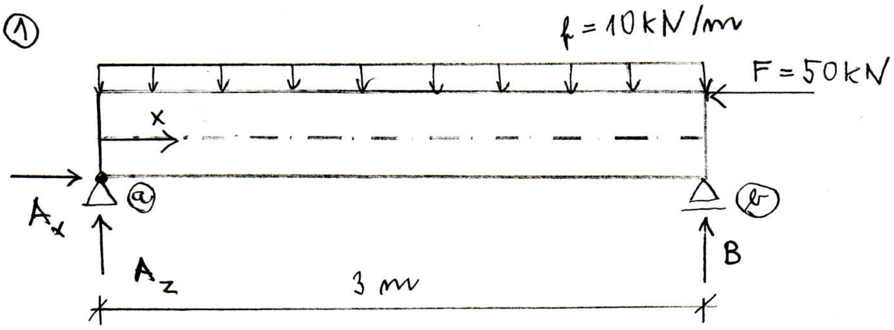
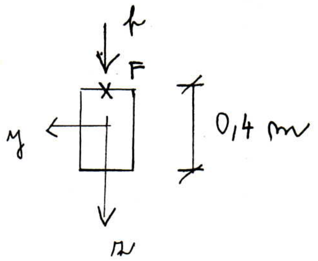
Vnitřní síly – opakování

①



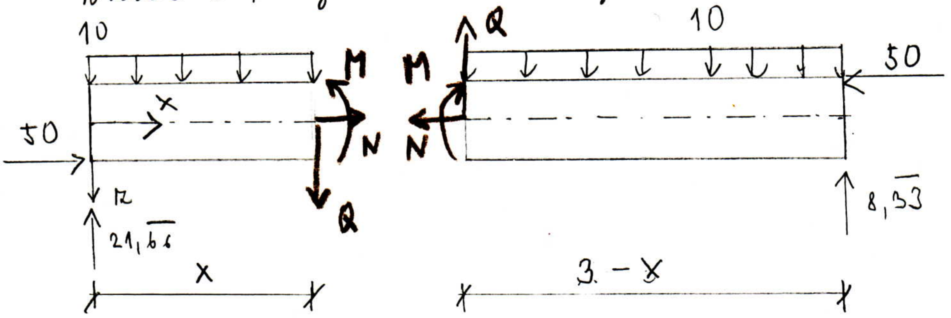


reakce: $A_x = 50\,kN$

$\curvearrowleft a \quad 3 \cdot B + 50 \cdot 0{,}4 - 3 \cdot 10 \cdot 1{,}5 = 0 \quad \rightarrow \quad B = 8{,}\overline{33}\,kN$

$A_z = 21{,}\overline{66}\,kN$

vnitřní síly k těžišťovým osám!



$N = -50\,kN$

$21{,}\overline{66} - 10x - Q = 0$

$Q = 21{,}\overline{66} - 10x$

$50 \cdot 0{,}2 - 21{,}\overline{66} \cdot x + 10 \cdot x \cdot \frac{x}{2} + M = 0$

$M = -10 + 21{,}\overline{66}\,x - 5x^2$

$Q(0) = 21{,}\overline{66}\,kN$

$Q(3) = -8{,}\overline{33}\,kN$

$M(0) = -10\,kNm$

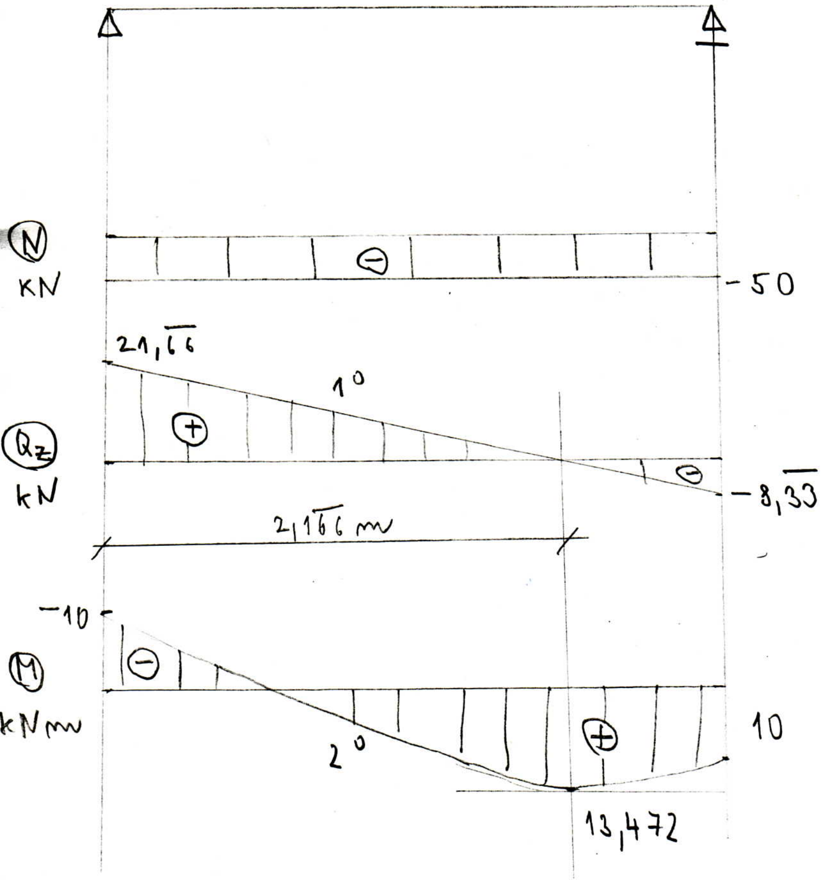
$M(3) = -10 + 21{,}\overline{66} \cdot 3 - 5 \cdot 3^2 = 10\,kNm$

$Q = 0 = 21{,}\overline{66} - 10x_m$

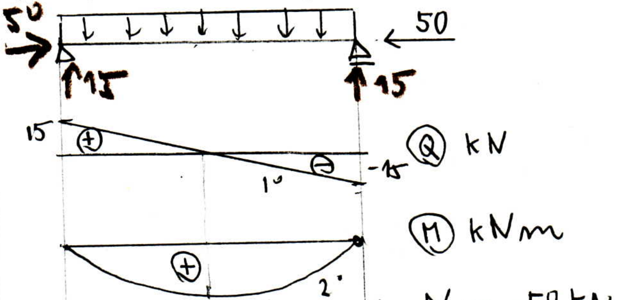
$\rightarrow x_m = 2{,}1\overline{66}\,m$

$M_{max} = -10 + 21{,}\overline{66} \cdot 2{,}1\overline{66} - 5 \cdot 2{,}1\overline{66}^2$

$M_{max} = 13{,}472\,kNm$



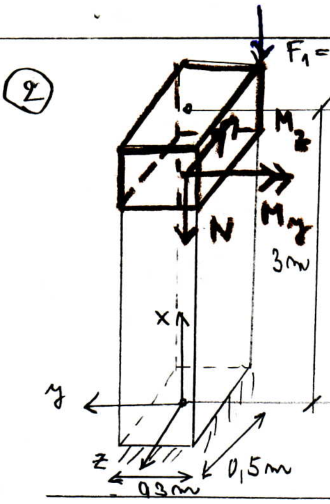
POZNÁMKA

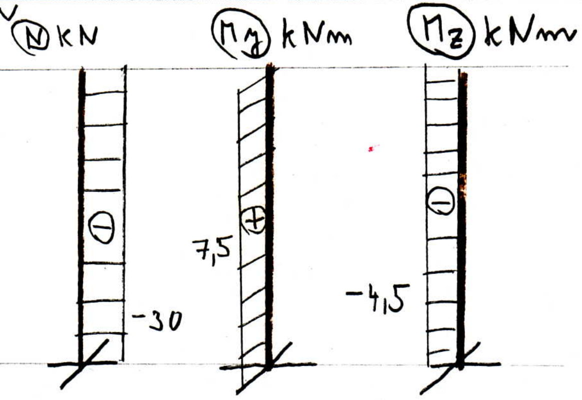


$15 \cdot \frac{3}{2} - 10 \cdot \frac{3}{2} \cdot \frac{3}{4} = 11{,}25 = \frac{1}{8} f l^2$

$N_x = -50\,kN$

②





Ⓜ VŽDY K TAŽENÝM VLÁKNŮM

$$\underline{N = -30 kN}$$

$$M_y - 30 \cdot \frac{0,5}{2} = 0$$

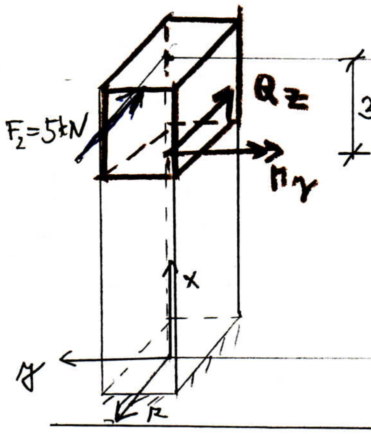
$$\underline{M_y = 7,5 kNm}$$

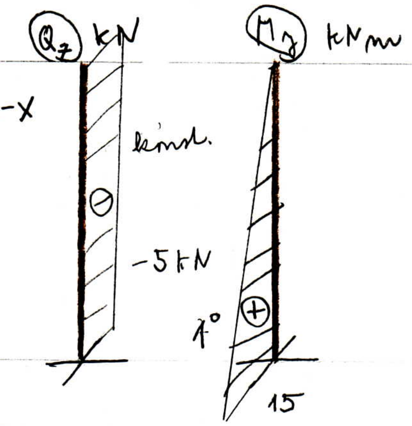
vykreslit v rovině $\boxed{xz}$!

$$M_z + 30 \cdot \frac{0,3}{2} = 0$$

$$\underline{M_z = -4,5 kNm}$$

vykreslit v rovině $\boxed{xy}$





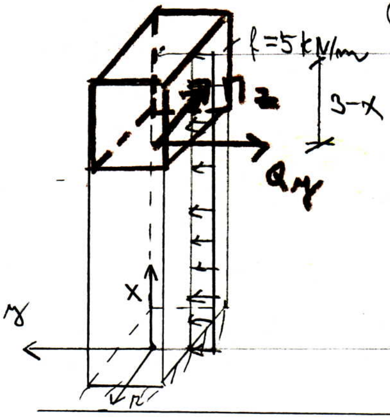
$$\underline{Q_z = -5kN}$$

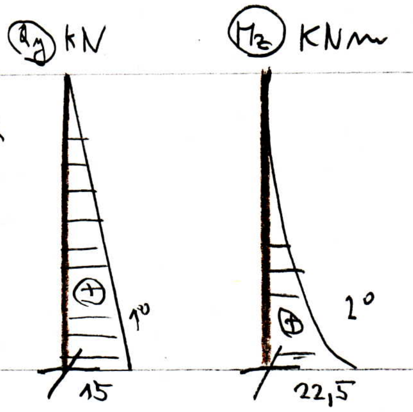
$$M_y - 5 \cdot (3-x) = 0$$

$$\underline{M_y = 15 - 5x}$$

$$M_y(0) = 15 kN$$

$$M_y(3) = 0 kN$$





$$Q_y(0) = 15 \quad Q_y(3) = 0 kN$$

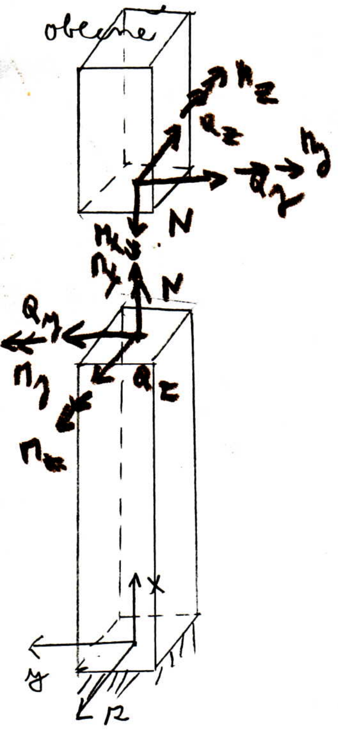
$$\underline{Q_y = 5(3-x) = 15 - 5x}$$

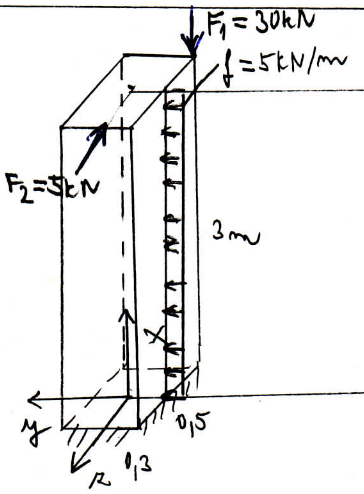
$$M_z - 5(3-x) \cdot \frac{(3-x)}{2} = 0$$

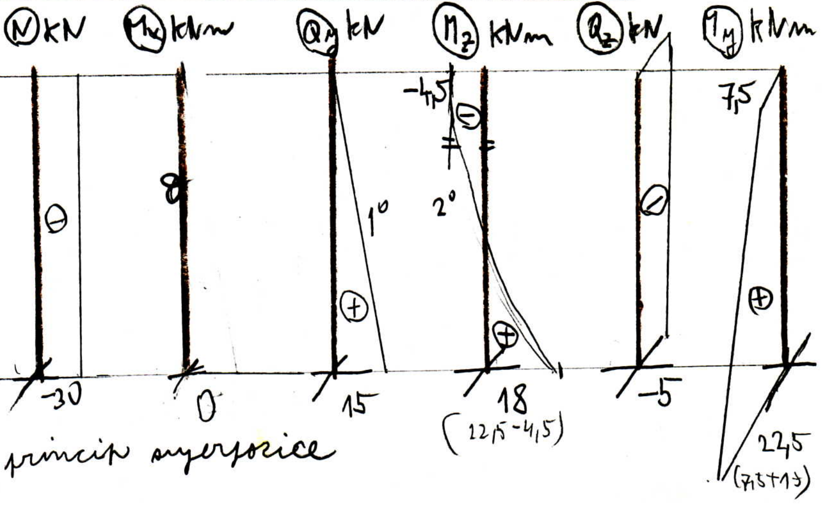
$$\underline{M_z = \frac{5}{2}(3-x)^2}$$

$$M_z(0) = \frac{5}{2} \cdot 3^2 = 22,5 kNm$$

$$M_z(3) = 0$$







princip superpozice

$$\underline{Q_z = -5kN}$$

$$\underline{M_y} = 7,5 + 15 - 5x = \underline{22,5 - 5x}$$

$$\underline{Q_y = 5(3-x)}$$

$$\underline{M_z = -4,5 + \frac{5}{2}(3-x)^2}$$

$$\boxed{\frac{dM_y}{dx} = -5 = \underline{Q_z}}$$

$$\boxed{\frac{dM_z}{dx} = 5(3-x) \cdot (-1) = -\underline{Q_y}} \quad \left(f_z = -\frac{dQ}{dx}\right)$$

• Momenty setrvačnosti rovinných obrazců

těžiště $y_T = \frac{S_z}{A} = \frac{\int y\,dA}{A}$ S... statický moment

$z_T = \frac{S_y}{A} = \frac{\int z\,dA}{A}$

momenty setrvačnosti

$$I_y = \int z^2\,dA$$

$$I_z = \int y^2\,dA$$

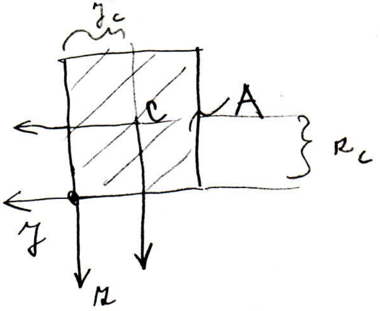
$$D_{yz} = \int yz\,dA$$

Steinerova věta → čl. doplňky

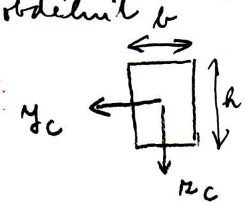
$$I_y = I_{yc} + z_c^2 A$$

$$I_z = I_{zc} + y_c^2 A$$

$$D_{yz} = D_{y_c z_c} + y_c z_c A$$



obdélník

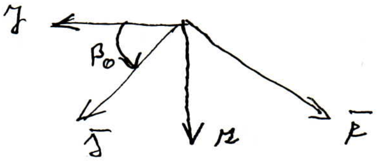


$$I_{y_c} = \frac{1}{12} b h^3$$

$$I_{z_c} = \frac{1}{12} h b^3$$

$$D_{y_c z_c} = 0$$

hlavní momenty setrvačnosti $D_{\bar{y}\bar{z}} = 0$



$$\operatorname{tg} 2\beta_0 = \frac{2D_{yz}}{I_z - I_y}$$

β_0 ... kladný po směru hodinových ručiček

$$I_{\bar{y}} = I_y \cdot \cos^2\beta + I_z \cdot \sin^2\beta - D_{yz} \sin 2\beta$$

$$I_{\bar{z}} = I_y \sin^2\beta + I_z \cdot \cos^2\beta + D_{yz} \sin 2\beta$$

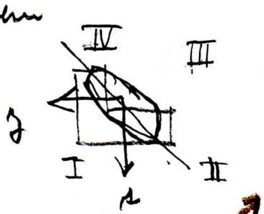
nebo

$$I_{1,2} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_z - I_y}{2}\right)^2 + D_{yz}^2}$$

MAX. POLOMĚR ~~EL.~~ SET. prochází kvadrantem

$D_{yz} < 0$ II IV

$D_{yz} > 0$ I III

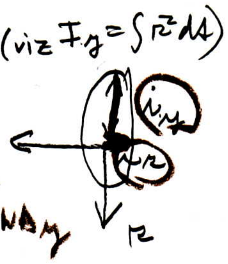


elipsa setrvačnosti → rozložení hmoty

• poloměry setrvačnosti $i_y = \sqrt{\frac{I_y}{A}}$, $i_z = \sqrt{\frac{I_z}{A}}$

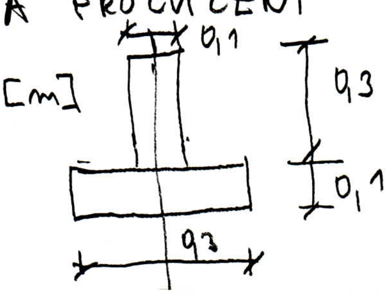
VYNESU NA z (viz $I_y = \int z^2 dA$)

VYNESU NA y

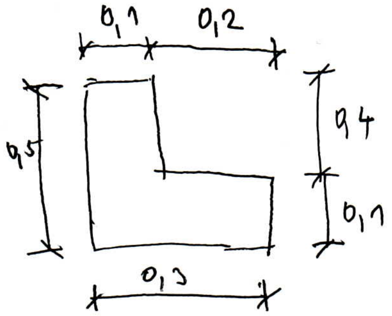


NA PROCVIČENÍ

[m]



- A
- těžiště
- I_y, I_z
- i_y, i_z



- A
- těžiště
- I_y, I_z, D_{yz}
- β_0
- I_{y0}, I_{z0} hlavní
- i_y, i_z