missed, any sophisticated theory whatever becomes pointless and cannot responsibly be implemented.