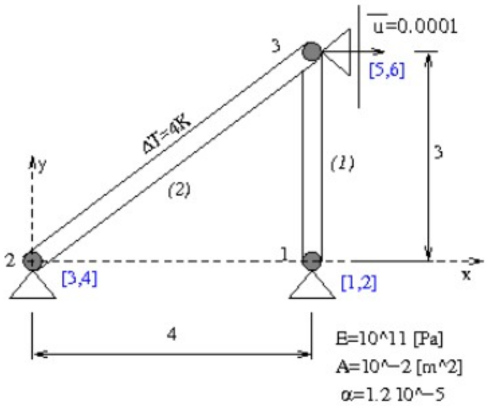

Nodal forces and the right-hand side vector from temperature loading

Constitutive equation:

$\sigma = E(\varepsilon - \varepsilon_0)$

where the initial strain (thermal strain - dilatation) from the thermal loading is: $\varepsilon_0 = \alpha \Delta T$,

where α is the coefficient of thermal dilatation and the temperature change is ΔT.

From the equilibrium equation, $F_{1x}^l + N = 0, -N + F_{2x}^l = 0$): it leads

$$\begin{Bmatrix} F_{1x}^l \\ F_{2x}^l \end{Bmatrix} = \frac{EA}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} + EA \begin{Bmatrix} \varepsilon_0 \\ -\varepsilon_0 \end{Bmatrix}$$

Vector of thermal loading: ní $f^{T,l} = -EA\varepsilon_0 \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

Extension into 2D and transformation into the global coordinate system:

$$f^{T,g} = -EA\varepsilon_0 \begin{Bmatrix} \cos(\psi) \\ \sin(\psi) \\ -\cos(\psi) \\ -\sin(\psi) \end{Bmatrix}$$

Calculation of strain and stress (axial force):

Strain derivation for a general bar in 2D

$$\varepsilon = \frac{u^l_{2x} - u^l_{1x}}{l}$$

Local displacements are expressed by the global ones:

$$u^l_{jx} = u^g_{jx}\cos(\phi) + u^g_{jy}\sin(\phi), \quad j = 1, 2$$

Introduction into the strain – displacement gives:

$$\varepsilon = \frac{1}{l}(u^g_{1x}(-\cos(\phi)) + u^g_{1y}(-\sin(\phi)) + u^g_{2x}(\cos(\phi)) + u^g_{2y}(\sin(\phi)))$$

Stress and axial force for the linear elastic bar:

$$\sigma = E\varepsilon$$

$$N = A\sigma = EA\varepsilon$$