Dynamic stiffness matrices

Definition of end displacements, rotations, forces and moments

Figure 1: Coordinate system.

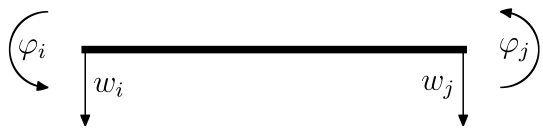

Figure 2: End displacements and rotations.

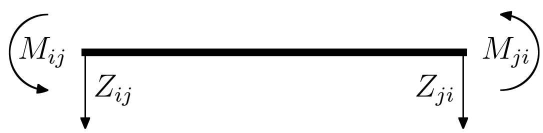

Figure 3: End forces and moments.

Relationship between end generalized displacements and generalized forces

$$\boldsymbol{K}(\lambda)\boldsymbol{d} = \boldsymbol{f} \tag{1}$$

Fixed beam

Figure 4: Fixed beam.

$$\boldsymbol{d}^T = (w_i, \varphi_i, w_j, \varphi_j) \tag{2}$$

$$\boldsymbol{f}^T = (Z_{ij}, M_{ij}, Z_{ji}, M_{ji}) \tag{3}$$

$$\boldsymbol{K} = \begin{pmatrix} \frac{EI}{l^3}F_6(\lambda) & \frac{EI}{l^2}F_4(\lambda) & \frac{EI}{l^3}F_5(\lambda) & -\frac{EI}{l^2}F_3(\lambda) \\ \frac{EI}{l^2}F_4(\lambda) & \frac{EI}{l}F_2(\lambda) & \frac{EI}{l^2}F_3(\lambda) & \frac{EI}{l}F_1(\lambda) \\ \frac{EI}{l^3}F_5(\lambda) & \frac{EI}{l^2}F_3(\lambda) & \frac{EI}{l^3}F_6(\lambda) & -\frac{EI}{l^2}F_4(\lambda) \\ -\frac{EI}{l^2}F_3(\lambda) & \frac{EI}{l}F_1(\lambda) & -\frac{EI}{l^2}F_4(\lambda) & \frac{EI}{l}F_2(\lambda) \end{pmatrix} \tag{4}$$

Fixed-hinged beam

Figure 5: Fixed-hinged beam.

$$\boldsymbol{d}^T = (w_i, \varphi_i, w_j) \tag{5}$$

$$\boldsymbol{f}^T = (Z_{ij}, M_{ij}, Z_{ji}) \tag{6}$$

$$\boldsymbol{K} = \begin{pmatrix} \frac{EI}{l^3}F_{11}(\lambda) & \frac{EI}{l^2}F_9(\lambda) & \frac{EI}{l^3}F_{10}(\lambda) \\ \frac{EI}{l^2}F_9(\lambda) & \frac{EI}{l}F_7(\lambda) & \frac{EI}{l^2}F_8(\lambda) \\ \frac{EI}{l^3}F_{10}(\lambda) & \frac{EI}{l^2}F_8(\lambda) & \frac{EI}{l^3}F_{12}(\lambda) \end{pmatrix} \tag{7}$$

Hinged-fixed beam

Figure 6: Hinged-fixed beam.

$$\boldsymbol{d}^T = (w_i, w_j, \varphi_j) \tag{8}$$

$$\boldsymbol{f}^T = (Z_{ij}, Z_{ji}, M_{ji}) \tag{9}$$

$$\boldsymbol{K} = \begin{pmatrix} \frac{EI}{l^3}F_{12}(\lambda) & \frac{EI}{l^3}F_{10}(\lambda) & -\frac{EI}{l^2}F_8(\lambda) \\ \frac{EI}{l^3}F_{10}(\lambda) & \frac{EI}{l^3}F_{11}(\lambda) & -\frac{EI}{l^2}F_9(\lambda) \\ -\frac{EI}{l^2}F_8(\lambda) & -\frac{EI}{l^2}F_9(\lambda) & \frac{EI}{l}F_7(\lambda) \end{pmatrix} \tag{10}$$

Frequency (Kloušek's) functions

frequency parameter

$$\lambda = l\sqrt[4]{\frac{\mu\omega^2}{EI}} \tag{11}$$

where l is the length, μ is the weight of unit length, ω is the prescribed circular frequency, E is the Young's modulus of elasticity and I is the moment of inertia of the cross section.

$$F_1(\lambda) = -\lambda\frac{\sinh\lambda - \sin\lambda}{\cosh\lambda\cos\lambda - 1} \tag{12}$$

$$F_2(\lambda) = -\lambda\frac{\cosh\lambda\sin\lambda - \sinh\lambda\cos\lambda}{\cosh\lambda\cos\lambda - 1} \tag{13}$$

$$F_3(\lambda) = -\lambda^2\frac{\cosh\lambda - \cos\lambda}{\cosh\lambda\cos\lambda - 1} \tag{14}$$

$$F_4(\lambda) = \lambda^2\frac{\sinh\lambda\sin\lambda}{\cosh\lambda\cos\lambda - 1} \tag{15}$$

$$F_5(\lambda) = \lambda^3\frac{\sinh\lambda + \sin\lambda}{\cosh\lambda\cos\lambda - 1} \tag{16}$$

$$F_6(\lambda) = -\lambda^3\frac{\cosh\lambda\sin\lambda + \sinh\lambda\cos\lambda}{\cosh\lambda\cos\lambda - 1} \tag{17}$$

$$F_7(\lambda) = \lambda\frac{2\sinh\lambda\sin\lambda}{\cosh\lambda\sin\lambda - \sinh\lambda\cos\lambda} \tag{18}$$

$$F_8(\lambda) = \lambda^2\frac{\sinh\lambda + \sin\lambda}{\cosh\lambda\sin\lambda - \sinh\lambda\cos\lambda} \tag{19}$$

$$F_9(\lambda) = -\lambda^2\frac{\cosh\lambda\sin\lambda + \sinh\lambda\cos\lambda}{\cosh\lambda\sin\lambda - \sinh\lambda\cos\lambda} \tag{20}$$

$$F_{10}(\lambda) = -\lambda^3\frac{\cosh\lambda + \cos\lambda}{\cosh\lambda\sin\lambda - \sinh\lambda\cos\lambda} \tag{21}$$

$$F_{11}(\lambda) = \lambda^3\frac{2\cosh\lambda\cos\lambda}{\cosh\lambda\sin\lambda - \sinh\lambda\cos\lambda} \tag{22}$$

$$F_{12}(\lambda) = \lambda^3\frac{\cosh\lambda\cos\lambda + 1}{\cosh\lambda\sin\lambda - \sinh\lambda\cos\lambda} \tag{23}$$