

List of functions for optimization algorithms testing

February 16, 2009

1 Mathematical formulation of test functions

- F1:

$$f(x) = 2(x - 0.75)^2 + \sin(5\pi x - 0.4\pi) - 0.125 \quad (1)$$

where

$$0 \leq x \leq 1$$

- F3:

$$f(x) = - \sum_{j=1}^5 [j \sin[(j+1)x + j]] \quad (2)$$

where

$$-10 \leq x \leq 10$$

- Branin:

$$f(x, y) = a(y - bx^2 + cx - d)^2 + h(1 - f) \cos x + h \quad (3)$$

where

$$a = 1, b = 5.1/4\pi^2, c = 5/\pi, d = 6,$$

$$h = 10, f = 1/8\pi, -5 \leq x \leq 10, 0 \leq y \leq 15$$

- Camelback:

$$f(x, y) = \left(4 - 2.1x^2 + \frac{x^4}{3}\right)x^2 + xy + (-4 + 4y^2)y^2 \quad (4)$$

where

$$-3 \leq x \leq 3, -2 \leq y \leq 2$$

- Goldprice:

$$f(x, y) = \begin{bmatrix} 1 + (x + y + 1)^2(19 - 14x + 3x^2 - 14y + 6xy + 3y^2) \\ 30 + (2x - 3y)^2(18 - 32x + 12x^2 + 48y - 36xy + 27y^2) \end{bmatrix} \quad (5)$$

where

$$-2 \leq x \leq 2, -2 \leq y \leq 2$$

- PShubert 1 and 2:

$$f(x, y) = \left\{ \sum_{i=1}^5 i \cos[(i+1)x + i] \right\} \cdot \left\{ \sum_{i=1}^5 i \cos[(i+1)y + i] \right\} + \beta[(x + 1.42513)^2 + (y + 0.80032)^2] \quad (6)$$

where

$$-10 \leq x \leq 10, -10 \leq y \leq 10,$$

for PShubert1: $\beta = 0.5$

for PShubert2: $\beta = 1.0$

- Quartic:

$$f(x, y) = \frac{x^4}{4} - \frac{x^2}{2} + \frac{x}{10} + \frac{y^2}{2} \quad (7)$$

where

$$-10 \leq x \leq 10, -10 \leq y \leq 10$$

- Shubert:

$$f(x, y) = \left\{ \sum_{i=1}^5 i \cos[(i+1)x + i] \right\} \cdot \left\{ \sum_{i=1}^5 i \cos[(i+1)y + i] \right\} \quad (8)$$

where

$$-10 \leq x \leq 10, -10 \leq y \leq 10$$

- Hartman 1:

$$f(x_1, x_2, x_3) = - \sum_{i=1}^4 c_i e^{-\sum_{j=1}^3 a_{ij}(x_i - p_{ij})^2} \quad (9)$$

where

$$0 \leq x_i \leq 1, i = 1, \dots, 3$$

$$x = (x_1, \dots, x_3), p_i = (p_{i1}, \dots, p_{i3}), a_i = (a_{i1}, \dots, a_{i3})$$

i	a_{ij}			c_i	p_{ij}		
1	3.0	10.0	30.0	1.0	0.36890	0.1170	0.2673
2	0.1	10.0	35.0	1.2	0.46990	0.4387	0.7470
3	3.0	10.0	30.0	3.0	0.10910	0.8732	0.5547
4	0.1	10.0	35.0	3.2	0.03815	0.5743	0.8828

- Shekel 1,2 and 3:

$$f(x) = - \sum_{i=1}^m \frac{1}{(x - a_i)^T (x - a_i) + c_i} \quad (10)$$

where

$$0 \leq x_j \leq 10,$$

for Shekel1: $m = 5$,
 for Shekel2: $m = 7$,
 for Shekel3: $m = 10$

$$x = (x_1, x_2, x_3, x_4)^T, a_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4})^T$$

i	a_{ij}				c_i
1	4.0	4.0	4.0	4.0	0.1
2	1.0	1.0	1.0	1.0	0.2
3	8.0	8.0	8.0	8.0	0.2
4	6.0	6.0	6.0	6.0	0.4
5	3.0	7.0	3.0	7.0	0.4
6	2.0	9.0	2.0	9.0	0.6
7	5.0	5.0	3.0	3.0	0.6
8	8.0	1.0	8.0	1.0	0.7
9	6.0	2.0	6.0	2.0	0.5
10	7.0	3.6	7.0	3.6	0.5

- Hartman 2:

$$f(x_1, \dots, x_6) = - \sum_{i=1}^4 c_i e^{-\sum_{j=1}^6 a_{ij}(x_i - p_{ij})^2} \quad (11)$$

where

$$0 \leq x_j \leq 1, j = 1, \dots, 6$$

$$x = (x_1, \dots, x_6), p_i = (p_{i1}, \dots, p_{i6}), a_i = (a_{i1}, \dots, a_{i6})$$

i	a_{ij}						c_i
1	10.00	3.00	17.00	3.50	1.70	8.00	1.0
2	0.05	10.00	17.00	0.10	8.00	14.00	1.2
3	3.00	3.50	1.70	10.00	17.00	8.00	3.0
4	17.00	8.00	0.05	10.00	0.01	14.00	3.2

i	p_{ij}					
1	0.1312	0.1696	0.5569	0.0124	0.8283	0.5886
2	0.2329	0.4135	0.8307	0.3736	0.1004	0.9991
3	0.2348	0.1451	0.3522	0.2883	0.3047	0.6650
4	0.4047	0.8828	0.8732	0.5743	0.1091	0.0381

- Hosc 45:

$$f(x) = 2 - \frac{1}{n!} \prod_{i=1}^n x_i \quad (12)$$

where

$$x = (x_1, \dots, x_n), 0 \leq x_i \leq 1, n = 10$$

- Brown 1:

$$f(x) = \left[\sum_{i \in J} (x_i - 3) \right]^2 + \sum_{i \in J} [10^{-3}(x_i - 3)^2 - (x_i - x_{i+1}) + e^{20(x_i - x_{i+1})}] \quad (13)$$

where

$$J = \{1, 3, \dots, 19\}, -1 \leq x_i \leq 4, 1 \leq i \leq 20, x = (x_1, \dots, x_{20})^T$$

- Brown 3:

$$f(x) = \sum_{i=1}^{19} [(x_i^2)^{(x_{i+1}^2+1)} + (x_{i+1}^2)^{(x_i^2+1)}] \quad (14)$$

$$x = (x_1, \dots, x_{20})^T, -1 \leq x_i \leq 4, 1 \leq i \leq 20$$

- F5n:

$$f(x) = (\pi/20) \cdot \left\{ 10 \sin^2(\pi y_1) + \sum_{i=1}^{19} [(y_i - 1)^2 \cdot (1 + 10 \sin^2(\pi y_i + 1))] + (y_{20} - 1)^2 \right\} \quad (15)$$

where

$$x = (x_1, \dots, x_{20})^T, -10 \leq x_i \leq 10, y_i = 1 + 0.25(x_i - 1)$$

- F10n:

$$f(x) = (\pi/20) \cdot \left\{ 10 \sin^2(\pi x_1) + \sum_{i=1}^{19} [(x_i - 1)^2 \cdot (1 + 10 \sin^2(\pi x_{i+1}))] + (x_{20} - 1)^2 \right\} \quad (16)$$

where

$$x = (x_1, \dots, x_{20})^T, -10 \leq x_i \leq 10$$

- F15n:

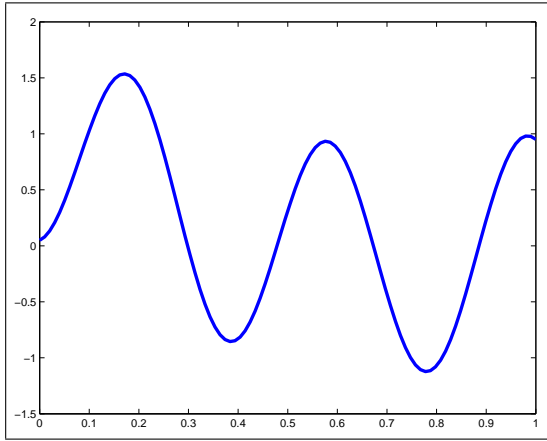
$$f(x) = (1/10) \cdot \left\{ \sin^2(3\pi x_1) + \sum_{i=1}^{19} [(x_i - 1)^2 (1 + \sin^2(3\pi x_{i+1}))] + (1/10)(x_{20} - 1)^2 [1 + \sin^2(2\pi x_{20})] \right\} \quad (17)$$

where

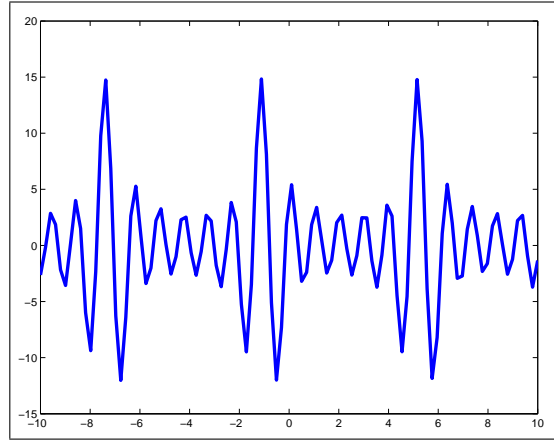
$$x = (x_1, \dots, x_{20})^T, -10 \leq x_i \leq 10$$

2 Graphical illustration of test function with one or two variables

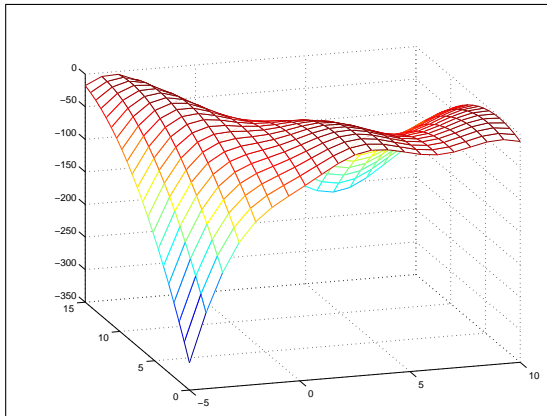
Functions on the following figures are again multiplied by (-1) in order to be maximized and to make the extremes better visible.



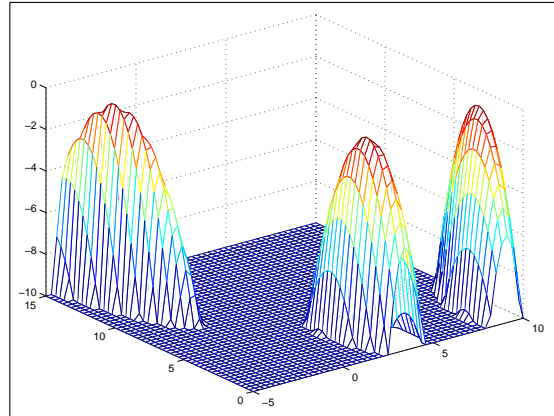
F1



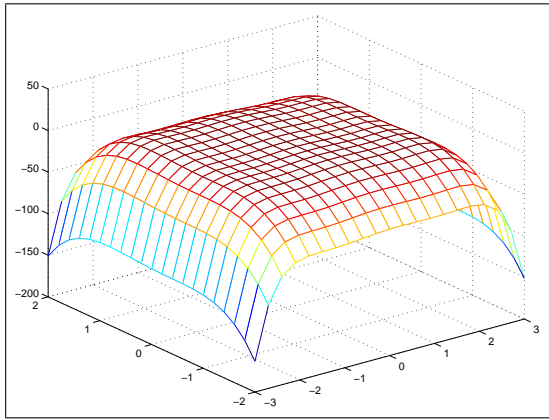
F3



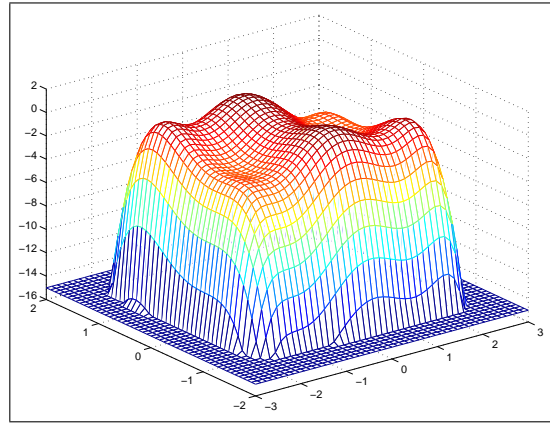
Branin



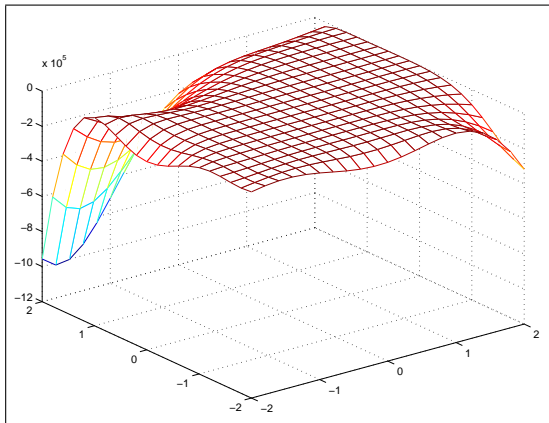
Branin - detailed



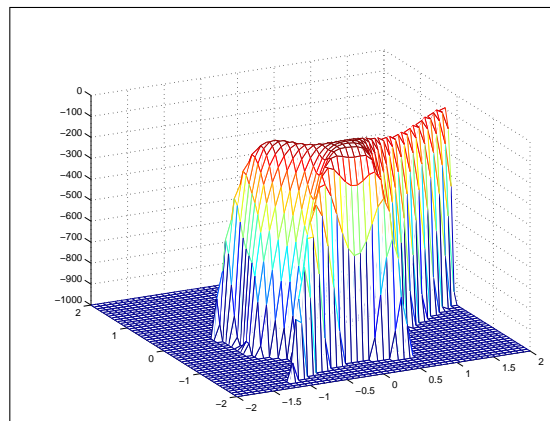
Camelback



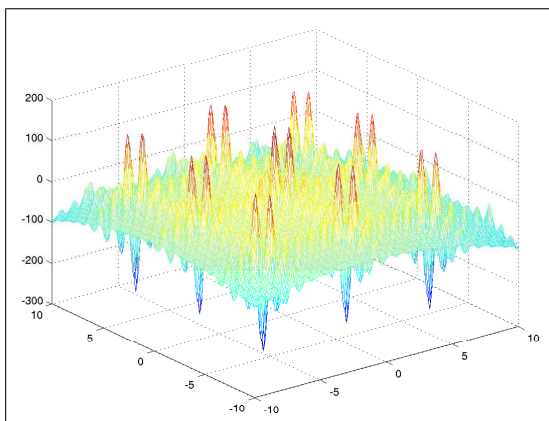
Camelback - detailed



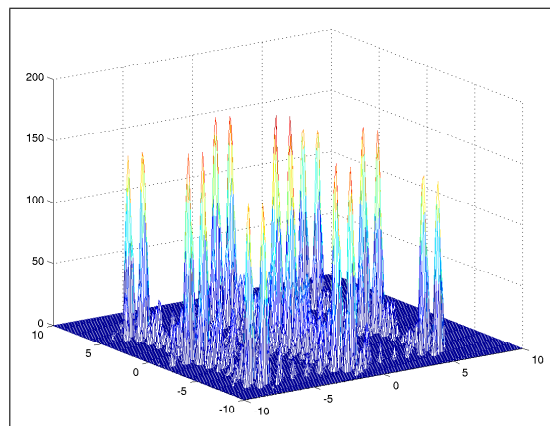
Goldprice



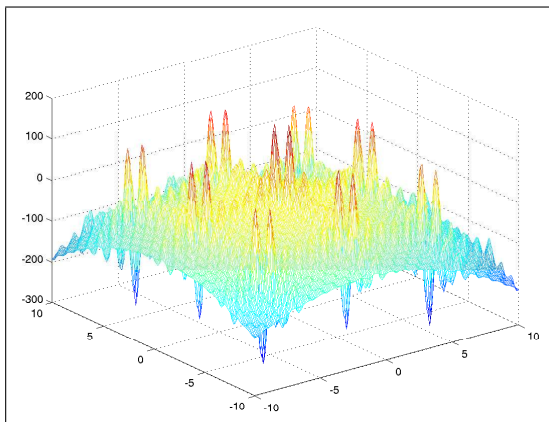
Goldprice - detailed



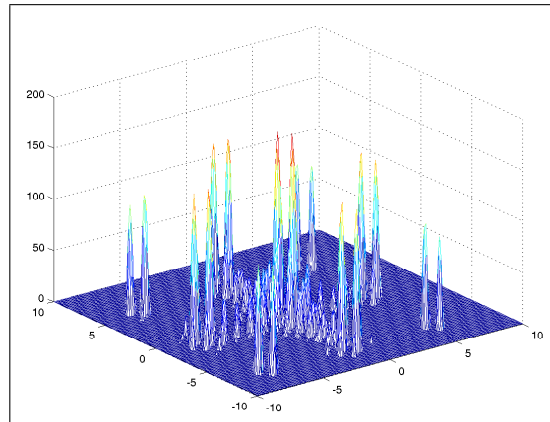
PShubert1



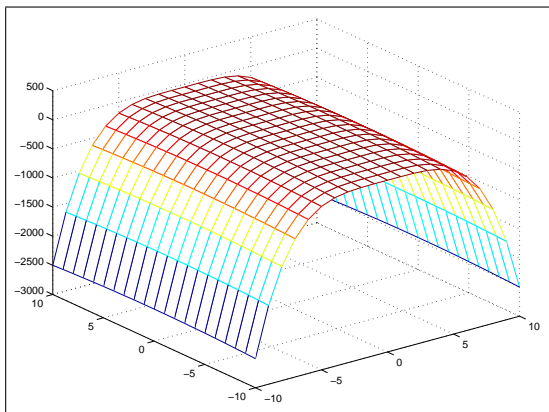
PShubert1 - detailed



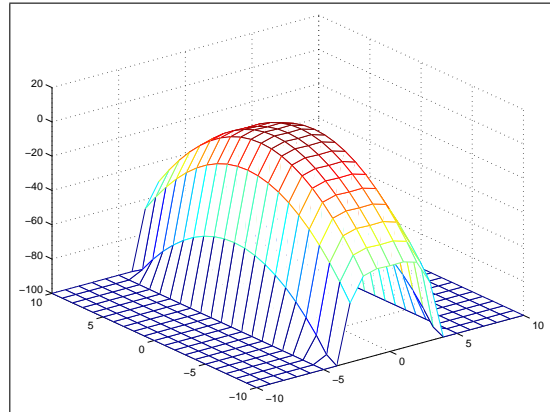
PShubert2



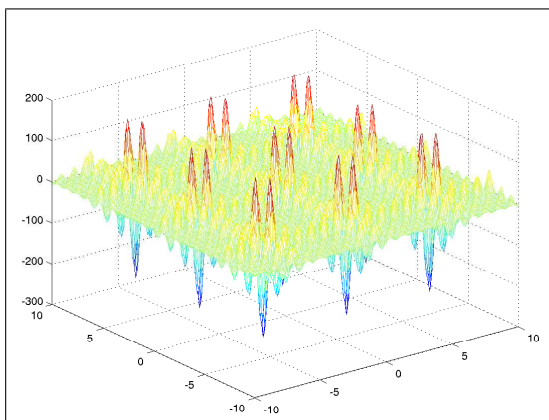
PShubert2 - detailed



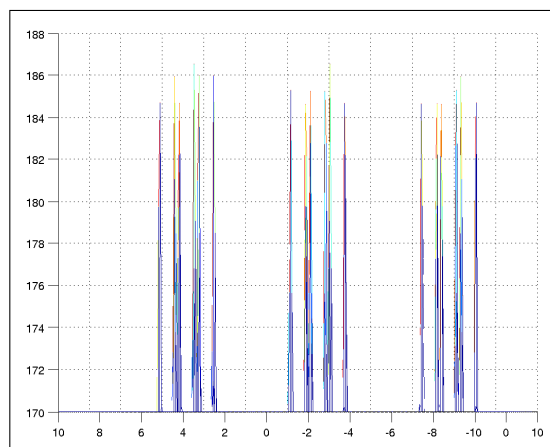
Quartic



Quartic - detailed



Shubert



Shubert - detailed