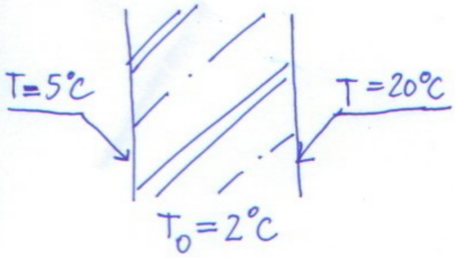
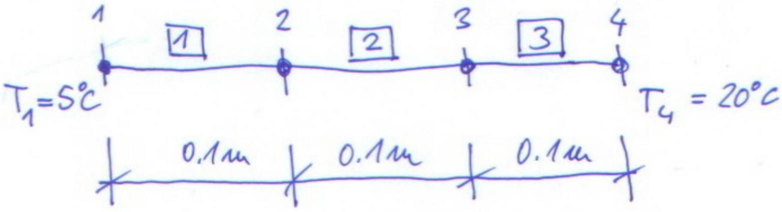
NONSTATIONARY HEAT CONDUCTION – 1D



INITIAL CONDITION: $T_0 = 2°C$



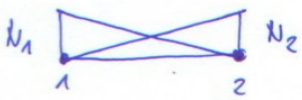
$$\underline{K}\,\underline{r} + \underline{C}\,\underline{\dot{r}} = \underline{f}$$

CONSTANT MATIRIAL PARAMETERS

$$\lambda = 2\ \mathrm{W \cdot m^{-1} K^{-1}}$$
$$\rho = 2500\ \mathrm{kg/m^3}$$
$$c = 1000\ \mathrm{J \cdot kg^{-1} K^{-1}}$$

$(A = 0{,}1\,\mathrm{m^2})$

SHAPE FUNCTIONS:



ELEM. COND. MATRIX:

$$\underline{K}^e_\Omega = \int_\Omega \underline{B}^{eT} \lambda\, \underline{B}^e \, d\Omega = \frac{\lambda}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix}$$

COND. MATRIX OF THE STRUCTURE:

$$\underline{K} = \begin{bmatrix} 40 & -20 \\ -20 & 40 \end{bmatrix}$$

ELEM. CAPACITY MATRIX:

$$\underline{C}^e_\Omega = \int_\Omega \underline{N}^{eT} \rho c\, \underline{N}^e \, d\Omega = \int_\Omega \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \rho c \begin{bmatrix} N_1 ; N_2 \end{bmatrix} d\Omega = \rho c \int_l \begin{bmatrix} N_1 N_1 & N_1 N_2 \\ N_2 N_1 & N_2 N_2 \end{bmatrix} d\Omega$$

$$\underline{C}^e_\Omega = \rho c l \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix}$$

$$\underline{C}^e_{1,2,3} = 250\,000 \begin{array}{c} \\ \left[\begin{array}{cc} 1/3 & 1/6 \\ 1/6 & 1/3 \end{array}\right] \\ \begin{array}{cc} 0 & 1 \\ 1 & 2 \\ 2 & 0 \end{array} \end{array} \begin{array}{ccc} 0 & 1 & 2 \\ 1 & 2 & 0 \end{array}$$

CAPACITY MATRIX OF THE STRUCTURE:

LOCALIZATION

$$\underline{C} = 250\,000 \begin{array}{c} \left[\begin{array}{cc} 2/3 & 1/6 \\ 1/6 & 2/3 \end{array}\right] \\ \begin{array}{cc} 1 & 2 \end{array} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

2/

CONTRIBUTION FROM THE DIRICHLE B. Cs. TO THE RHS:

$$\underline{d} = \begin{Bmatrix} -100 \\ -400 \end{Bmatrix} \qquad \underline{f} = -\underline{d} = \begin{Bmatrix} 100 \\ 400 \end{Bmatrix}$$

TIME INTEGRATION:

$$\underbrace{\left[\underline{K}\tau + \frac{\underline{C}}{\Delta t}\right]}_{M_1} \underline{r}_i = \underline{f}_{i-1}(1-\tau) + \underline{f}_i \tau + \underbrace{\left[\frac{\underline{C}}{\Delta t} + \underline{K}(1-\tau)\right]}_{M_2} \underline{r}_{i-1}$$

SET UP:

$\tau = 0.5 \qquad \Delta t = 60s$

B.Cs. ARE CONSTANT IN TIME:

$$\underline{f}_{i-1}(1-\tau) + \underline{f}_i \tau = \underline{f}$$

$$\underline{M}_1 = \begin{bmatrix} 40 & -20 \\ -20 & 40 \end{bmatrix} \cdot 0.5 + \frac{250\,000}{60} \begin{bmatrix} 2/3 & 1/6 \\ 1/6 & 2/3 \end{bmatrix} = \begin{bmatrix} 2797.78 & 684.445 \\ 684.445 & 2797.78 \end{bmatrix}$$

$$\underline{M}_2 = \frac{250000}{60} \begin{bmatrix} 2/3 & 1/6 \\ 1/6 & 2/3 \end{bmatrix} - \begin{bmatrix} 40 & -20 \\ -20 & 40 \end{bmatrix} \cdot 0.5 = \begin{bmatrix} 2757.78 & 704.445 \\ 704.445 & 2757.78 \end{bmatrix}$$

STEP No. 1:

$$\underline{M}_1 \underline{r}_1 = \underline{f} + \underline{M}_2 \underline{r}_0 \qquad \underline{r}_0 = \begin{Bmatrix} 2 \\ 2 \end{Bmatrix} \quad \text{INITIAL CONDITIONS:}$$

$$\underline{M}_1 \underline{r}_1 = \underline{h}_1$$

$$\underline{h}_1 = \begin{Bmatrix} 100 \\ 400 \end{Bmatrix} + \begin{bmatrix} 2757.78 & 704.445 \\ 704.445 & 2757.78 \end{bmatrix} \begin{Bmatrix} 2 \\ 2 \end{Bmatrix} = \begin{Bmatrix} 7024.45 \\ 7324.45 \end{Bmatrix}$$

SOLUTION:

$$\underline{r}_1 = \begin{Bmatrix} 1.98933 \\ 2.13128 \end{Bmatrix}$$

STEP No. 2:

$$\underline{M}_1 \underline{r}_2 = \underline{\mathcal{R}}_2 \qquad \underline{\mathcal{R}}_2 = \begin{Bmatrix} 7087.5 \\ 7678.97 \end{Bmatrix}$$

SOLUTION:

$$\underline{r}_2 = \begin{Bmatrix} 1.98033 \\ 2.2602 \end{Bmatrix}$$

STEP No. 3:

$$\underline{M}_1 \underline{r}_3 = \underline{\mathcal{R}}_3$$

SOLUTION:

$$\underline{r}_3 = \begin{Bmatrix} 1.97294 \\ 2.38682 \end{Bmatrix}$$

!! TEMPERATURE AT NODE 2 IS DECREASING !!

THIS IS PHYSICALY INCORRECT.

IT IS CAUSED BY THE NUMERICA SOLUTION VIA FEM.

THE SOLUTION CAN BE THE DIAGONALIZATION OF THE EL. CAPACITY MATRIX

STEP No. 4

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STEP No. 5

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TILL THE END TIME OR STEADY STATE

$$\underline{C}^e = 250\,000 \left[\begin{array}{c|c} 1/3 & 1/6 \\ \hline 1/6 & 1/3 \end{array}\right] = 250\,000 \left[\begin{array}{c|c} 1/3+1/6 & 0 \\ \hline 0 & 1/3+1/6 \end{array}\right] =$$

DIAGONALIZED MATRIX:

$$\underline{C}^e_{diag.} = 250\,000 \left[\begin{array}{c|c} 1/2 & 0 \\ \hline 0 & 1/2 \end{array}\right]$$