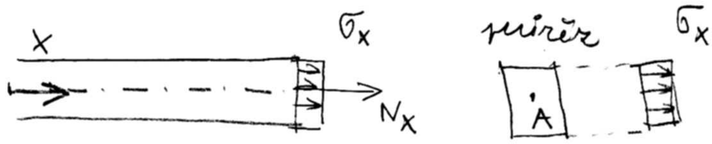
úkol 1

• vnitřní síly : pouze $N_x \neq 0$ (ostatní nulové)

↓

napětí $\sigma_x = \frac{N_x}{A}$



• deformace

$$\varepsilon_x = \frac{\sigma_x}{E} + \alpha \Delta t$$ ... fyzikální rovnice

$$\left(\varepsilon_x = \frac{N_x}{EA} + \alpha \Delta t\right)$$

E ... modul pružnosti

α ... souč. teplotní roztažnosti

Δt ... změna teploty

$$\varepsilon_x = \frac{du}{dx}$$ ... geometrická rovnice

↓

$$\frac{du}{dx} = \frac{N_x}{EA} + \alpha \Delta t$$

• posunutí ↓

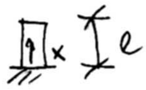
$$u = \int \left(\frac{N_x}{EA} + \alpha \Delta t\right) dx$$

zvláštní případ : $N_x, E, A, \alpha, \Delta t$ → tedy ε_x jsou konstantní v intervalu

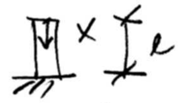
$$u = \frac{N_x}{EA} \cdot x + \alpha \Delta t x + c$$

↓

z okrajových podmínek

např. $x = 0 \rightarrow u = 0 \Rightarrow c = 0$ 

POZOR



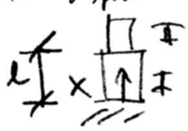
$u(x=l) = 0$ c = ...

• celkové prodloužení intervalu

$$\Delta l = u(l) - u(0)$$

$$\boxed{\Delta l = \frac{N_x \cdot l}{E \cdot A} + \alpha \Delta t \cdot l}$$

nebo podmínek spojitosti

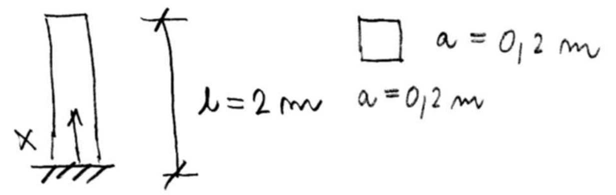


$u_I(x=l) = u_{II}(x=l)$

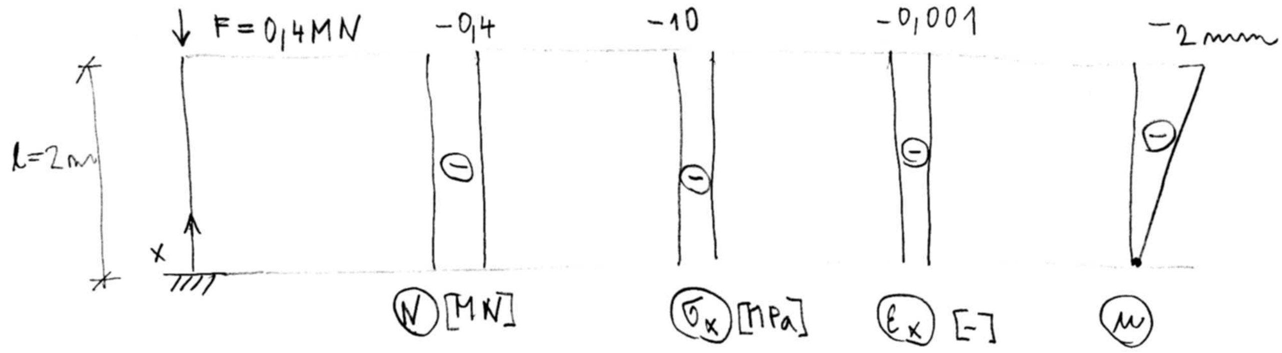
① Určete průběhy $N_x, \sigma_x, \varepsilon_x, u$ na dřevěném sloupu ($E = 10\,000$ MPa, $\alpha = 3 \cdot 10^{-6}$ [K^{-1}], $\gamma = 5$ kN/m^3) od zatížení

a) silou F b) vlastní tíhou c) konstantní změnou teploty

d) nerovnoměrná teplota



a) silou F



$$\sigma_x = \frac{N}{A} = \frac{-0{,}4}{0{,}2^2} = -10 \text{ MPa}$$

$$\varepsilon_x = \frac{\sigma_x}{E} = -\frac{10}{10000} = -0{,}001 \ [-]$$

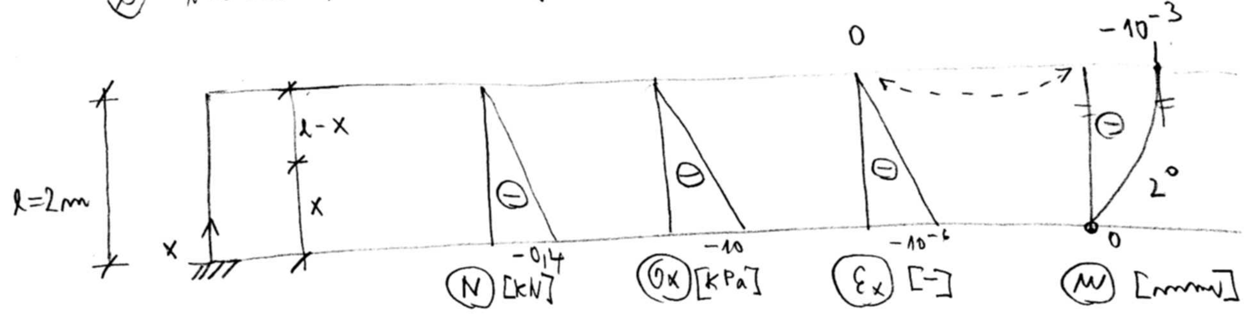
$$\frac{du}{dx} = -0{,}001 \ (= \varepsilon_x)$$

$u = -0{,}001x + c$ ... lineární fce

O.P.: $u(0) = 0 \rightarrow c = 0$

$u(l) = \Delta l = -0{,}001 \cdot 2 = -0{,}002$ m

b) vlastní tíhou: objem. tíha $\gamma = 5$ kN/m^3



- tíha části prutu $(l-x)$ $\gamma \cdot a^2(l-x)$
- normálová síla $\underline{N_x = -\gamma \cdot a^2(l-x)} = -5 \cdot 0{,}2^2(2-x)$ ... lin. fce

$N_x(x=0) = -5 \cdot 0{,}2^2 \cdot 2 = -0{,}4$ kN ... maximum

$N_x(x=2) = 0$

- $\sigma_x = \frac{N_x}{A} = -\frac{\gamma \cdot \cancel{a^2}(l-x)}{\cancel{a^2}} = -5(2-x)$ ... lin. fce, max $\sigma_x(0) = -10$ kPa
- $\varepsilon_x = \frac{\sigma_x}{E} = \frac{-\gamma(l-x)}{E} = \frac{-5 \cdot (2-x)}{10 \cdot 10^6}$ ... lin fce, max $\varepsilon_x(0) = -1 \cdot 10^{-6}$ [−]
- $\frac{du}{dx} = -\frac{\gamma(l-x)}{E} \rightarrow u = -\frac{\gamma}{E}\left(lx - \frac{x^2}{2}\right) + c$ O.P.: $u(0) = 0 \rightarrow c = 0$

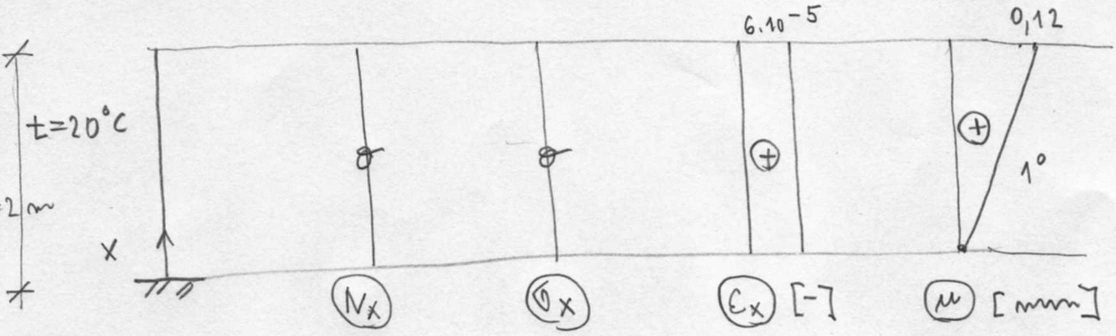
$u = -\frac{\gamma}{E}\left(lx - \frac{x^2}{2}\right) = -\frac{5}{10^7}\left(2x - \frac{x^2}{2}\right)$ ... kvadr. fce

$\max u(l) = -\frac{\gamma}{E}\left(l^2 - \frac{l^2}{2}\right) = -\frac{\gamma}{E}\cdot\frac{l^2}{2} = -\frac{5}{10^7}\cdot\frac{2^2}{2} = -10^{-6}\,m = \underline{-10^{-3}\,mm}$

↓

$\left(\varepsilon_x = \frac{du}{dx} = 0\right)$

c) rovnoměrné ohřátí $\Delta t = 20°C$ (nebo $20°K$ ... teplotní rozdíl !)



$N_x = 0 \Rightarrow \sigma_x = 0$ ... staticky určitá kce − změnou teploty – roztažení × zkrácení

$\varepsilon_x = \cancel{\frac{\sigma_x}{E}}^{\,0} + \alpha\Delta t = 3.10^{-6}.20 = 6.10^{-5}$ [-]

$\frac{du}{dx} = 6.10^{-5} \rightarrow \underline{u = 6.10^{-5}x} + \cancel{C}^{\,0}$ o.p. $u(0) = 0 \rightarrow C = 0$

$\max u(x = 2m) = 6.10^{-5}.2 = 1{,}2.10^{-4}\,m = 0{,}12\,mm$

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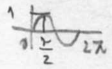
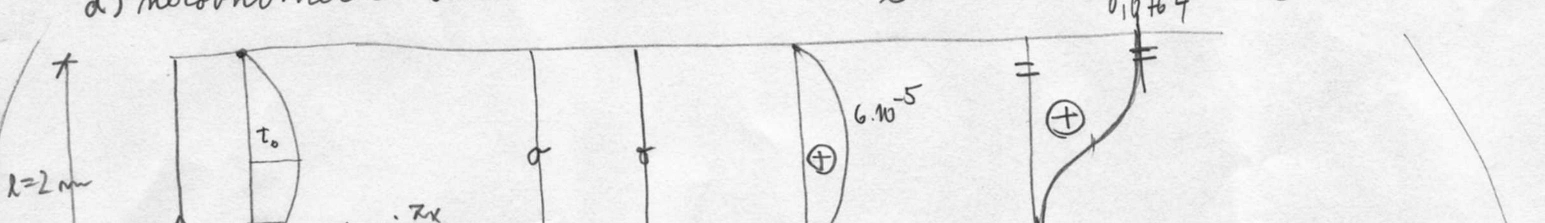
doplněk

d) nerovnoměrné ohřátí $\Delta t = t_0 . \sin\frac{\pi x}{l}$; $t_0 = 20°C$





$N_x = 0,\ \sigma_x = 0$

$\varepsilon_x = \alpha\Delta t = \alpha . t_0 . \sin\frac{\pi x}{l}$ $\qquad \varepsilon_x\left(x = \frac{l}{2} = 1m\right) = 3.10^{-6}.20.\cancel{\sin\frac{\pi . l}{l\,2}}^{\,1} = 6.10^{-5}$ [-]

$\frac{du}{dx} = \varepsilon_x = \alpha . t_0 . \sin\frac{\pi x}{l} \rightarrow u = \int \alpha t_0 \sin\frac{\pi x}{l}dx$ $\quad$ subst: $a = \frac{\pi x}{l}$ $\quad da = \frac{\pi}{l}dx$, $\quad dx = \frac{l}{\pi}.da$

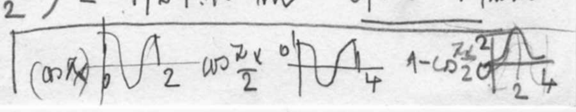
$\underline{u = -\alpha . t_0 . \cos\left(\frac{\pi x}{l}\right) . \frac{l}{\pi} + C}$

o.p.: $x = 0$ ... $u = 0 \rightarrow 0 = -\alpha t_0 . 1 . \frac{l}{\pi} + C \rightarrow \underline{C = \alpha . t_0 . \frac{l}{\pi}}$

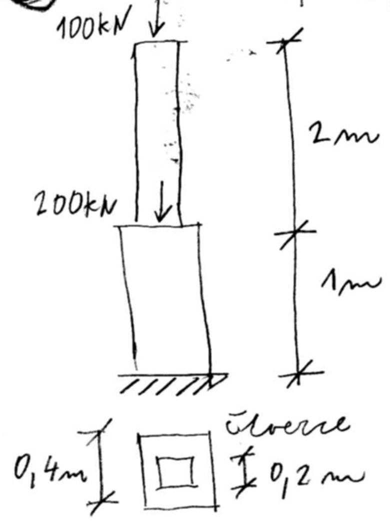
$u = -\alpha t_0 \frac{l}{\pi} \cos\frac{\pi x}{l} + \alpha t_0 \frac{l}{\pi} = \underline{\alpha t_0 \frac{l}{\pi}\left(1 - \cos\frac{\pi x}{l}\right)}$

extrémy: $u(0) = 0$

$\max u(x = 2m) = 3.10^{-6}.20.\frac{2}{\pi}\Big(\underbrace{1 - \cos\frac{\pi . 2}{2}}_{\frac{1-(-1)}{2}}\Big) = 7{,}64.10^{-5}\,m = \underline{0{,}0764}\,mm$



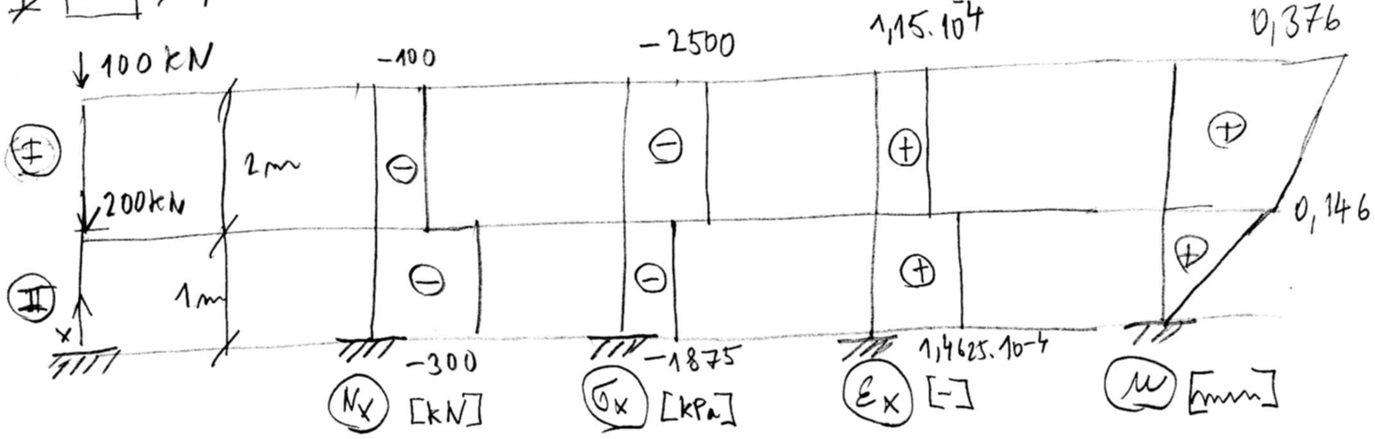
(2) Průběh $N_x, \sigma_x, \varepsilon_x, u$? celkové prodloužení $\Delta l = ?$



- stupňovitý sloup se 2 břemeny
- ohřátí $\Delta t = 20°C$

$$E = 2.10^4 \text{ MPa}$$

$$\alpha = 12.10^{-6} \, (K^{-1})$$



I

$$\sigma_x^{I} = \frac{N_x^{I}}{A^{I}} = \frac{-100}{0,2^2} = -2500 \text{ kPa}$$

II

$$\sigma_x^{II} = \frac{N_x^{II}}{A^{II}} = \frac{-300}{0,4^2} = -1875 \text{ kPa}$$

$$\varepsilon_x^{I} = \frac{\sigma_x^{I}}{E} + \alpha\Delta t = \frac{-2,5}{2.10^4} + 12.10^{-6}.20 = 1,15.10^{-4}$$

$$\varepsilon_x^{II} = \frac{\sigma_x^{II}}{E} + \alpha\Delta t = \frac{-1,875}{2.10^4} + 12.10^{-6}.20 = 1,4625.10^{-4}$$

$$u^{I} = \int \varepsilon_x^{I} dx = \varepsilon_x^{I}.x + c_1$$

$$u^{I} = 1,15.10^{-4}x + c_1$$

$$u^{II} = \int \varepsilon_x^{II} dx = \varepsilon_x^{II}.x + c_2$$

$$\boxed{u^{II} = 1,4625.10^{-4}x} + c_2 \rightarrow 0$$

okrajová podmínka: $\boxed{u^{II}(0) = 0} \rightarrow c_2 = 0$

podmínka spojitosti: $\boxed{u^{I}(x=1m) = u^{II}(x=1m)}$

$$1,15.10^{-4}.1 + c_1 = 1,4625.10^{-4}.1 \rightarrow c_1 = +0,3125.10^{-4}$$

$$\boxed{u^{I} = 1,15.10^{-4}x + 0,3125.10^{-4}}$$

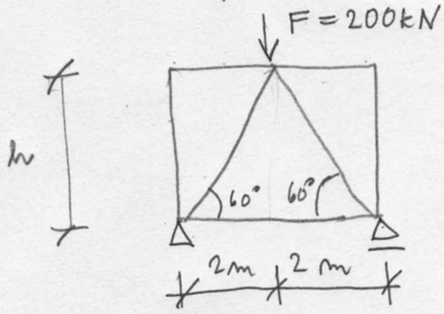
$$u(x=1) = \underbrace{1,15.10^{-4}.1 + 0,3125.10^{-4}}_{u^{I}(1)} = \underbrace{1,4625.10^{-4}.1}_{u^{II}(1)} = 0,146 \text{ mm}$$

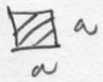
$$u(x=3m) = u^{II}(3m) = 1,15.10^{-4}.3 + 0,3125.10^{-4} = 3,7625.10^{-4} \text{ m} = 0,376 \text{ mm} = \Delta l$$

KONTROLA: $\Delta l = \frac{N_x^{I}}{EA^{I}} l_I + \frac{N_x^{II}}{EA^{II}} l_{II} + \alpha\Delta t\, l = \frac{-100}{2.10^7.0,2^2}.2 - \frac{300}{2.10^7.0,4^2}.1 + 12.10^{-6}.20.3 = 0,37625$ mm

③ Dřevěná příhradová konstrukce

- všechny pruty stejného čtvercového průřezu
- určete rozměr strany čtverce, aby v tažených i tlačených prutech nebylo překročeno normálové napětí $\bar{\sigma} = 10\,\mathrm{MPa}$.

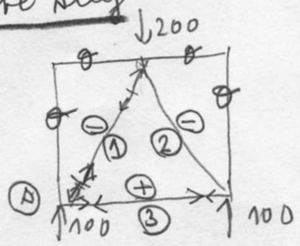


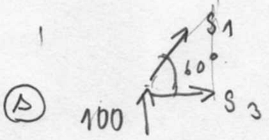


$a = ?$

$$\mathrm{tg}\,60° = \frac{h}{2} \rightarrow h = 2 \cdot \mathrm{tg}\,60° = 3{,}464\,\mathrm{m}$$

osové síly





$\uparrow: \; 100 + S_1 \cdot \sin 60° = 0$

$\Rightarrow \underline{S_1 = -115{,}47\,\mathrm{kN}}$ (tlak)

$\rightarrow: \; S_3 + S_1 \cdot \cos 60° = 0$

$\Rightarrow S_3 = -S_1 \cdot \cos 60° = -(-115{,}47) \cdot \cos 60°$

$\underline{S_3 = +57{,}735\,\mathrm{kN}}$ (tah)

$$|S_1| > |S_3|$$

navrhnu na S_1 (tlak)

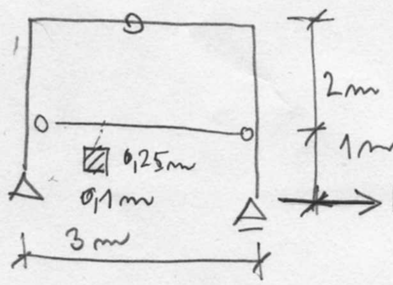
$\underline{N = S_1 = -115{,}47\,\mathrm{kN}}$

$N = \sigma_x \cdot A \qquad |\sigma_x| = \dfrac{|N_x|}{A} \leq \bar{\sigma}$

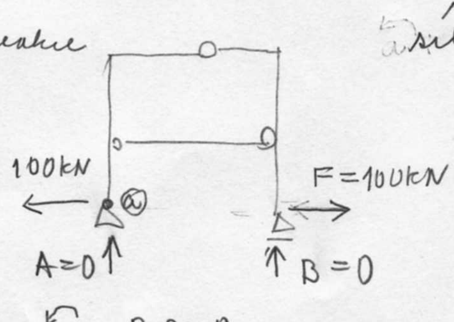
$$\Rightarrow A \geq \frac{|N_x|}{\bar{\sigma}} = \frac{115{,}47\,\mathrm{kN}}{10\,000\,\mathrm{kPa}} = \underline{0{,}011547\,\mathrm{m}^2}$$

$$a \geq \sqrt{A} \geq \sqrt{0{,}011547} = \underline{0{,}1075\,\mathrm{m}} \quad \boxed{\text{strana } a \geq 0{,}1075\,\mathrm{m}}$$

④ napětí σ_x v dřev. táhlu ?

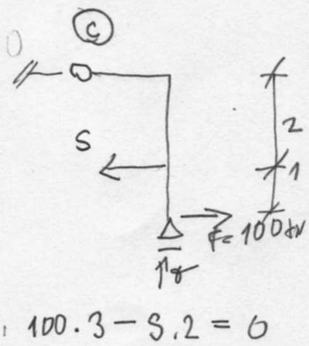


reakce



$\curvearrowleft a: \; B \cdot 3 = 0$

síla S



$\curvearrowleft c: \; 100 \cdot 3 - S \cdot 2 = 0$

$\underline{S = 150\,\mathrm{kN} = N}$

$$\underline{\underline{\sigma_x}} = \frac{N}{A} = \frac{150}{0{,}1 \cdot 0{,}25} = 6000\,\mathrm{kPa} = \underline{6\,\mathrm{MPa}}$$