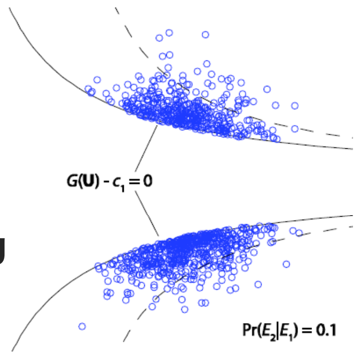
[ISUME, CTU Prague, May 2, 2011]



What's the point of modelling uncertainty in engineering?

Optimal decision making under uncertainty

Daniel Straub
Engineering Risk Analysis Group
TU München

Technische Universität München

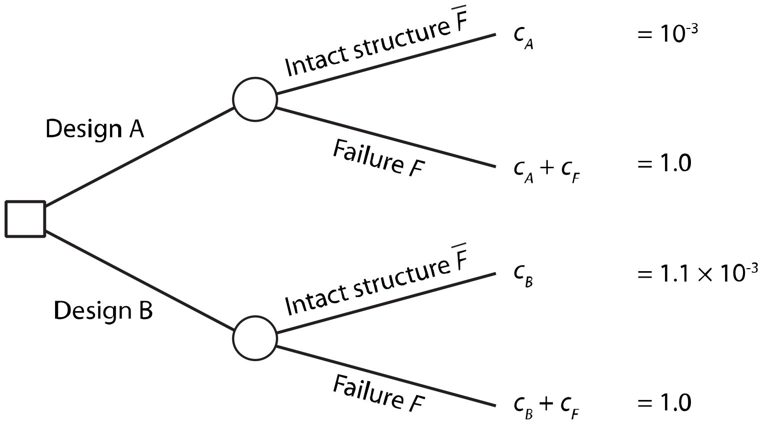
What is the value of a probabilistic analysis?

- Probabilistic analysis provides
  a more accurate description of the system
- For engineers, this is not an inherent benefit

- The improved system description might support
  the identification of better engineering solutions
- This is the benefit

2

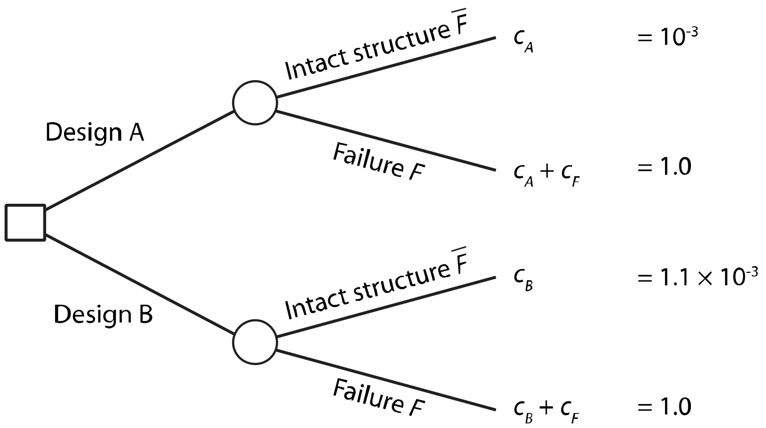
A basic engineering problem



3

A basic engineering problem

$$\mathrm{E}[C|Design = i] = \Pr(F|Design = i)\, c_F + c_i$$

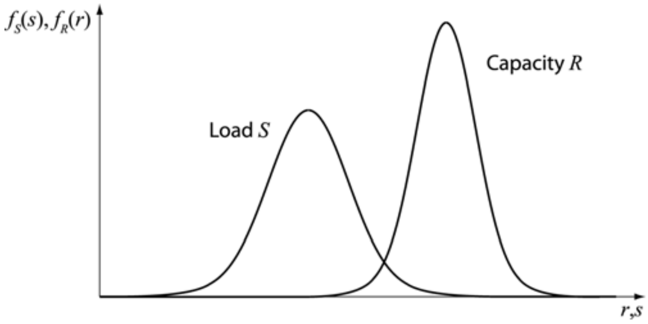


4

A basic engineering problem

$$\mathrm{E}[C|Design = i] = \Pr(F|Design = i)\, c_F + c_i$$

- Basic failure definition: $F = \{R \leq S\}$



5

A basic engineering problem

$$\mathrm{E}[C|Design = i] = \Pr(F|Design = i)\, c_F + c_i$$

$$\Pr(F|Design = i) = \Phi\left(-\frac{\mu_{R_i} - \mu_S}{\sqrt{\sigma_{R_i}^2 + \sigma_S^2}}\right)$$

Mean and standard deviation of the load	$\mu_S = 100\text{kN}, \sigma_S = 25\text{kN}$
Mean and standard deviation of the capacity for design A	$\mu_{R_A} = 300\text{kN}, \sigma_{R_A} = 45\text{kN}$
Mean and standard deviation of the capacity for design B	$\mu_{R_B} = 300\text{kN}, \sigma_{R_B} = 30\text{kN}$
Acceptable probability of failure	$p_F^T = 10^{-4}$

6

Design A is the optimal choice (as identified with a probabilistic analysis)

$$a_p = \arg\min_{i=A,B} \mathrm{E}[C|Design = i]$$

$$\text{subject to } \Pr(F|Design = i) \leq p_F^T$$

$$\mathrm{E}[C|Design = A] = 1.05 \cdot 10^{-3}$$

$$\mathrm{E}[C|Design = B] = 1.10 \cdot 10^{-3}$$

7

The alternative is a deterministic code-based approach

- Select the cheapest design complying with the code:

$$a_d = \arg\min_{i=A,B} c_i$$

$$\text{subject to } \underbrace{\gamma_S S_k \leq \frac{R_{ik}}{\gamma_R}}_{\text{Code criterion}}$$

8

The alternative is a deterministic code-based approach

- Select the cheapest design complying with the code:

$$a_d = \arg\min_{i=A,B} c_i$$

$$\text{subject to } \gamma_S S_k \leq \frac{R_{ik}}{\gamma_R}$$

Partial safety factor for loads	$\gamma_S = 1.5$
Partial safety factor for capacity	$\gamma_R = 1.3$
Characteristic values for loads	$S_k = \mu_S$
Characteristic values for capacities	$R_{ik} = \mu_{R_i} - 1.64\sigma_{R_i}$

9

The alternative is a deterministic code-based approach

- Select the cheapest design complying with the code:

$$a_d = \arg\min_{i=A,B} c_i$$

$$\text{subject to } \underbrace{\gamma_S S_k \leq \frac{R_{ik}}{\gamma_R}}_{\text{Code criterion}}$$

- Both comply → Design A is selected

10

The probabilistic analysis provides no benefit in this case

- Both analyses lead to the same design
- Conditional value of information (CVI) of the probabilistic analysis is zero

Value of information conditional on different design situations

- Same design options A and B
- Loading environment can change: Vary μ_S and σ_S (= $0.25\mu_S$)

Value of information conditional on different design situations

- Same design options A and B
- Loading environment can change: Vary μ_S and σ_S (= $0.25\mu_S$)

- For given $M_S=\mu_S$, find the optimal design
  - according to the deterministic analysis:
$$\begin{cases} a_d = \arg\min_{i=A,B} c_i \\ \text{subject to } \gamma_S S_k \leq \dfrac{R_{ik}}{\gamma_R} \end{cases}$$
  - according to the probabilistic analysis:
$$\begin{cases} a_p = \arg\min_{i=A,B} \mathrm{E}[C|Design = i] \\ \text{subject to } \Pr(F|Design = i) \leq p_F^T \end{cases}$$

13

Value of information conditional on different design situations

The conditional value of information is:

$$CVI(\mu_S) = \underbrace{\mathrm{E}[C|Design = a_d \cap \mathrm{M}_S = \mu_S]}_{\text{Expected cost with deterministic design}} - \underbrace{\mathrm{E}\big[C\big|Design = a_p \cap \mathrm{M}_S = \mu_S\big]}_{\text{Expected cost with probabilistic design}}$$

14

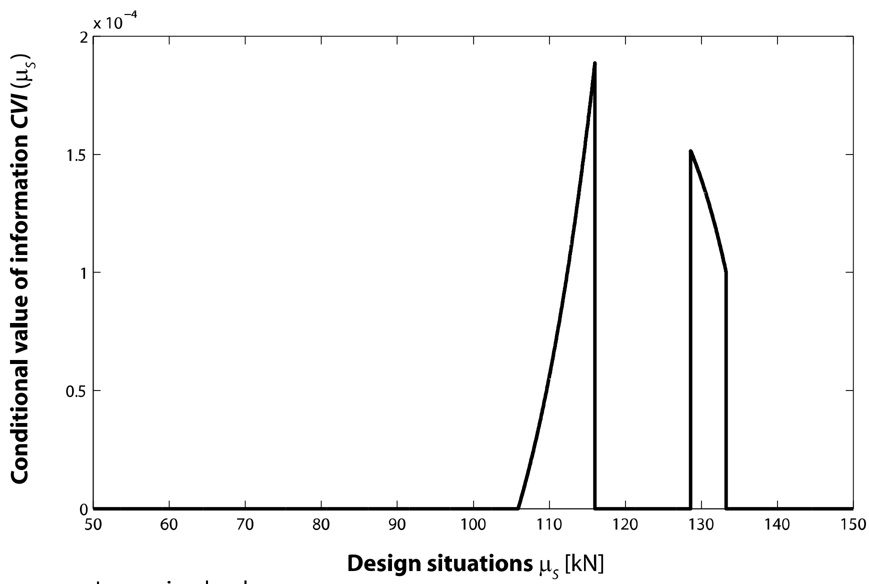
Value of information conditional on different design situations

The conditional value of information is:

$$CVI(\mu_S) = \underbrace{\mathrm{E}[C|Design = a_d \cap \mathrm{M}_S = \mu_S]}_{\text{Expected cost with }\underline{\text{deterministic}}\text{ design}} - \underbrace{\mathrm{E}\left[C\middle|Design = a_p \cap \mathrm{M}_S = \mu_S\right]}_{\text{Expected cost with }\underline{\text{probabilistic}}\text{ design}}$$

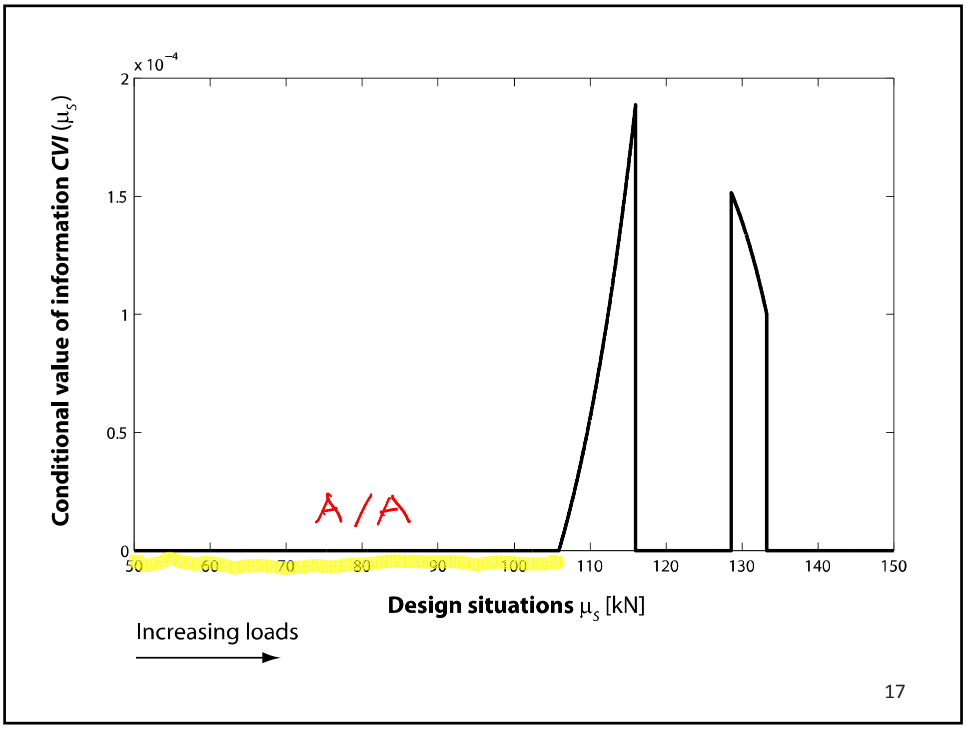
- The CVI cannot be negative!

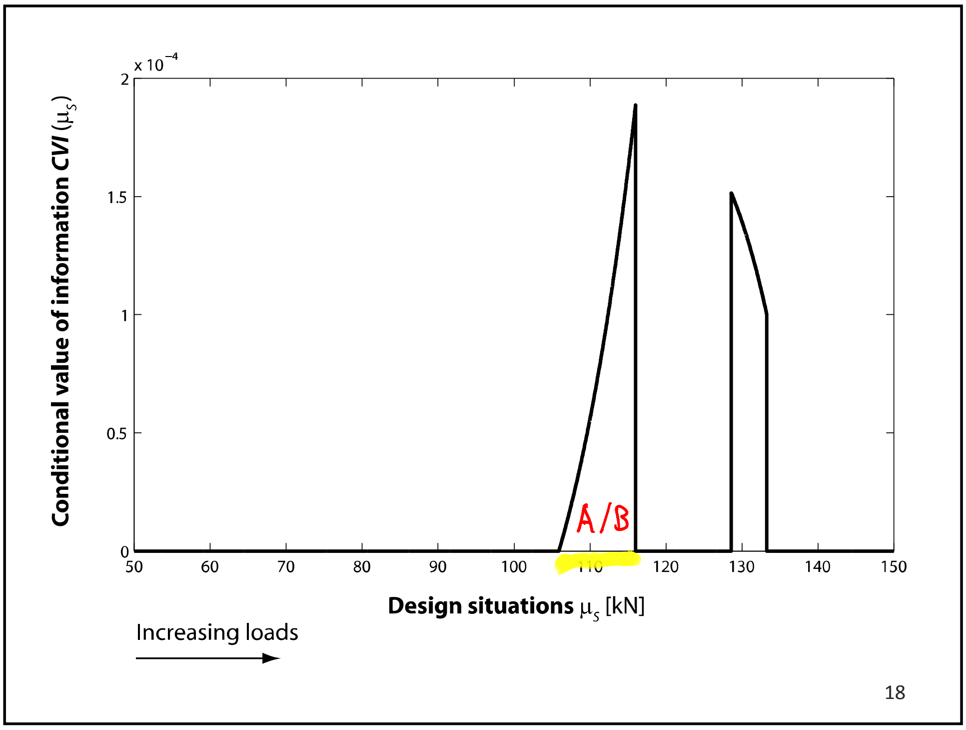
15

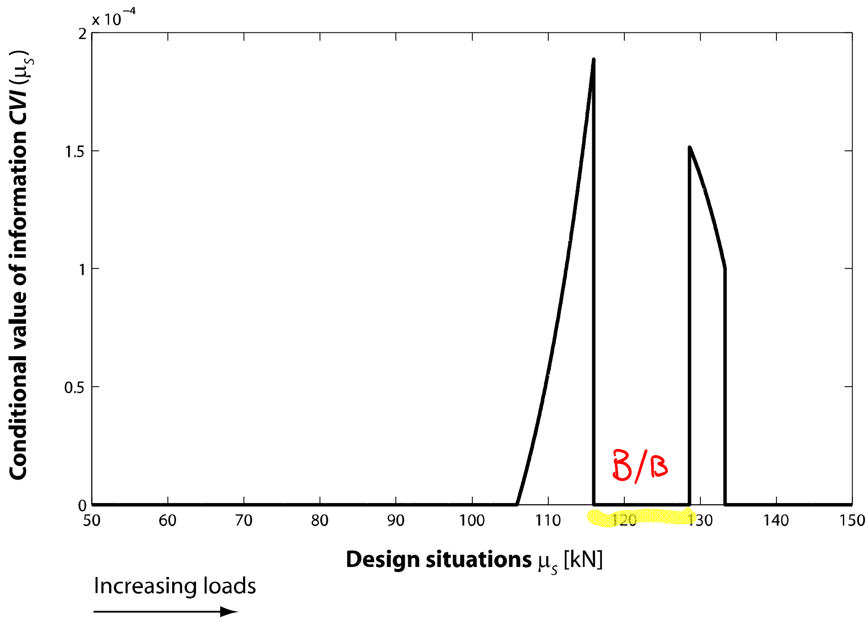


Increasing loads →

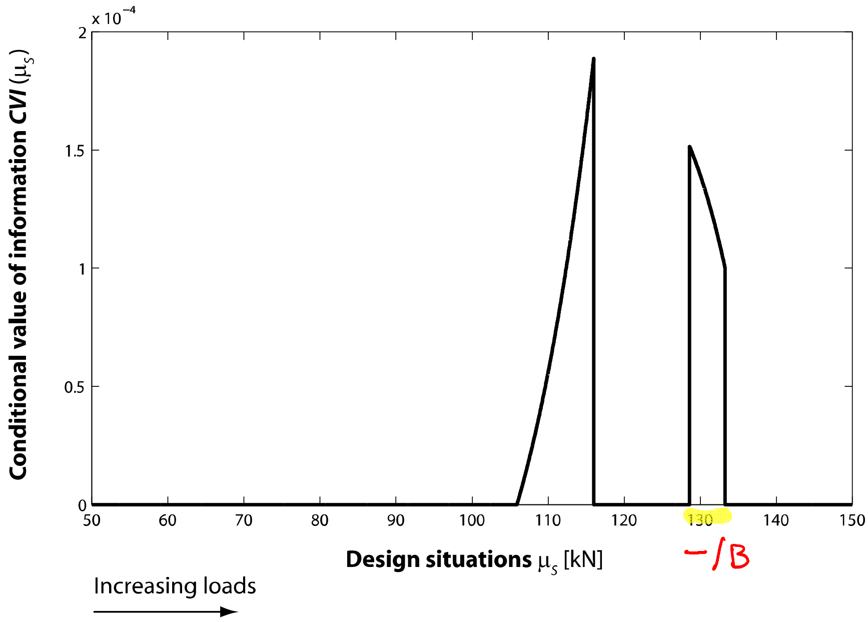
16


x 10^-4
Conditional value of information CVI (μS)
Design situations μS [kN]
A / A
Increasing loads
17

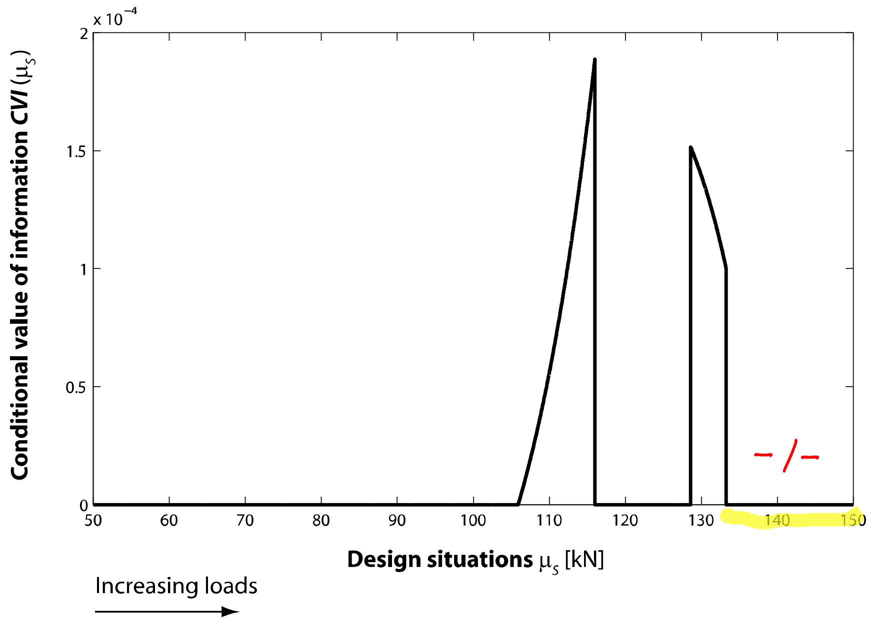

x 10^-4
Conditional value of information CVI (μS)
Design situations μS [kN]
A / B
Increasing loads
18


x 10^-4
Conditional value of information CVI (μs)
Design situations μs [kN]
B/B
Increasing loads

19


x 10^-4
Conditional value of information CVI (μs)
Design situations μs [kN]
–/B
Increasing loads

20



21

Value of information of the probabilistic analysis

- The value of information is the expected value of the CVI with respect to all possible design situations:

$$VI = \int_{\mathrm{M}_S} CVI(\mu_S) f_{\mathrm{M}_S}(\mu_S) \mathrm{d}\mu_S$$

22

Value of information of the probabilistic analysis

- The value of information is the expected value of the CVI with respect to the possible design situations:

$$VI = \int_{\mathrm{M}_S} CVI(\mu_S) f_{\mathrm{M}_S}(\mu_S) \mathrm{d}\mu_S$$

- Assuming a uniform distribution of M_S in the range [50kN,150kN], we obtain

$$VI = 1.4 \cdot 10^{-5}$$

- (The cost of the design/construction is in the order of 10^{-3})

23

When is a probabilistic analysis useful <u>in practice</u>?

(Some lessons to be learnt from the example)

- The analysis must be able to identify better solutions than a deterministic analysis
- The benefit of the better solution must be significantly higher than the <u>cost of the analysis</u>

- Useful for problems
  - In which the phenomena cannot be adequately modeled deterministically
    - In the presence of large uncertainties and/or non-linear effects
    - When dealing with collecting information
  - Where the potential benefit is huge (e.g. optimization of aircraft design)

24

When is a probabilistic analysis useful <u>in practice</u>?

(Some lessons to be learnt from the example)

- The analysis must be able to identify better solutions than a deterministic analysis
- The benefit of the better solution must be significantly higher than the <u>cost of the analysis</u>

- Useful for problems
  - In which the phenomena cannot be adequately modeled deterministically
    - In the presence of large uncertainties and/or non-linear effects
    - **When dealing with collecting information**
  - Where the potential benefit is huge (e.g. optimization of aircraft design)

26

Value of information theory:

- Raiffa H., and R. Schlaifer (1961), *Applied Statistical Decision Theory,* Cambridge University Press, Cambridge.
- Benjamin, J. R., and C. A. Cornell (1970), *Probability, statistics, and decision for civil engineers*, McGraw-Hill, New York.
- Straub, D. (2004), *Generic Approaches to Risk Based Inspection Planning for Steel Structures*, PhD thesis, ETH Zürich.

27

What are we doing?
Decisions in complex systems under conditions of uncertainty

Aging of the infrastructure system:
- Monitoring & Inspection
- Maintenance
- Replacement / redesign

Natural hazards in the system „built environment“
- Prevention
- Emergency response
- Rehabilitation

Safety in the system „society“
- Target reliability
- Prescriptive limits
- Service life duration

Three applications

a. Avalanche risk analysis
b. Dependence in earthquake fragility modelling
c. Planning of inspections in offshore structures

29

Avalanche risk assessment

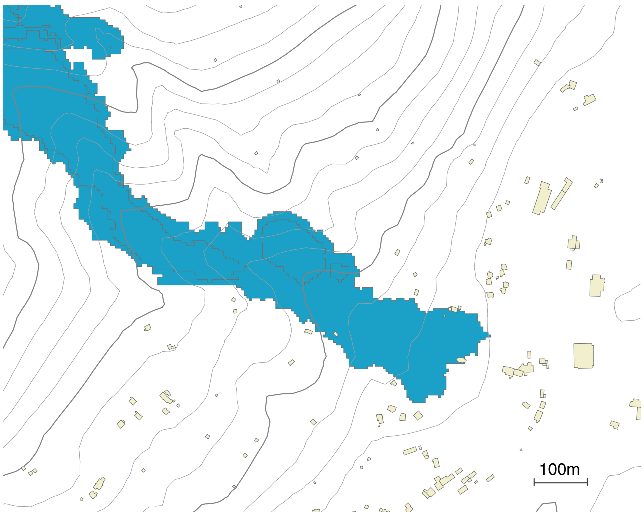
- Where is it safe to build?
- Where should protection measures be implemented?
- When should roads be closed / buildings be evacuated?

Source: Kt. St. Gallen, Switzerland

30

Avalanche risk analysis

Avalanche model:

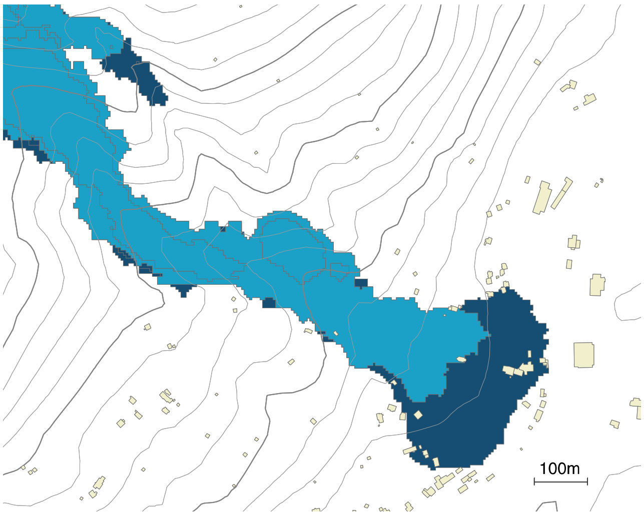


Straub D., Grêt-Regamey A. (2006). *Cold Regions Science and Technology*, 46(3) , pp. 192-203.

31

Avalanche risk analysis

- Parameter uncertainty
- E.g. friction parameter μ

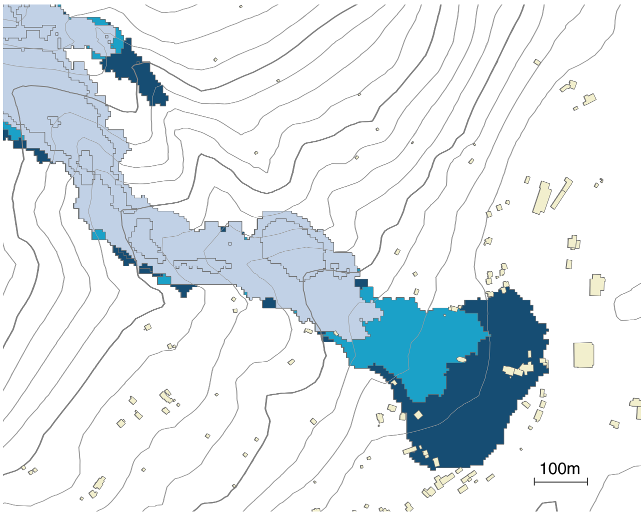


Straub D., Grêt-Regamey A. (2006). *Cold Regions Science and Technology*, 46(3) , pp. 192-203.

32

Avalanche risk analysis

- Parameter uncertainty
- E.g. friction parameter μ

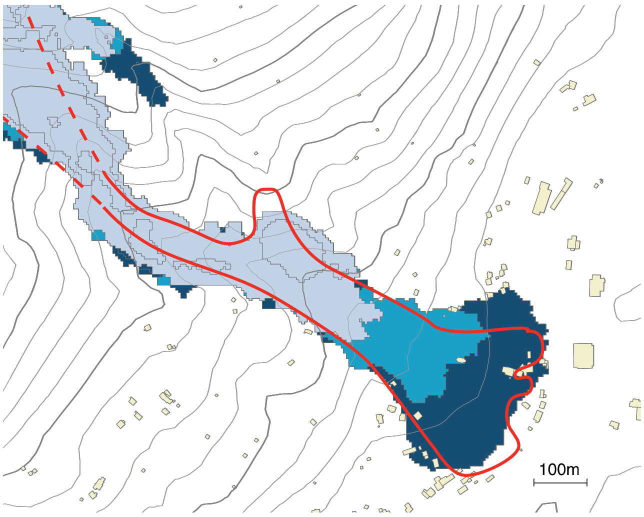


Straub D., Grêt-Regamey A. (2006). *Cold Regions Science and Technology*, 46(3) , pp. 192-203.

33

Avalanche risk analysis

- Observations available
  (here 50 years)

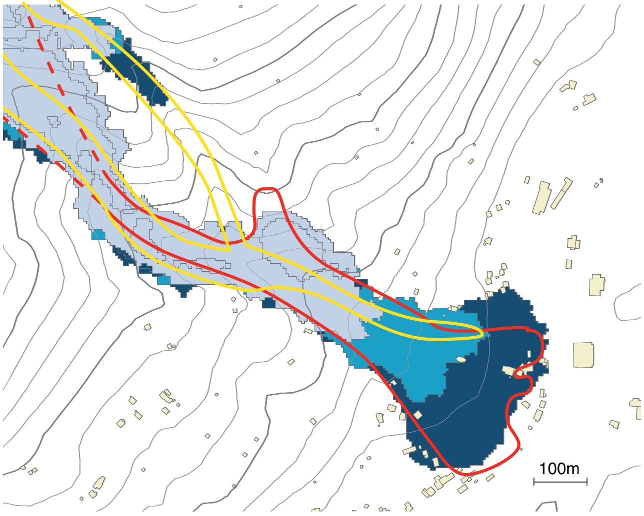


Straub D., Grêt-Regamey A. (2006). *Cold Regions Science and Technology*, 46(3) , pp. 192-203.

34

Avalanche risk analysis

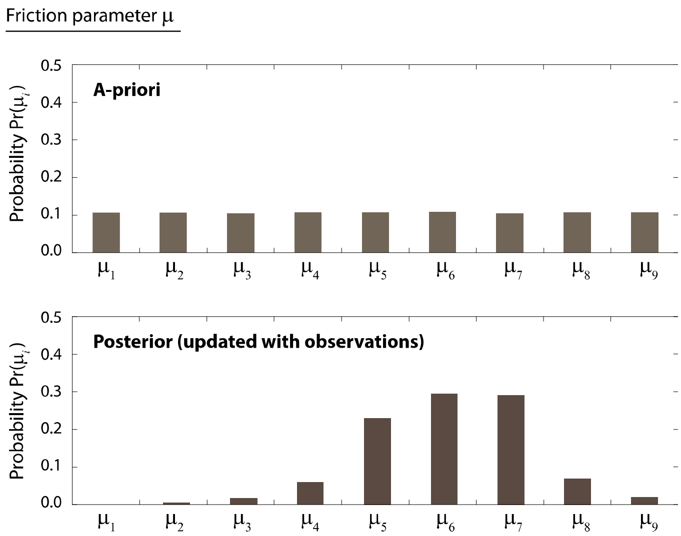
- Observations available
  (here 50 years)



Straub D., Grêt-Regamey A. (2006). *Cold Regions Science and Technology*, 46(3) , pp. 192-203.

35

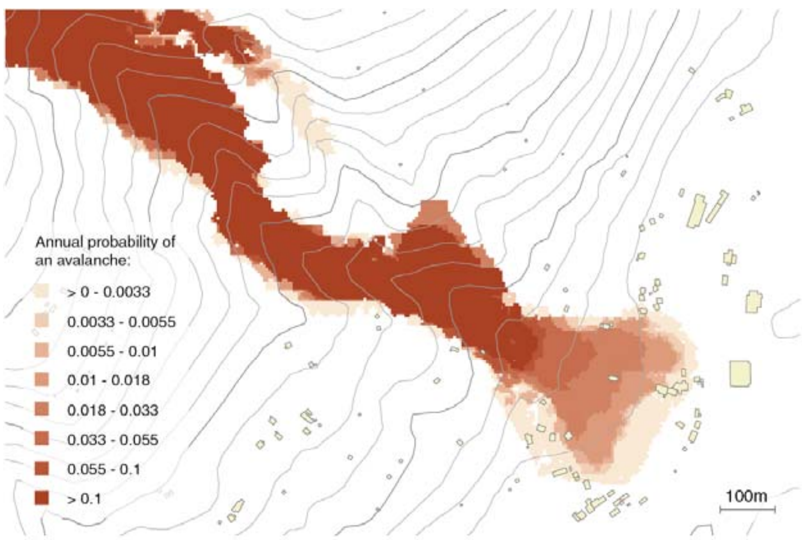
Avalanche risk analysis – Bayesian updating



Straub D., Grêt-Regamey A. (2006). *Cold Regions Science and Technology*, 46(3) , pp. 192-203.

36

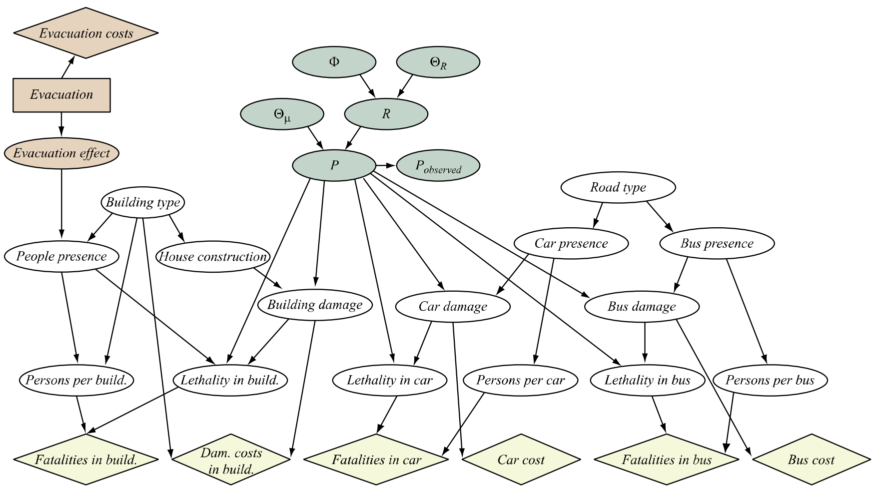
Results in a probabilistic hazard map



Straub D., Grêt-Regamey A. (2006). *Cold Regions Science and Technology*, 46(3) , pp. 192-203.

37

Bayesian networks for avalanche risk assessment

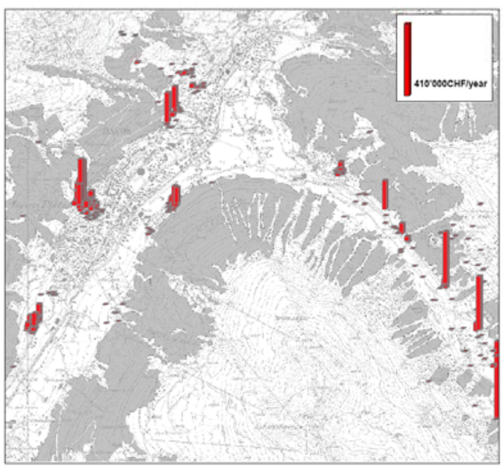


Grêt-Regamey A., Straub D. (2006). *Natural Hazards and Earth System Sciences,* 6(6), pp. 911-926.

38

Implementation of the BN models in software is straightforward

- Implementation in a GIS environment
- Regional risk analysis



Grêt-Regamey A., Straub D. (2006). *Natural Hazards and Earth System Sciences,* 6(6), pp. 911-926.

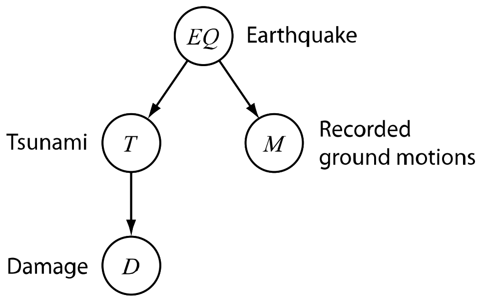
39

Modelling dependence in Earthquake fragility

(Statistical dependence is not captured by simple analyses)

Bayesian network is a powerful modeling tool

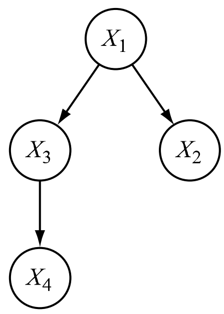
- Tsunami warning example:



Straub D., (2010). Lecture Notes in Engineering Risk Analysis. TU München

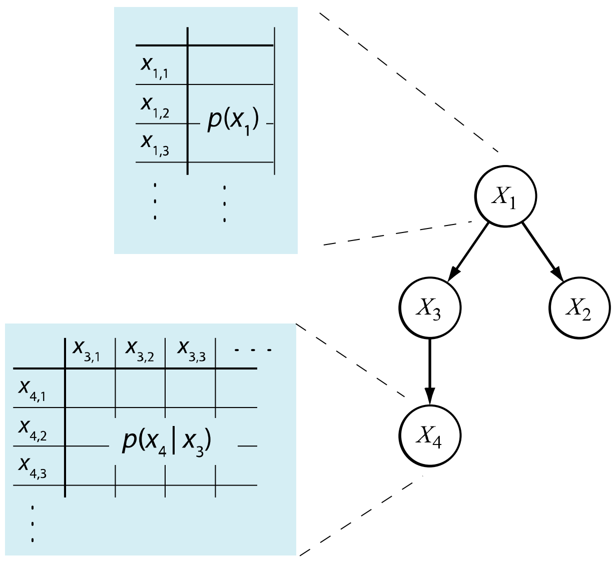
Bayesian network in a nutshell

- Probabilistic models based on directed acyclic graphs
- Models the joint probability distribution of a set of variables



42

Bayesian network in a nutshell



43

Bayesian network in a nutshell

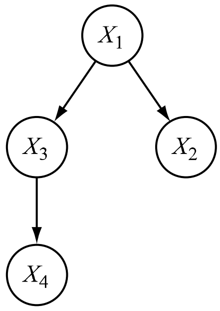
- Efficient factoring of the joint probability distribution into conditional (local) distributions given the parents

Here:

$$p(x_1, x_2, x_3, x_4) =$$
$$= p(x_1)\,p(x_2 \mid x_1)\,p(x_3 \mid x_1)\,p(x_4 \mid x_3)$$

General:

$$p(\mathbf{x}) = \prod_{i=1}^{n} p[x_i \mid pa(x_i)]$$



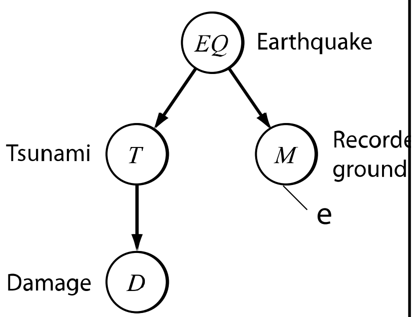
44

Bayesian network in a nutshell

- Facilitates Bayesian updating when additional information (evidence) is available

E.g.:

$$p(x_3 \mid e_2) = \frac{p(e_2, x_3)}{p(e_2)}$$
$$= \frac{\sum_{X_1} p(x_1)\,p(e_2 \mid x_1)\,p(x_3 \mid x_1)}{\sum_{X_2} p(x_1)\,p(e_2 \mid x_1)}$$

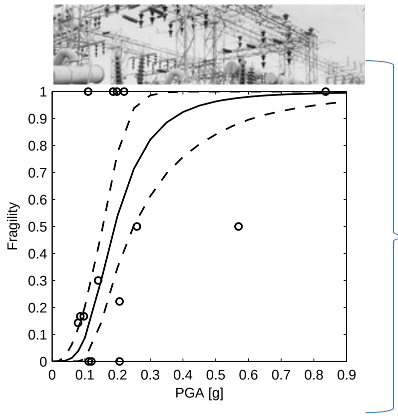


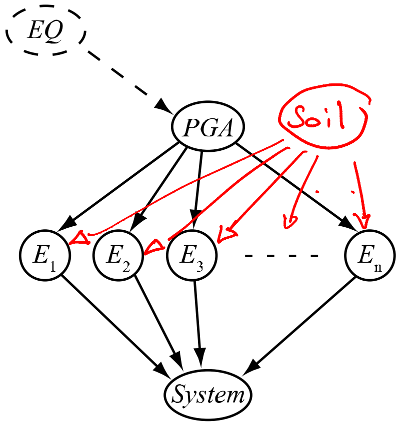
Straub D., (2010). Lecture Notes in Engineering Risk Analysis. TU München

45

Modelling with BN:
System dependence through common factors

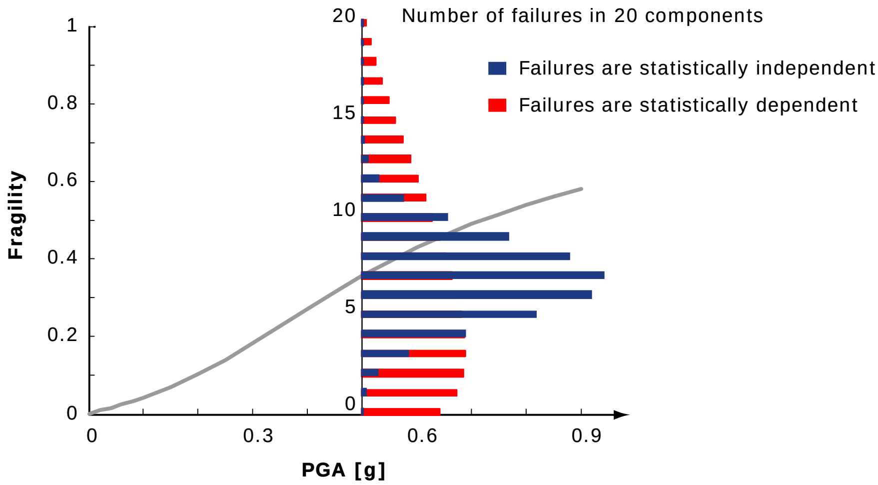
- Performance of an electrical substation during an EQ





46

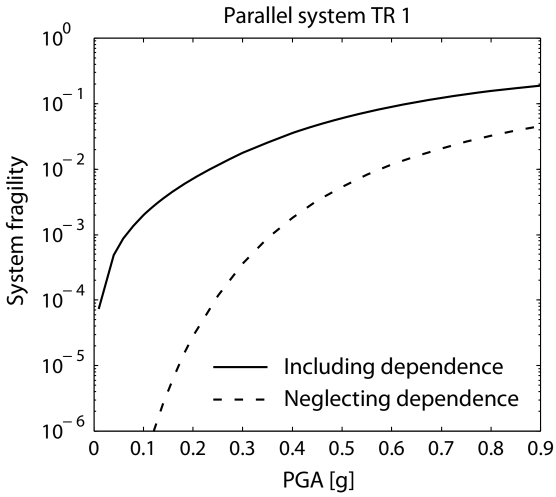
Can we observe the statistical dependence ?



47

When accounting for dependence, the system fragility strongly increases

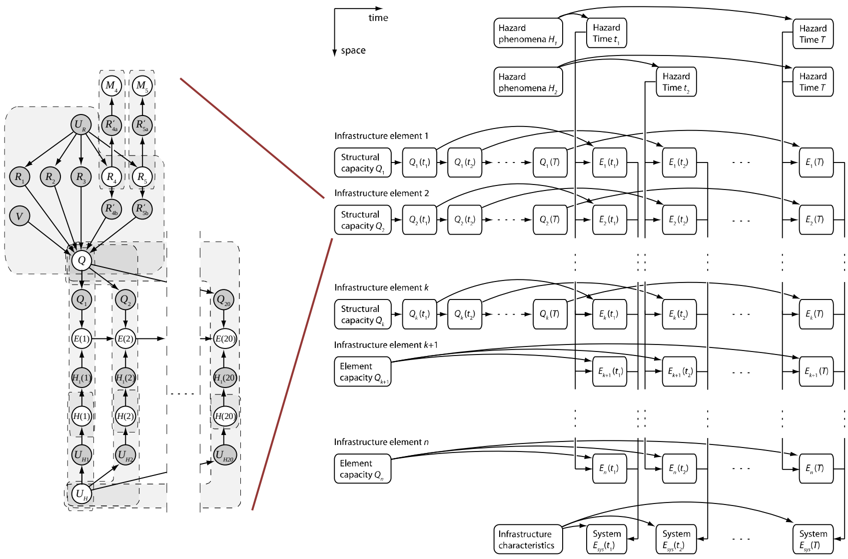
- Redundant system: (parallel system with 5 components)



Straub D., Der Kiureghian A. (2008). *Structural Safety*, 30(4), pp. 320-366.

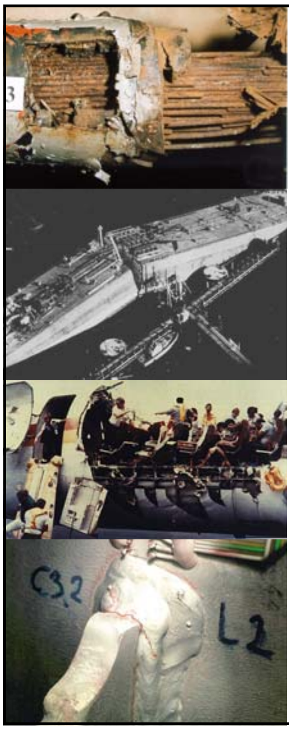
49

EQ: Modeling systems and portfolio of structures



Straub D., Der Kiureghian A., (2010). *Journal of Engineering Mechanics*

50



Risk-based inspection, maintenance, repair planning

- Structures deteriorate with time
- Deterioration is associated with large uncertainty
- Inspections are performed to reduce uncertainty

→ The effect of inspections (and monitoring) can only be appraised probabilistically

- Applications:
  - Offshore structures subject to fatigue, corrosion, scour, ship impact, …
  - Process systems subject to corrosion, erosion, SCC, etc…
  - Concrete structures (tunnels, bridges) subject to corrosion of the reinforcement
  - Aircraft structures

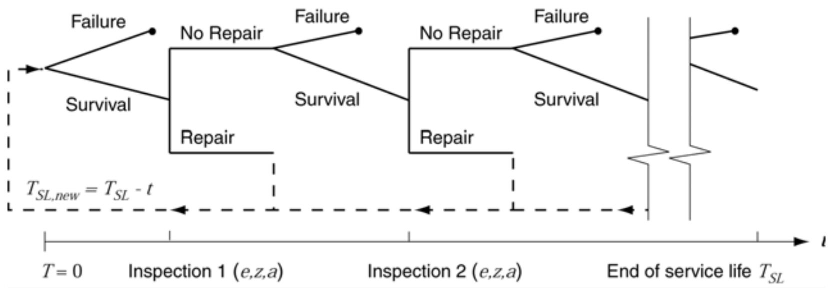
51

Optimizing inspection strategies

- Deterioration of offshore steel structures and pipelines
- Goal: Optimize sub-sea inspections

Plan and optimize inspections

- We model the entire service life through event trees:

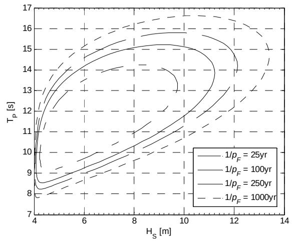


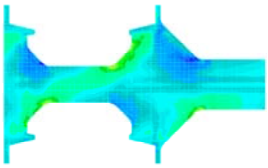
54

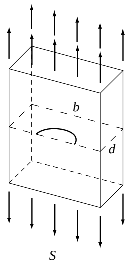
Probabilistic deterioration modelling

- Fracture mechanics based probabilistic models of crack growth:

Fatigue loads | Structural response | Crack growth







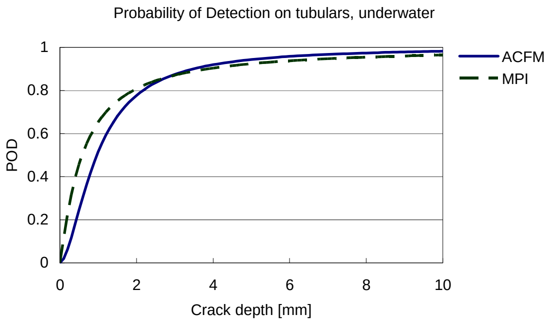
$$\frac{da}{dN} = C_{P,a}\left(\Delta K_a\left(a,c\right)\right)^{m_{fm}}$$

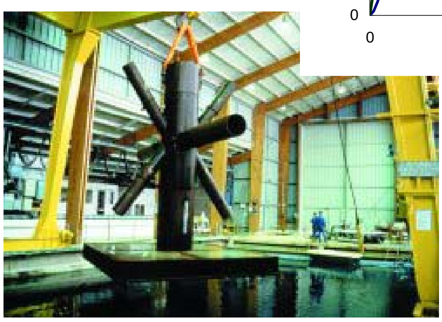
$$\frac{dc}{dN} = C_{P,c}\left(\Delta K_c\left(a,c\right)\right)^{m_{fm}}$$

55

Inspection modeling

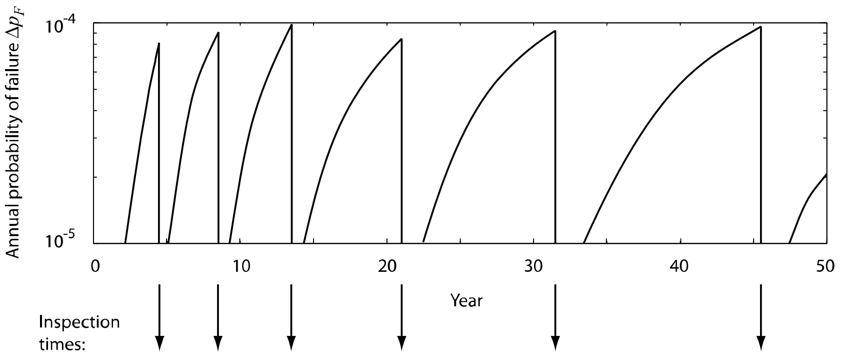
- Inspections are also modeled qualitatively





57

Probability of failure as a function of time and the influence of inspection

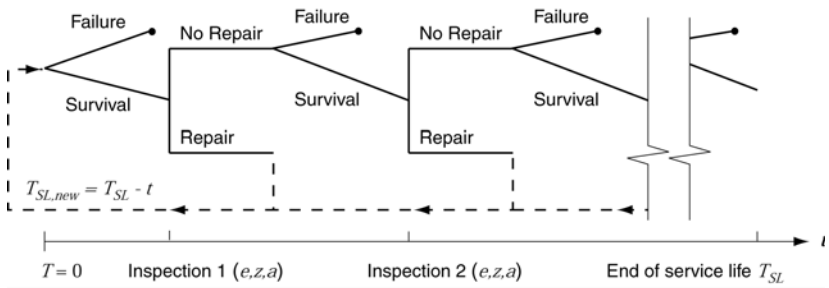


Straub D., Faber M.H. (2006). *Computer-Aided Civil and Infrastructure Engineering*, 21(3), pp. 179-192.

58

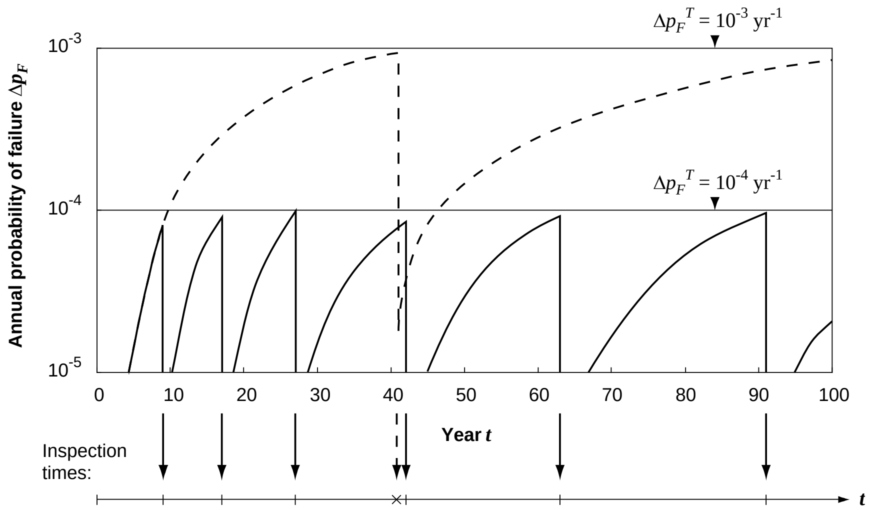
Plan and optimize inspections

- We model the entire service life through event trees:



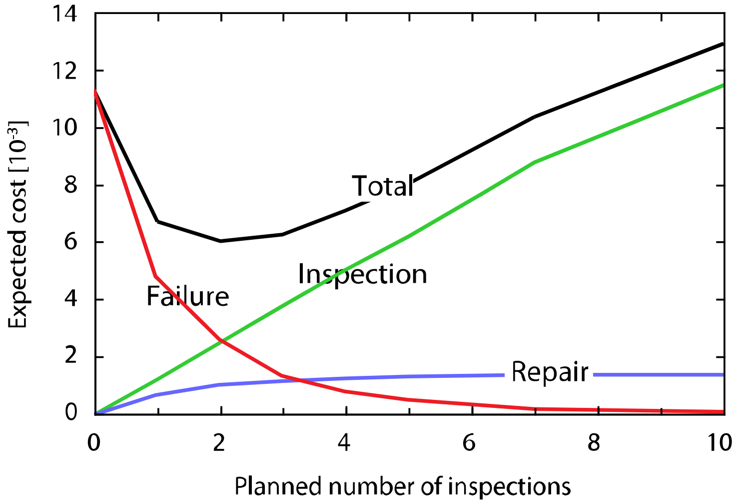
59

The maximum probability of failure determines the number of inspections



60

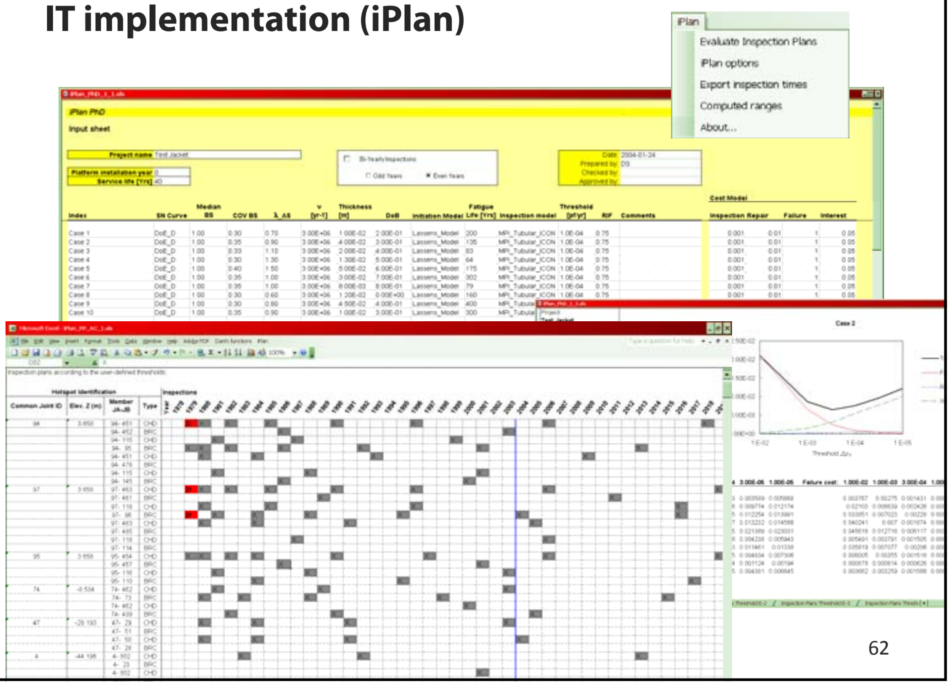
Optimization



Straub D., Faber M.H. (2004). *J. of Offshore Mechanics and Arctic Engineering*, 126(3), pp. 265-271.

61

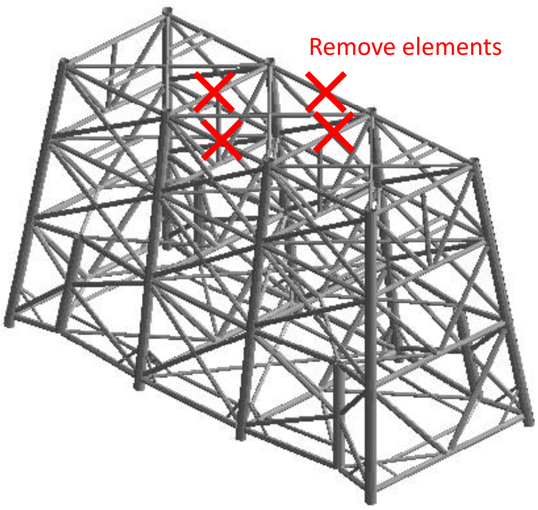
IT implementation (iPlan)



62

Structural importance

- How to dermine redundancy?
- Deterministic approach is not sufficient (most components are not part of the dominant mechanism)
- (Simplified) probabilistic approach is needed

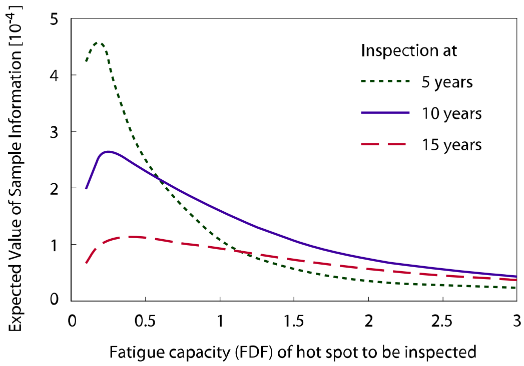


Straub D., Der Kiureghian A. (2011) *J. of Structural Engineering*, in print.

63

Optimize inspections in the structural systems

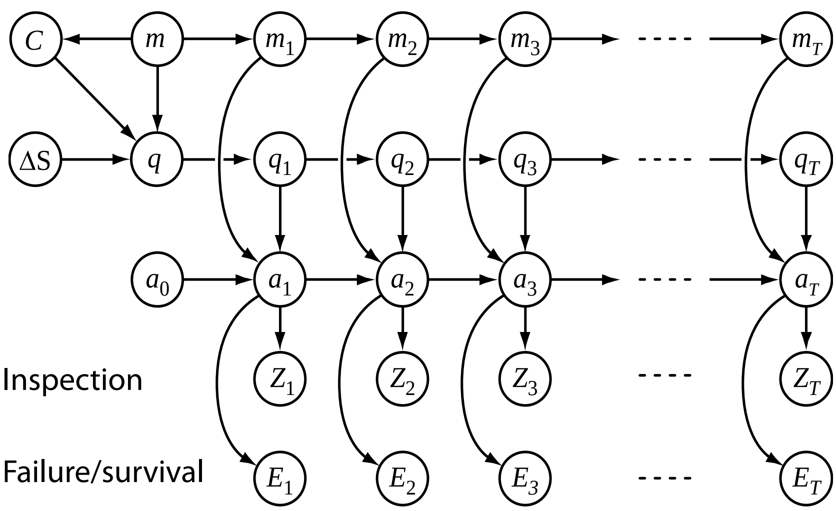
Single component: 1-5 Decision variables
Structure: 100 -1000 Decision variables
→ Heuristic method based on Value-of-Information



Straub D., Faber M.H. (2005). *Structural Safety*, 27(4), pp 335-355.

64

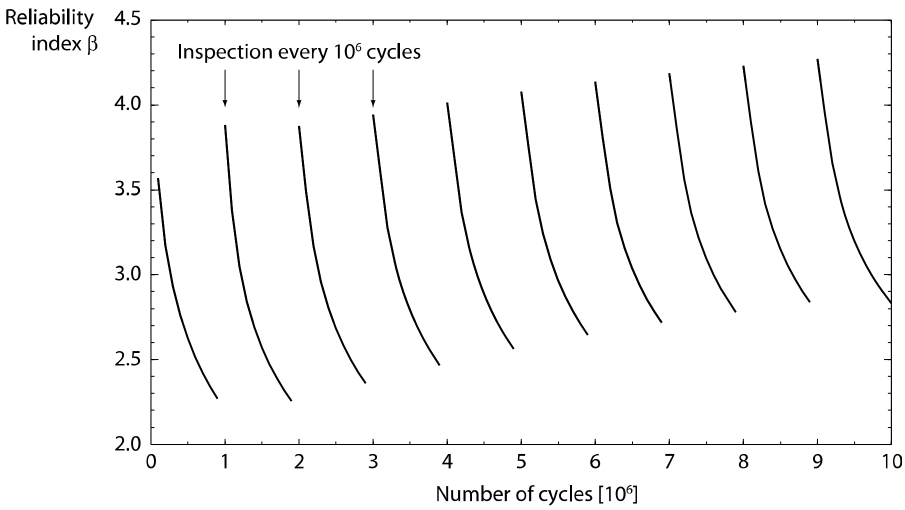
DBN model for deterioration modeling



Straub D. (2009). *Journal of Engineering Mechanics*, **135**(10), pp. 1089-1099

65

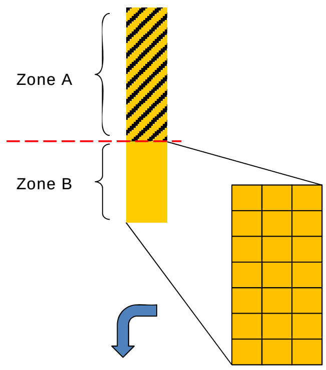
Bayesian updating is robust AND efficient

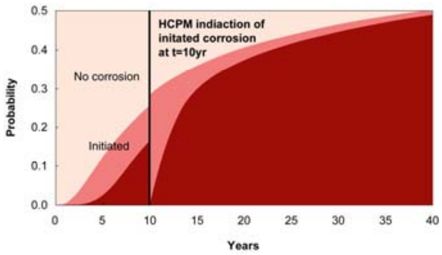


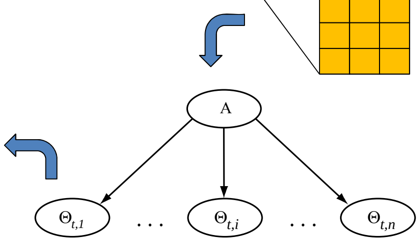
Straub D. (2009). *Journal of Engineering Mechanics*, **135**(10), pp. 1089-1099

66

Monitoring, Inspection and Maintenance for Concrete Structures







Straub D., et al. (2009). *Structure and Infrastructure Engineering*,

67

Therefore,

… simplified (engineering) models are often sufficient for making optimal decisions

… but probabilistic analysis can provide useful insights and help making better decisions

… if we ensure that the benefit of the analysis outweighs the cost of the analysis

68

questions or comments?

straub@tum.de / www.era.bv.tum.de

(D. Straub, July 2006)

69