• jeden ohybový moment nenulový $M_y \neq 0$ nebo $M_z \neq 0$ ($y, z$ ... hlavní osy)
(ostatní vnitřní síly nulové $N_x = 0$)

· obecně ohyb prutů → výpočet napětí

$$\sigma_x = \frac{N_x}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

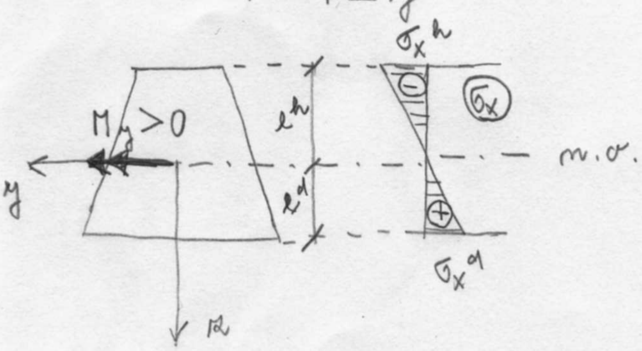
platí pro hlavní těžišťové osy ($D_{yz} = 0$)

## $M_y \neq 0$

$$\boxed{\sigma_x = \frac{M_y}{I_y} \cdot z}$$

• neutrální osa: $\sigma_x = 0 \rightarrow z = 0$

$n.o. \equiv y$




• napětí v krajních vláknech

$$\max|\sigma_x^h| = \frac{|M_y|}{W_y^h}$$

$$\max|\sigma_x^d| = \frac{|M_y|}{W_y^d}$$

průřezové moduly

$$W_y^h = \frac{I_y}{e^h} \qquad [m^3]$$

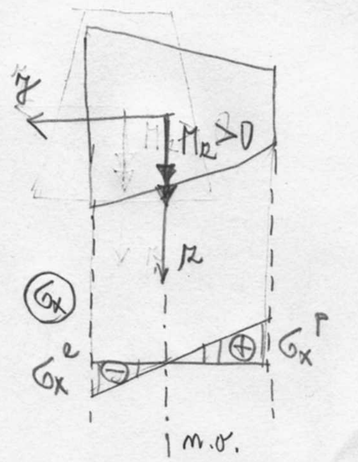
$$W_y^d = \frac{I_y}{e^d} \qquad [m^3]$$

## $M_z \neq 0$

$$\boxed{\sigma_x = -\frac{M_z}{I_z} \cdot y}$$

· neutrální osa: $\sigma_x = 0 \rightarrow y = 0$

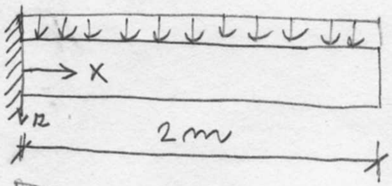
$n.o. \equiv z$



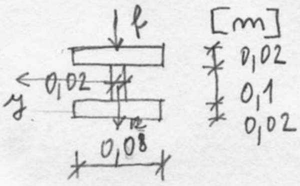

$$\max|\sigma_x^p| = \frac{|M_z|}{W_z^p}$$

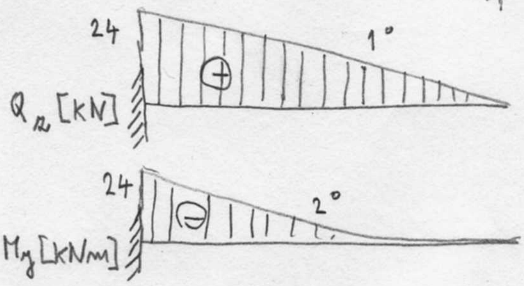
$$\max|\sigma_x^l| = \frac{|M_z|}{W_z^l}$$

① Určete průběh $\sigma_x$ ve vetknutém průřezu




$f = 12\ kN/m$

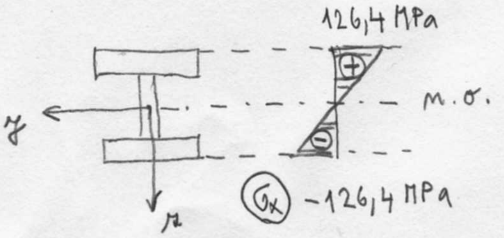



$$I_y = \frac{1}{12}\left(0{,}08\ 0{,}14^3 - 0{,}06\cdot 0{,}1^3\right) = 1{,}329\cdot 10^{-5}\ m^4$$

$$M_y = -12\cdot 2\cdot 1 = -24\ kNm$$

$$\sigma_x = \frac{M_y}{I_y}\cdot z = \frac{-24}{1{,}329\cdot 10^{-5}}\, z = -1805{,}9\cdot 10^3\, z$$




$$\sigma_x^h = -1805{,}9\cdot 10^3\cdot(-0{,}07) = 126{,}4\cdot 10^3\ kPa$$

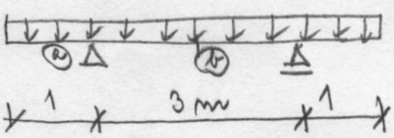
$$\sigma_x^d = -1805{,}9\cdot 10^3\cdot 0{,}07 = -126{,}4\cdot 10^3\ kPa$$

alternativně symetrický

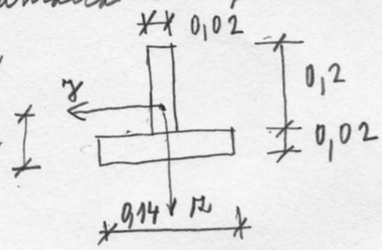
$$W_y^h = W_y^d = \frac{I_y}{e^{d,h}} = \frac{1{,}329\cdot 10^{-5}}{0{,}07} = 1{,}8986\cdot 10^{-4}\ m^3$$

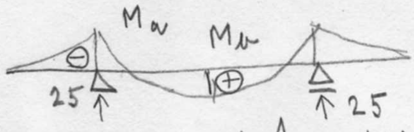
$$\max_{\min} |\sigma_x| = \frac{|M_y|}{W_y} = \frac{24}{1{,}8986\cdot 10^{-4}} = 126{,}4\cdot 10^3\ kPa$$

② Určete průběh $\sigma_x$ v nejvíce namáhaném průřezu




$f = 10\ kN/m$




$$e = \frac{0{,}14\cdot 0{,}02\cdot 0{,}01 + 0{,}02\cdot 0{,}2\cdot 0{,}12}{0{,}14\cdot 0{,}02 + 0{,}02\cdot 0{,}2} = 0{,}075\ m$$

$$M_a = -10\cdot 1\cdot \frac{1}{2} = -5\ kNm$$

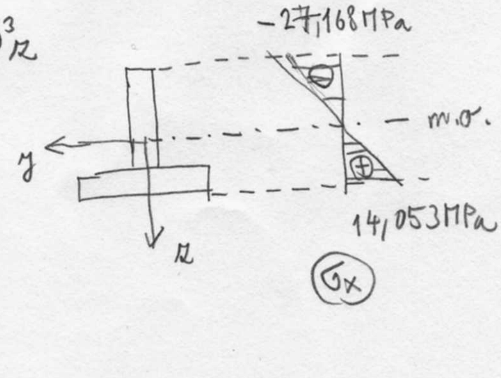
$$M_b = -10\cdot 2{,}5\cdot \frac{2{,}5}{2} + 25\cdot 1{,}5 = \underline{6{,}25\ kNm}$$

$$I_y = \frac{1}{12}\cdot 0{,}14\cdot 0{,}02^3 + 0{,}14\cdot 0{,}02\cdot 0{,}065^2 + \frac{1}{12}\cdot 0{,}02\cdot 0{,}2^3 + 0{,}02\cdot 0{,}2\cdot 0{,}045^2 = 3{,}336\cdot 10^{-5}\ m^4$$

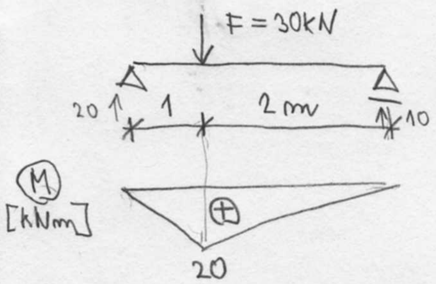
$$\sigma_x = \frac{M_y}{I_y}\cdot z = \frac{6{,}25}{3{,}336\cdot 10^{-5}}\cdot z = 187{,}37\cdot 10^3 z$$

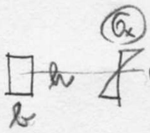
$$\sigma_{xd} = 187{,}37\cdot 10^3\cdot 0{,}075 = 14\ 053\ kPa$$

$$\sigma_{xh} = 187{,}37\cdot 10^3(-0{,}145) = -27\ 168\ kPa$$



(3) Navrhněte obdélníkový průřez s poměrem stran $b:h = 5:7$ tak, aby v krajních vláknech nejvíce namáhaného průřezu nepřesáhlo napětí hodnotu $\bar{\sigma} = 10\,\mathrm{MPa}$.



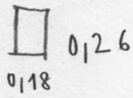

$\max M_y = 20\,\mathrm{kNm}$

$$|\sigma_x| = \frac{|M_y|}{I_y}|z| = \frac{|M_y|}{\frac{1}{12}bh^3}\cdot\frac{h}{2} = \frac{|M_y|}{\frac{1}{6}bh^2} = \frac{|M_y|}{W_y} \le \bar{\sigma} = 10\,\mathrm{MPa}$$

$\rightarrow$ pro obdélník $W_y = \frac{1}{6}bh^2$

$b/h = 5/7 \Rightarrow b = 5/7\,h \rightarrow W_y = \frac{5}{42}h^3$

$$W_y \ge \frac{|M_y|}{\bar{\sigma}} \rightarrow h^3 \ge \frac{42}{5}\cdot\frac{|M_y|}{\bar{\sigma}} = \frac{42}{5}\cdot\frac{20}{10\,000} = 0{,}0168$$



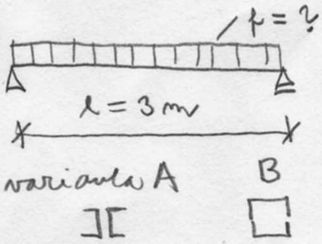

$h \ge \sqrt[3]{0{,}0168} = 0{,}256\,\mathrm{m}$ $\quad b = \frac{5}{7}\cdot 0{,}256 = 0{,}183\,\mathrm{m}$

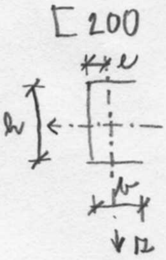
Návrh: $h = 0{,}26\,\mathrm{m}$ ; $b = 0{,}18\,\mathrm{m}$

Posouzení: $$\sigma_x^{MAX} = \frac{M_y}{W_y} = \frac{M_y}{\frac{1}{6}bh^2} = \frac{20\cdot 10^{-3}}{\frac{1}{6}\cdot 0{,}18\cdot 0{,}26^2} = 9{,}862\,\mathrm{MPa} < 10\,\mathrm{MPa} = \bar{\sigma}$$

VYHOVUJE

(4) Určete velikost zatížení nosníku tak, aby nebylo překročeno napětí v tahu a v tlaku $\bar{\sigma} = 160\,\mathrm{MPa}$. Porovnejte dvě varianty průřezu z válcovaných tyčí [200.




$A^{[} = 3220\,\mathrm{mm}^2$

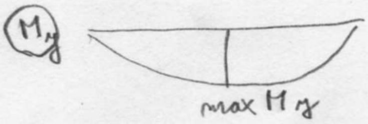
$b = 75\,\mathrm{mm}$

$h = 200\,\mathrm{mm}$

$e = 20{,}1\,\mathrm{mm}$

$I_y^{[} = 19{,}1\cdot 10^{-6}\,\mathrm{m}^4$

$I_z^{[} = 1{,}48\cdot 10^{-6}\,\mathrm{m}^4$




$\max M_y = \frac{1}{8}fl^2$ $\quad |\max \sigma_x| = \frac{M_y}{W_y} = \frac{\frac{1}{8}fl^2}{W_y} \le \bar{\sigma}$

$$\rightarrow f \le \frac{8\bar{\sigma}W_y}{l^2}$$

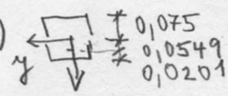
A)


$I_y = 2\cdot 19{,}1\cdot 10^{-6} = 3{,}82\cdot 10^{-5}\,\mathrm{m}^4$

$W_y = \frac{I_y}{\frac{h}{2}} = \frac{3{,}82\cdot 10^{-5}}{0{,}1} = 3{,}82\cdot 10^{-4}\,\mathrm{m}^3$

$$f \le \frac{8\cdot\bar{\sigma}W_y}{l^2} = \frac{8\cdot 160\cdot 10^3\cdot 3{,}82\cdot 10^{-4}}{3^2} = \underline{54{,}33\,\mathrm{kN/m}}$$

B)




$I_y = 2(I_z^{[} + A^{[}\cdot 0{,}0549^2) = 2(1{,}48\cdot 10^{-6} + 3220\cdot 10^{-6}\cdot 0{,}0549^2)$

$I_y = 2{,}237\cdot 10^{-5}\,\mathrm{m}^4$

$W_y = \frac{I_y}{0{,}075} = \frac{2{,}237\cdot 10^{-5}}{0{,}075} = 2{,}98\cdot 10^{-4}\,\mathrm{m}^3$

$$f \le \frac{8\bar{\sigma}\cdot W_y}{l^2} = \frac{8\cdot 160\cdot 10^3\cdot 2{,}98\cdot 10^{-4}}{3^2} = \underline{42{,}42\,\mathrm{kN/m}}$$