

$$E = 10 \text{ kPa}$$

$$\nu = 0.1$$

$$A = 1 \text{ m}$$

$$f_x = 1 \text{ kN/m}^2$$

CONSTITUTIVE EQ. - PLANE STRESS:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\sigma_x = 1 \text{ kPa} = \frac{10}{1-(0.1)^2} \cdot 1 \cdot a$$

$$\Rightarrow a = \frac{1-(0.1)^2}{10} = 0.099 = \epsilon_x$$

$$\sigma_y = \frac{10}{1-(0.1)^2} \cdot 0.1 \cdot 0.099 = 0.1 \text{ kPa}$$

DEF. SHAPE:



$$u = a \cdot x$$

$$v = \theta$$

KINEMATIC EQUATIONS:

$$\epsilon_x = \frac{\partial u}{\partial x} = a$$

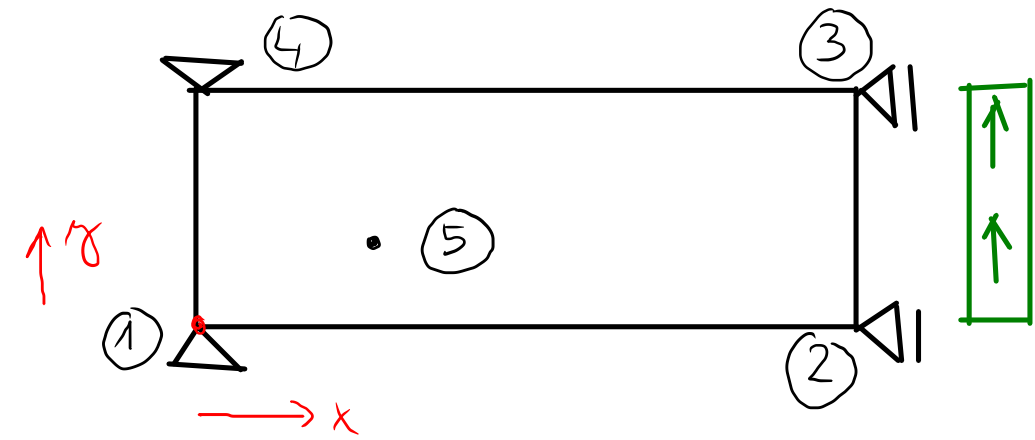
$$\epsilon_y = \frac{\partial v}{\partial y} = \theta$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \theta$$

DISPLACEMENTS:

$$u_2 = \epsilon_x \cdot x_2 = 0.099 \cdot 6 = \underline{0.594 \text{ m}}$$

$$u_5 = \epsilon_x \cdot x_5 = 0.099 \cdot 2 = \underline{0.198 \text{ m}}$$



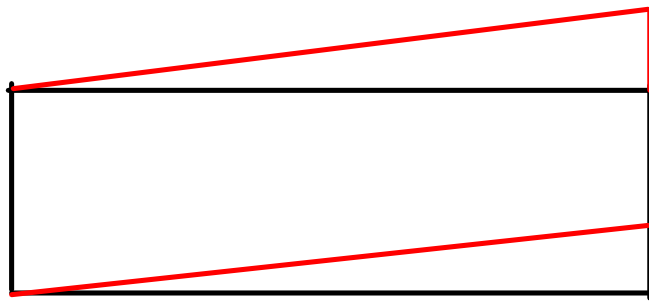
$$q_y = 1 \text{ kN/m}^2$$

$$\tau_{xy} = \frac{E}{1-\nu^2} \cdot \frac{(1-\nu)}{2} \cdot a = 1 \text{ kPa}$$

$$\Rightarrow a = \frac{(1-(0.1)^2)}{10} \cdot \frac{2}{(1-0.1)} = 0.22 = \gamma_{xy}$$

$$\tau_{xy} = \dots 1 \text{ kPa}$$

DEF. SHAPE :



$$u = \theta$$

$$v = a \cdot x$$

$$\epsilon_x = \frac{\partial u}{\partial x} = \theta$$

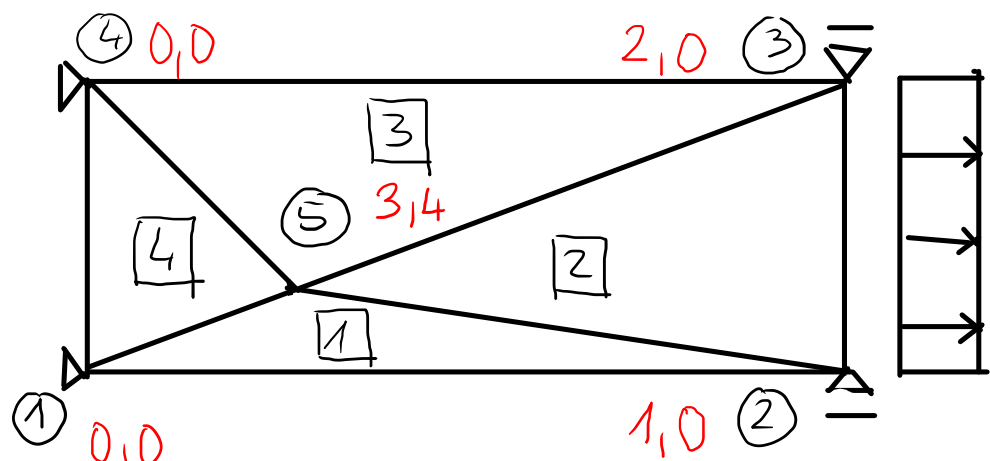
$$\epsilon_y = \frac{\partial v}{\partial y} = \theta$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = a$$

DISPLACEMENTS:

$$v_2 = \gamma_{xy} \cdot x_2 = 0.22 \cdot 6 = \underline{1.32 \text{ m}}$$

$$v_5 = \gamma_{xy} \cdot x_5 = 0.22 \cdot 2 = \underline{0.44 \text{ m}}$$



kč.:

FROM KINEMATIC EQS.:

$$\underline{B}^e = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \theta & \frac{\partial N_2}{\partial x} & \theta & \frac{\partial N_3}{\partial x} & \theta \\ \theta & \frac{\partial N_1}{\partial y} & \theta & \frac{\partial N_2}{\partial y} & \theta & \frac{\partial N_3}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} \end{bmatrix}$$

$$= \begin{bmatrix} \gamma_{23} & 0 & \gamma_{31} & 0 & \gamma_{12} & 0 \\ 0 & \chi_{32} & 0 & \chi_{13} & \theta & \chi_{21} \\ \chi_{32} & \gamma_{23} & \chi_{13} & \gamma_{31} & \chi_{21} & \gamma_{12} \end{bmatrix}$$

u, v

$$\underline{K} \underline{r} = \underline{f}$$

$$\underline{u}^e = \underline{N}^{eT} \underline{r}_n^e$$

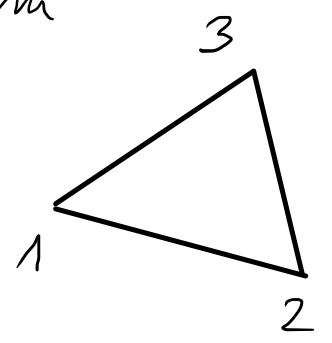
$$\underline{N}^e = \{N_1, N_2, N_3\}$$

$$\underline{v}^e = \underline{N}^{eT} \underline{r}_v^e$$

$$\underline{r}^e = \{u_1, v_1, u_2, v_2, u_3, v_3\}$$

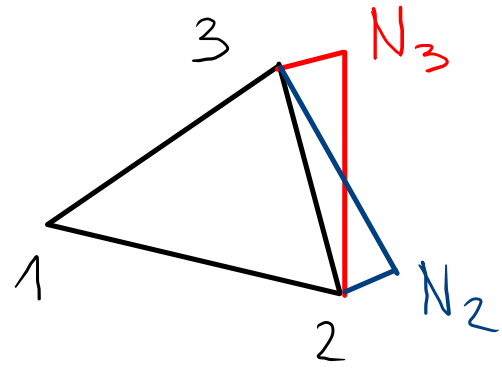
$$\underline{\epsilon}^e = \underline{B}^{eT} \underline{r}^e$$

$$\underline{\sigma}^e = \underline{D}^e \underline{B}^{eT} \underline{r}^e$$



$$\underline{K}^e = \int_{\Omega^e} \underline{B}^{eT} \underline{D}^e \underline{B}^e d\Omega = \underline{B}^{eT} \underline{D}^e \underline{B}^e \int_{\Omega^e} d\Omega = A \cdot \underline{B}^{eT} \underline{D}^e \underline{B}^e \Rightarrow \text{IN MATLAB}$$

RIGHT-HAND SIDE VECTOR:



$$\underline{f}_x^e = \int_{x_2}^{x_3} \begin{Bmatrix} \theta \\ N_2 \\ N_3 \end{Bmatrix} \bar{p}_x ds = \frac{1}{2} l \begin{Bmatrix} 0 \\ \bar{p}_x \\ \bar{p}_x \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1.5 \\ 1.5 \end{Bmatrix} \begin{matrix} 0 \\ 1 \\ 2 \end{matrix}$$

$$\underline{f}_y^e = \int_{x_2}^{x_3} \begin{Bmatrix} \theta \\ N_2 \\ N_3 \end{Bmatrix} \bar{p}_y ds = \dots$$

$$\underline{f}^e = \begin{Bmatrix} 0 \\ 0 \\ 1.5 \\ \theta \\ 1.5 \\ 0 \end{Bmatrix} \begin{matrix} 3 \\ 4 \\ 1 \\ 0 \\ 2 \\ 0 \end{matrix}$$

(CODE NUMBERS)