Numerická analýza transportních procesů - NTP2

Přednáška č. 5

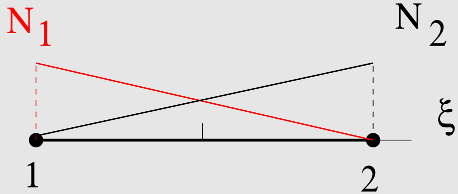
Konečné prvky a aproximačních funkce

(zdroj - mauál programu SIFEL)

Prvky pro 1D úlohy

“Tyčový” prvek s lineárními aproximačními funkcemi (2 uzly)

Aproximační funkce:



$$N_1^{(1)} = \frac{1}{2}(1-\xi) \; , \qquad (1)$$

$$N_2^{(1)} = \frac{1}{2}(1+\xi) \; , \qquad (2)$$

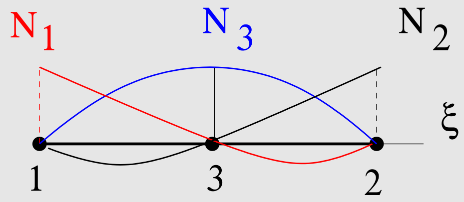
Parciální derivace vhledem k ξ

$$\frac{\partial N_1^{(1)}}{\partial \xi} = -\frac{1}{2} \; , \qquad (3)$$

$$\frac{\partial N_2^{(1)}}{\partial \xi} = \frac{1}{2} \; , \qquad (4)$$

"Tyčový" prvek s kvadratickými aproximačními funkcemi (3 uzly)

Aproximační funkce:



$$N_1^{(1)} = \frac{1}{2}(\xi - 1)\xi \ , \tag{5}$$

$$N_2^{(1)} = \frac{1}{2}(1 + \xi)\xi \ , \tag{6}$$

$$N_3^{(1)} = 1 - \xi^2 \ . \tag{7}$$

Parciální derivace vhledem k ξ

$$\frac{\partial N_1^{(1)}}{\partial \xi} = \xi - \frac{1}{2} \ , \tag{8}$$

$$\frac{\partial N_2^{(1)}}{\partial \xi} = \xi + \frac{1}{2} \ , \tag{9}$$

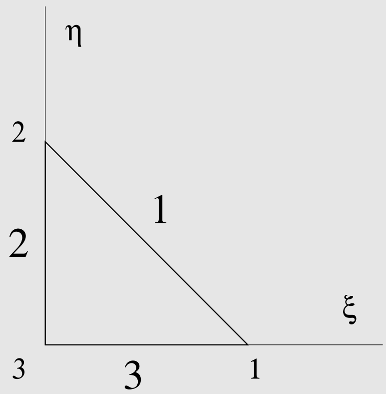
$$\frac{\partial N_3^{(1)}}{\partial \xi} = -2\xi \ . \tag{10}$$

Prvky pro 2D úlohy

Trojúhelníkový prvek s lineárními aproximačními funkcemi (3 uzly)

Číslování uzlů a hran prvku:

hrany	uzly
1	1, 2
2	2, 3
3	3, 1



Trojúhelníkový prvek s lineárními aproximačními funkcemi (3 uzly)

Aproximační funkce:

$$N_1^{(1)} = \xi \; , \tag{11}$$

$$N_2^{(1)} = \eta \; , \tag{12}$$

$$N_3^{(1)} = 1 - \xi - \eta \; . \tag{13}$$

Parciální derivace vhledem k ξ

$$\frac{\partial N_1^{(1)}}{\partial \xi} = 1 \; , \tag{14}$$

$$\frac{\partial N_2^{(1)}}{\partial \xi} = 0 \; , \tag{15}$$

$$\frac{\partial N_3^{(1)}}{\partial \xi} = -1 \; . \tag{16}$$

Parciální derivace podle η

$$\frac{\partial N_1^{(1)}}{\partial \eta} = 0 \; , \tag{17}$$

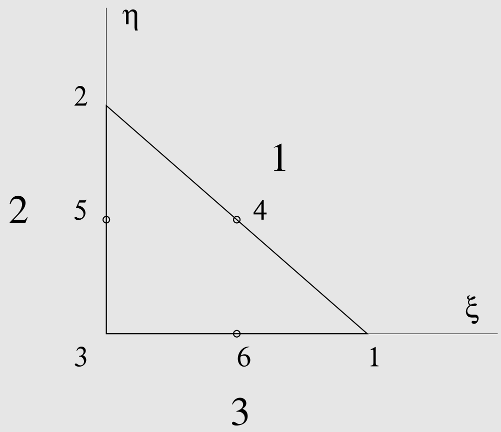
$$\frac{\partial N_2^{(1)}}{\partial \eta} = 1 \; , \tag{18}$$

$$\frac{\partial N_3^{(1)}}{\partial \eta} = -1 \; . \tag{19}$$

Trojúhelníkový prvek s kvadratickými aproximačními funkcemi (6 uzlů)

Číslování uzlů a hran prvku:

hrany	uzly
1	1, 2, 4
2	2, 3, 5
3	3, 1, 6



Trojúhelníkový prvek s kvadratickými aproximačními funkcemi (6 uzlů)

Aproximační funkce:

$$N_1^{(2)} = 2\xi(\xi - 0.5) \; , \tag{20}$$

$$N_2^{(2)} = 2\eta(\eta - 0.5) \; , \tag{21}$$

$$N_3^{(2)} = 2(1 - \xi - \eta)(0.5 - \xi - \eta) \; . \tag{22}$$

$$N_4^{(2)} = 4\xi\eta \; , \tag{23}$$

$$N_5^{(2)} = 4\eta(1 - \xi - \eta) \; , \tag{24}$$

$$N_6^{(2)} = 4\xi(1 - \xi - \eta) \; , \tag{25}$$

Trojúhelníkový prvek s kvadratickými aproximačními funkcemi (6 uzlů)

Parciální derivace vhledem k ξ

$$\frac{\partial N_1^{(2)}}{\partial \xi} = 4\xi - 1 \,, \tag{26}$$

$$\frac{\partial N_2^{(2)}}{\partial \xi} = 0 \,, \tag{27}$$

$$\frac{\partial N_3^{(2)}}{\partial \xi} = 4\xi + 4\eta - 3 \,, \tag{28}$$

$$\frac{\partial N_4^{(2)}}{\partial \xi} = 4\eta \,, \tag{29}$$

$$\frac{\partial N_5^{(2)}}{\partial \xi} = -4\eta \,, \tag{30}$$

$$\frac{\partial N_6^{(2)}}{\partial \xi} = 4 - 8\xi - 4\eta \,. \tag{31}$$

Trojúhelníkový prvek s kvadratickými aproximačními funkcemi (6 uzlů)

Parciální derivace podle η

$$\frac{\partial N_1^{(2)}}{\partial \eta} = 0 \; , \tag{32}$$

$$\frac{\partial N_2^{(2)}}{\partial \eta} = 4\eta - 1 \; , \tag{33}$$

$$\frac{\partial N_3^{(2)}}{\partial \eta} = 4\xi + 4\eta - 3 \; , \tag{34}$$

$$\frac{\partial N_4^{(2)}}{\partial \eta} = 4\xi \; , \tag{35}$$

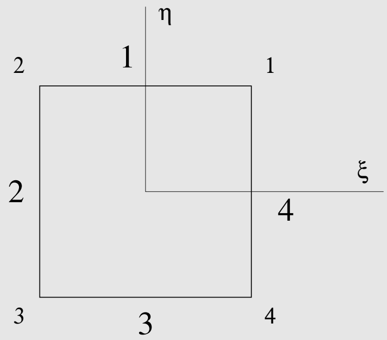
$$\frac{\partial N_5^{(2)}}{\partial \eta} = 4 - 4\xi - 8\eta \; , \tag{36}$$

$$\frac{\partial N_6^{(2)}}{\partial \eta} = -4\xi \; . \tag{37}$$

Čtyřúhelníkový prvek s lineárními aproximačními funkcemi (4 uzly)

Číslování uzlů a hran prvku:

hrany	uzly
1	1, 2
2	2, 3
3	3, 4
4	4, 1



Čtyřúhelníkový prvek s lineárními aproximačními funkcemi (4 uzly)

Bi-lineární aproximační funkce:

$$N_1^{(1)} = \frac{1}{4}(1+\xi)(1+\eta) \ , \tag{38}$$

$$N_2^{(1)} = \frac{1}{4}(1-\xi)(1+\eta) \ , \tag{39}$$

$$N_3^{(1)} = \frac{1}{4}(1-\xi)(1-\eta) \ , \tag{40}$$

$$N_4^{(1)} = \frac{1}{4}(1+\xi)(1-\eta) \ . \tag{41}$$

Parciální derivace podle ξ

$$\frac{\partial N_1^{(1)}}{\partial \xi} = \frac{1}{4}(1+\eta) \ , \tag{42}$$

$$\frac{\partial N_2^{(1)}}{\partial \xi} = -\frac{1}{4}(1+\eta) \ , \tag{43}$$

$$\frac{\partial N_3^{(1)}}{\partial \xi} = -\frac{1}{4}(1-\eta) \ , \tag{44}$$

$$\frac{\partial N_4^{(1)}}{\partial \xi} = \frac{1}{4}(1-\eta) \ . \tag{45}$$

Čtyřúhelníkový prvek s lineárními aproximačními funkcemi (4 uzly)

Parciální derivace podle η

$$\frac{\partial N_1^{(1)}}{\partial \eta} = \frac{1}{4}(1+\xi) \ , \quad (46)$$

$$\frac{\partial N_2^{(1)}}{\partial \eta} = \frac{1}{4}(1-\xi) \ , \quad (47)$$

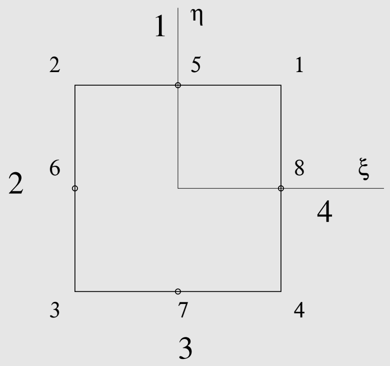
$$\frac{\partial N_3^{(1)}}{\partial \eta} = -\frac{1}{4}(1-\xi) \ , \quad (48)$$

$$\frac{\partial N_4^{(1)}}{\partial \eta} = -\frac{1}{4}(1+\xi) \ . \quad (49)$$

Čtyřúhelníkový prvek s kvadratickými aproximačními funkcemi (8 uzlů)

Číslování uzlů a hran prvku:

hrany	uzly
1	1, 2, 5
2	2, 3, 6
3	3, 4, 7
4	4, 1, 8



Čtyřúhelníkový prvek s kvadratickými aproximačními funkcemi (8 uzlů)

Bi-kvadratické aproximační funkce:

$$N_1^{(2)} = \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1)\ , \tag{50}$$

$$N_2^{(2)} = \frac{1}{4}(1-\xi)(1+\eta)(-\xi+\eta-1)\ , \tag{51}$$

$$N_3^{(2)} = \frac{1}{4}(1-\xi)(1-\eta)(-\xi-\eta-1)\ , \tag{52}$$

$$N_4^{(2)} = \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1)\ , \tag{53}$$

$$N_5^{(2)} = \frac{1}{2}(1-\xi^2)(1+\eta)\ , \tag{54}$$

$$N_6^{(2)} = \frac{1}{2}(1-\xi)(1-\eta^2)\ , \tag{55}$$

$$N_7^{(2)} = \frac{1}{2}(1-\xi^2)(1-\eta)\ , \tag{56}$$

$$N_8^{(2)} = \frac{1}{2}(1+\xi)(1-\eta^2)\ . \tag{57}$$

Čtyřúhelníkový prvek s kvadratickými aproximačními funkcemi (8 uzlů)

Parciální derivace podle ξ

$$\frac{\partial N_1^{(2)}}{\partial \xi} = \frac{1}{4}(1+\eta)(\xi+\eta-1) + \frac{1}{4}(1+\xi)(1+\eta) \ , \quad (58)$$

$$\frac{\partial N_2^{(2)}}{\partial \xi} = -\frac{1}{4}(1+\eta)(-\xi+\eta-1) - \frac{1}{4}(1-\xi)(1+\eta) \ , \quad (59)$$

$$\frac{\partial N_3^{(2)}}{\partial \xi} = -\frac{1}{4}(1-\eta)(-\xi-\eta-1) - \frac{1}{4}(1-\xi)(1-\eta) \ , \quad (60)$$

$$\frac{\partial N_4^{(2)}}{\partial \xi} = \frac{1}{4}(1-\eta)(\xi-\eta-1) + \frac{1}{4}(1+\xi)(1-\eta) \ , \quad (61)$$

$$\frac{\partial N_5^{(2)}}{\partial \xi} = -\xi(1+\eta) \ , \quad (62)$$

$$\frac{\partial N_6^{(2)}}{\partial \xi} = -\frac{1}{2}(1-\eta^2) \ , \quad (63)$$

$$\frac{\partial N_7^{(2)}}{\partial \xi} = -\xi(1-\eta) \ , \quad (64)$$

$$\frac{\partial N_8^{(2)}}{\partial \xi} = \frac{1}{2}(1-\eta^2) \ . \quad (65)$$

Čtyřúhelníkový prvek s kvadratickými aproximačními funkcemi (8 uzlů)

Parciální derivace podle η

$$\frac{\partial N_1^{(2)}}{\partial \eta} = \frac{1}{4}(1+\xi)(\xi+\eta-1)+\frac{1}{4}(1+\xi)(1+\eta)\ , \tag{66}$$

$$\frac{\partial N_2^{(2)}}{\partial \eta} = \frac{1}{4}(1-\xi)(-\xi+\eta-1)+\frac{1}{4}(1-\xi)(1+\eta)\ , \tag{67}$$

$$\frac{\partial N_3^{(2)}}{\partial \eta} = -\frac{1}{4}(1-\xi)(-\xi-\eta-1)-\frac{1}{4}(1-\xi)(1-\eta)\ , \tag{68}$$

$$\frac{\partial N_4^{(2)}}{\partial \eta} = -\frac{1}{4}(1+\xi)(\xi-\eta-1)-\frac{1}{4}(1+\xi)(1-\eta)\ , \tag{69}$$

$$\frac{\partial N_5^{(2)}}{\partial \eta} = \frac{1}{2}(1-\xi^2)\ , \tag{70}$$

$$\frac{\partial N_6^{(2)}}{\partial \eta} = (1-\xi)(-\eta)\ , \tag{71}$$

$$\frac{\partial N_7^{(2)}}{\partial \eta} = -\frac{1}{2}(1-\xi^2)\ , \tag{72}$$

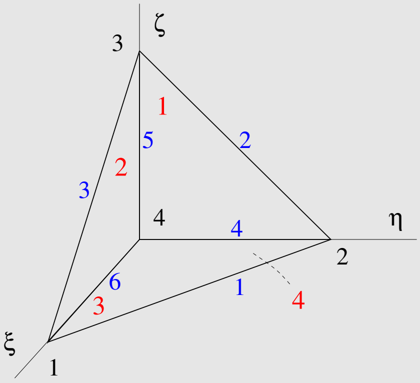
$$\frac{\partial N_8^{(2)}}{\partial \eta} = (1+\xi)(-\eta)\ . \tag{73}$$

Prvky pro 3D úlohy

Čtyřstěn s lineárními aproximačními funkcemi (4 uzly)

Číslování uzlů a ploch prvku:

plocha	uzly
1	3, 2, 4
2	1, 3, 4
3	2, 1, 4
4	1, 2, 3



Čtyřstěn s lineárními aproximačními funkcemi (4 uzly)

Aproximační funkce:

$$N_1^{(1)} = \xi \; , \tag{74}$$

$$N_2^{(1)} = \eta \; , \tag{75}$$

$$N_3^{(1)} = \zeta \; , \tag{76}$$

$$N_4^{(1)} = 1 - \xi - \eta - \zeta \; . \tag{77}$$

Parciální derivace vhledem k ξ

$$\frac{\partial N_1^{(1)}}{\partial \xi} = 1 \; , \tag{78}$$

$$\frac{\partial N_2^{(1)}}{\partial \xi} = 0 \; , \tag{79}$$

$$\frac{\partial N_3^{(1)}}{\partial \xi} = 0 \; . \tag{80}$$

$$\frac{\partial N_4^{(1)}}{\partial \xi} = -1 \; . \tag{81}$$

Čtyřstěn s lineárními aproximačními funkcemi (4 uzly)

Parciální derivace vhledem k η

$$\frac{\partial N_1^{(1)}}{\partial \eta} = 0 \,, \tag{82}$$

$$\frac{\partial N_2^{(1)}}{\partial \eta} = 1 \,, \tag{83}$$

$$\frac{\partial N_3^{(1)}}{\partial \eta} = 0 \,. \tag{84}$$

$$\frac{\partial N_4^{(1)}}{\partial \eta} = -1 \,. \tag{85}$$

Parciální derivace podle ζ

$$\frac{\partial N_1^{(1)}}{\partial \zeta} = 0 \,, \tag{86}$$

$$\frac{\partial N_2^{(1)}}{\partial \zeta} = 0 \,, \tag{87}$$

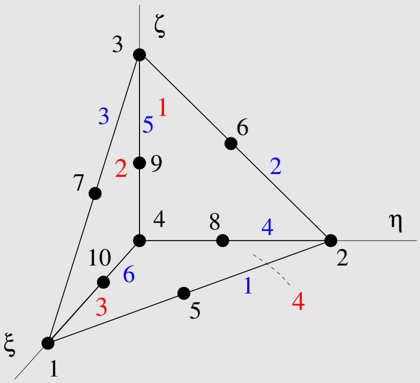
$$\frac{\partial N_3^{(1)}}{\partial \zeta} = 1 \,. \tag{88}$$

$$\frac{\partial N_4^{(1)}}{\partial \zeta} = -1 \,. \tag{89}$$

Čtyřstěn s kvadratickými aproximačními funkcemi (10 uzlů)

Číslování uzlů a ploch prvku:

plocha	uzly
1	3, 2, 4, 6, 8, 9
2	1, 3, 4, 7, 9, 10
3	2, 1, 4, 5, 10, 8
4	1, 2, 3, 5, 6, 7



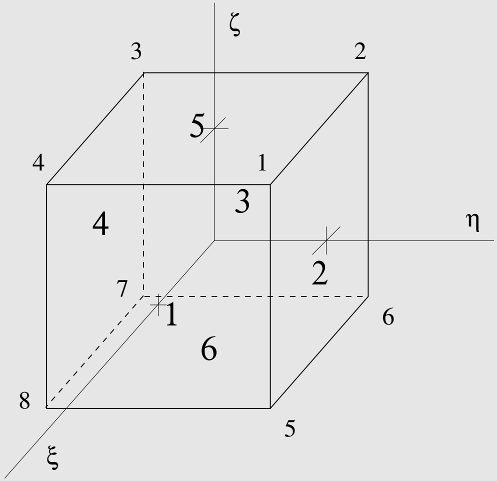
Šestistěn s lineárními aproximačními funkcemi (8 uzlů)

Číslování uzlů a ploch prvku:

plocha	uzly
1	1, 4, 8, 5
2	2, 1, 5, 6
3	3, 2, 6, 7
4	4, 3, 7, 8
5	1, 2, 3, 4
6	5, 6, 7, 8

Šestistěn s lineárními aproximačními funkcemi (8 uzlů)

Číslování uzlů a ploch prvku:



Šestistěn s lineárními aproximačními funkcemi (8 uzlů)

Tri-lineární aproximační funkce:

$$N_1^{(1)} = \frac{1}{8}(1+\xi)(1+\eta)(1+\zeta) \; , \quad (90)$$

$$N_2^{(1)} = \frac{1}{8}(1-\xi)(1+\eta)(1+\zeta) \; , \quad (91)$$

$$N_3^{(1)} = \frac{1}{8}(1-\xi)(1-\eta)(1+\zeta) \; , \quad (92)$$

$$N_4^{(1)} = \frac{1}{8}(1+\xi)(1-\eta)(1+\zeta) \; , \quad (93)$$

$$N_5^{(1)} = \frac{1}{8}(1+\xi)(1+\eta)(1-\zeta) \; , \quad (94)$$

$$N_6^{(1)} = \frac{1}{8}(1-\xi)(1+\eta)(1-\zeta) \; , \quad (95)$$

$$N_7^{(1)} = \frac{1}{8}(1-\xi)(1-\eta)(1-\zeta) \; , \quad (96)$$

$$N_8^{(1)} = \frac{1}{8}(1+\xi)(1-\eta)(1-\zeta) \; . \quad (97)$$

Šestistěn s lineárními aproximačními funkcemi (8 uzlů)

Parciální derivace podle ξ

$$\frac{\partial N_1^{(1)}}{\partial \xi} = \frac{1}{8}(1+\eta)(1+\zeta)\ , \quad (98)$$

$$\frac{\partial N_2^{(1)}}{\partial \xi} = -\frac{1}{8}(1+\eta)(1+\zeta)\ , \quad (99)$$

$$\frac{\partial N_3^{(1)}}{\partial \xi} = -\frac{1}{8}(1-\eta)(1+\zeta)\ , \quad (100)$$

$$\frac{\partial N_4^{(1)}}{\partial \xi} = \frac{1}{8}(1-\eta)(1+\zeta)\ , \quad (101)$$

$$\frac{\partial N_5^{(1)}}{\partial \xi} = \frac{1}{8}(1+\eta)(1-\zeta)\ , \quad (102)$$

$$\frac{\partial N_6^{(1)}}{\partial \xi} = -\frac{1}{8}(1+\eta)(1-\zeta)\ , \quad (103)$$

$$\frac{\partial N_7^{(1)}}{\partial \xi} = -\frac{1}{8}(1-\eta)(1-\zeta)\ , \quad (104)$$

$$\frac{\partial N_8^{(1)}}{\partial \xi} = \frac{1}{8}(1-\eta)(1-\zeta)\ . \quad (105)$$

Šestistěn s lineárními aproximačními funkcemi (8 uzlů)

Parciální derivace podle η

$$\frac{\partial N_1^{(1)}}{\partial \eta} = \frac{1}{8}(1+\xi)(1+\zeta) \ , \tag{106}$$

$$\frac{\partial N_2^{(1)}}{\partial \eta} = \frac{1}{8}(1-\xi)(1+\zeta) \ , \tag{107}$$

$$\frac{\partial N_3^{(1)}}{\partial \eta} = -\frac{1}{8}(1-\xi)(1+\zeta) \ , \tag{108}$$

$$\frac{\partial N_4^{(1)}}{\partial \eta} = -\frac{1}{8}(1+\xi)(1+\zeta) \ , \tag{109}$$

$$\frac{\partial N_5^{(1)}}{\partial \eta} = \frac{1}{8}(1+\xi)(1-\zeta) \ , \tag{110}$$

$$\frac{\partial N_6^{(1)}}{\partial \eta} = \frac{1}{8}(1-\xi)(1-\zeta) \ , \tag{111}$$

$$\frac{\partial N_7^{(1)}}{\partial \eta} = -\frac{1}{8}(1-\xi)(1-\zeta) \ , \tag{112}$$

$$\frac{\partial N_8^{(1)}}{\partial \eta} = -\frac{1}{8}(1+\xi)(1-\zeta) \ . \tag{113}$$

Šestistěn s lineárními aproximačními funkcemi (8 uzlů)

Parciální derivace podle ζ

$$\frac{\partial N_1^{(1)}}{\partial \zeta} = \frac{1}{8}(1+\xi)(1+\eta) \ , \tag{114}$$

$$\frac{\partial N_2^{(1)}}{\partial \zeta} = \frac{1}{8}(1-\xi)(1+\eta) \ , \tag{115}$$

$$\frac{\partial N_3^{(1)}}{\partial \zeta} = \frac{1}{8}(1-\xi)(1-\eta) \ , \tag{116}$$

$$\frac{\partial N_4^{(1)}}{\partial \zeta} = \frac{1}{8}(1+\xi)(1-\eta) \ , \tag{117}$$

$$\frac{\partial N_5^{(1)}}{\partial \zeta} = -\frac{1}{8}(1+\xi)(1+\eta) \ , \tag{118}$$

$$\frac{\partial N_6^{(1)}}{\partial \zeta} = -\frac{1}{8}(1-\xi)(1+\eta) \ , \tag{119}$$

$$\frac{\partial N_7^{(1)}}{\partial \zeta} = -\frac{1}{8}(1-\xi)(1-\eta) \ , \tag{120}$$

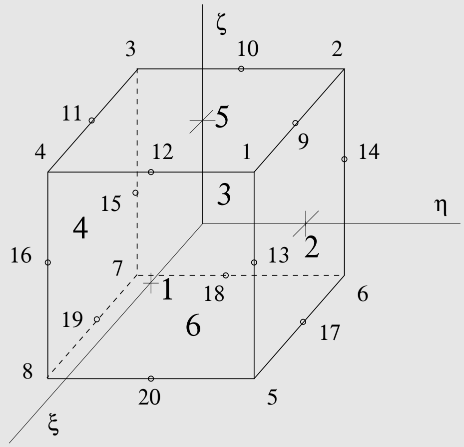
$$\frac{\partial N_8^{(1)}}{\partial \zeta} = -\frac{1}{8}(1+\xi)(1-\eta) \ . \tag{121}$$

Šestistěn s kvadratickými aproximačními funkcemi (20 uzlů)

Číslování uzlů a ploch prvku:

plocha	uzly
1	1, 4, 8, 5, 12, 16, 20, 13
2	2, 1, 5, 6, 9, 13, 17, 14
3	3, 2, 6, 7, 10, 14, 18, 15
4	4, 3, 7, 8, 11, 15, 19, 16
5	1, 2, 3, 4, 9, 10, 11, 12
6	5, 6, 7, 8, 17, 18, 19, 20

Šestistěn s kvadratickými aproximačními funkcemi (20 uzlů)



Šestistěn s kvadratickými aproximačními funkcemi (20 uzlů)

Tri-kvadratické aproximační funkce:

$$N_1^{(2)} = \frac{1}{8}(1+\xi)(1+\eta)(1+\zeta)(\xi+\eta+\zeta-2) \; , \tag{122}$$

$$N_2^{(2)} = \frac{1}{8}(1-\xi)(1+\eta)(1+\zeta)(-\xi+\eta+\zeta-2) \; , \tag{123}$$

$$N_3^{(2)} = \frac{1}{8}(1-\xi)(1-\eta)(1+\zeta)(-\xi-\eta+\zeta-2) \; , \tag{124}$$

$$N_4^{(2)} = \frac{1}{8}(1+\xi)(1-\eta)(1+\zeta)(\xi-\eta+\zeta-2) \; , \tag{125}$$

$$N_5^{(2)} = \frac{1}{8}(1+\xi)(1+\eta)(1-\zeta)(\xi+\eta-\zeta-2) \; , \tag{126}$$

$$N_6^{(2)} = \frac{1}{8}(1-\xi)(1+\eta)(1-\zeta)(-\xi+\eta-\zeta-2) \; , \tag{127}$$

$$N_7^{(2)} = \frac{1}{8}(1-\xi)(1-\eta)(1-\zeta)(-\xi-\eta-\zeta-2) \; , \tag{128}$$

$$N_8^{(2)} = \frac{1}{8}(1+\xi)(1-\eta)(1-\zeta)(\xi-\eta-\zeta-2) \; , \tag{129}$$

$$N_9^{(2)} = \frac{1}{4}(1-\xi^2)(1+\eta)(1+\zeta) \; , \tag{130}$$

$$N_{10}^{(2)} = \frac{1}{4}(1-\xi)(1-\eta^2)(1+\zeta) \; , \tag{131}$$

$$N_{11}^{(2)} = \frac{1}{4}(1-\xi^2)(1-\eta)(1+\zeta) \; , \tag{132}$$

$$N_{12}^{(2)} = \frac{1}{4}(1+\xi)(1-\eta^2)(1+\zeta) \ , \tag{133}$$

$$N_{13}^{(2)} = \frac{1}{4}(1+\xi)(1+\eta)(1-\zeta^2) \ , \tag{134}$$

$$N_{14}^{(2)} = \frac{1}{4}(1-\xi)(1+\eta)(1-\zeta^2) \ , \tag{135}$$

$$N_{15}^{(2)} = \frac{1}{4}(1-\xi)(1-\eta)(1-\zeta^2) \ , \tag{136}$$

$$N_{16}^{(2)} = \frac{1}{4}(1+\xi)(1-\eta)(1-\zeta^2) \ , \tag{137}$$

$$N_{17}^{(2)} = \frac{1}{4}(1-\xi^2)(1+\eta)(1-\zeta) \ , \tag{138}$$

$$N_{18}^{(2)} = \frac{1}{4}(1-\xi)(1-\eta^2)(1-\zeta) \ , \tag{139}$$

$$N_{19}^{(2)} = \frac{1}{4}(1-\xi^2)(1-\eta)(1-\zeta) \ , \tag{140}$$

$$N_{20}^{(2)} = \frac{1}{4}(1+\xi)(1-\eta^2)(1-\zeta) \ . \tag{141}$$

Šestistěn s kvadratickými aproximačními funkcemi (20 uzlů)

Parciální derivace podle ξ

$$\frac{\partial N_1^{(2)}}{\partial \xi} = \frac{1}{8}(1+\eta)(1+\zeta)(\xi+\eta+\zeta-2)+\frac{1}{8}(1+\xi)(1+\eta)(1+\zeta)\ , \quad (142)$$

$$\frac{\partial N_2^{(2)}}{\partial \xi} = -\frac{1}{8}(1+\eta)(1+\zeta)(-\xi+\eta+\zeta-2)-\frac{1}{8}(1-\xi)(1+\eta)(1+\zeta)\ , \quad (143)$$

$$\frac{\partial N_3^{(2)}}{\partial \xi} = -\frac{1}{8}(1-\eta)(1+\zeta)(-\xi-\eta+\zeta-2)-\frac{1}{8}(1-\xi)(1-\eta)(1+\zeta)\ , \quad (144)$$

$$\frac{\partial N_4^{(2)}}{\partial \xi} = \frac{1}{8}(1-\eta)(1+\zeta)(\xi-\eta+\zeta-2)+\frac{1}{8}(1+\xi)(1-\eta)(1+\zeta)\ , \quad (145)$$

$$\frac{\partial N_5^{(2)}}{\partial \xi} = \frac{1}{8}(1+\eta)(1-\zeta)(\xi+\eta-\zeta-2)+\frac{1}{8}(1+\xi)(1+\eta)(1-\zeta)\ , \quad (146)$$

$$\frac{\partial N_6^{(2)}}{\partial \xi} = -\frac{1}{8}(1+\eta)(1-\zeta)(-\xi+\eta-\zeta-2)-\frac{1}{8}(1-\xi)(1+\eta)(1-\zeta)\ , \quad (147)$$

$$\frac{\partial N_7^{(2)}}{\partial \xi} = -\frac{1}{8}(1-\eta)(1-\zeta)(-\xi-\eta-\zeta-2)-\frac{1}{8}(1-\xi)(1-\eta)(1-\zeta)\ , \quad (148)$$

$$\frac{\partial N_8^{(2)}}{\partial \xi} = \frac{1}{8}(1-\eta)(1-\zeta)(\xi-\eta-\zeta-2)+\frac{1}{8}(1+\xi)(1-\eta)(1-\zeta)\ , \quad (149)$$

$$\frac{\partial N_9^{(2)}}{\partial \xi} = -\frac{1}{2}\xi(1+\eta)(1+\zeta) \ , \quad (150)$$

$$\frac{\partial N_{10}^{(2)}}{\partial \xi} = -\frac{1}{4}(1-\eta^2)(1+\zeta) \ , \quad (151)$$

$$\frac{\partial N_{11}^{(2)}}{\partial \xi} = -\frac{1}{2}\xi(1-\eta)(1+\zeta) \ , \quad (152)$$

$$\frac{\partial N_{12}^{(2)}}{\partial \xi} = \frac{1}{4}(1-\eta^2)(1+\zeta) \ , \quad (153)$$

$$\frac{\partial N_{13}^{(2)}}{\partial \xi} = \frac{1}{4}(1+\eta)(1-\zeta^2) \ , \quad (154)$$

$$\frac{\partial N_{14}^{(2)}}{\partial \xi} = -\frac{1}{4}(1+\eta)(1-\zeta^2) \ , \quad (155)$$

$$\frac{\partial N_{15}^{(2)}}{\partial \xi} = -\frac{1}{4}(1-\eta)(1-\zeta^2) \ , \quad (156)$$

$$\frac{\partial N_{16}^{(2)}}{\partial \xi} = \frac{1}{4}(1-\eta)(1-\zeta^2) \ , \quad (157)$$

$$\frac{\partial N_{17}^{(2)}}{\partial \xi} = -\frac{1}{2}\xi(1+\eta)(1-\zeta) \ , \quad (158)$$

$$\frac{\partial N_{18}^{(2)}}{\partial \xi} = -\frac{1}{4}(1-\eta^2)(1-\zeta) \ , \quad (159)$$

$$\frac{\partial N_{19}^{(2)}}{\partial \xi} = -\frac{1}{2}\xi(1-\eta)(1-\zeta) \; , \tag{160}$$

$$\frac{\partial N_{20}^{(2)}}{\partial \xi} = \frac{1}{4}(1-\eta^2)(1-\zeta) \; . \tag{161}$$

Šestistěn s kvadratickými aproximačními funkcemi (20 uzlů)

Parciální derivace podle η

$$\frac{\partial N_1^{(2)}}{\partial \eta} = \frac{1}{8}(1+\xi)(1+\zeta)(\xi+\eta+\zeta-2) + \frac{1}{8}(1+\xi)(1+\eta)(1+\zeta) \ , \tag{162}$$

$$\frac{\partial N_2^{(2)}}{\partial \eta} = \frac{1}{8}(1-\xi)(1+\zeta)(-\xi+\eta+\zeta-2) + \frac{1}{8}(1-\xi)(1+\eta)(1+\zeta) \ , \tag{163}$$

$$\frac{\partial N_3^{(2)}}{\partial \eta} = -\frac{1}{8}(1-\xi)(1+\zeta)(-\xi-\eta+\zeta-2) - \frac{1}{8}(1-\xi)(1-\eta)(1+\zeta) \ , \tag{164}$$

$$\frac{\partial N_4^{(2)}}{\partial \eta} = -\frac{1}{8}(1+\xi)(1+\zeta)(\xi-\eta+\zeta-2) - \frac{1}{8}(1+\xi)(1-\eta)(1+\zeta) \ , \tag{165}$$

$$\frac{\partial N_5^{(2)}}{\partial \eta} = \frac{1}{8}(1+\xi)(1-\zeta)(\xi+\eta-\zeta-2) + \frac{1}{8}(1+\xi)(1+\eta)(1-\zeta) \ , \tag{166}$$

$$\frac{\partial N_6^{(2)}}{\partial \eta} = \frac{1}{8}(1-\xi)(1-\zeta)(-\xi+\eta-\zeta-2) + \frac{1}{8}(1-\xi)(1+\eta)(1-\zeta) \ , \tag{167}$$

$$\frac{\partial N_7^{(2)}}{\partial \eta} = -\frac{1}{8}(1-\xi)(1-\zeta)(-\xi-\eta-\zeta-2) - \frac{1}{8}(1-\xi)(1-\eta)(1-\zeta) \ , \tag{168}$$

$$\frac{\partial N_8^{(2)}}{\partial \eta} = -\frac{1}{8}(1+\xi)(1-\zeta)(\xi-\eta-\zeta-2) - \frac{1}{8}(1+\xi)(1-\eta)(1-\zeta) \ , \tag{169}$$

$$\frac{\partial N_9^{(2)}}{\partial \eta} = \frac{1}{4}(1-\xi^2)(1+\zeta) \ , \tag{170}$$

$$\frac{\partial N_{10}^{(2)}}{\partial \eta} = -\frac{1}{2}(1-\xi)\eta(1+\zeta) \ , \tag{171}$$

$$\frac{\partial N_{11}^{(2)}}{\partial \eta} = -\frac{1}{4}(1-\xi^2)(1+\zeta) \ , \tag{172}$$

$$\frac{\partial N_{12}^{(2)}}{\partial \eta} = -\frac{1}{2}(1+\xi)\eta(1+\zeta) \ , \tag{173}$$

$$\frac{\partial N_{13}^{(2)}}{\partial \eta} = \frac{1}{4}(1+\xi)(1-\zeta^2) \ , \tag{174}$$

$$\frac{\partial N_{14}^{(2)}}{\partial \eta} = \frac{1}{4}(1-\xi)(1-\zeta^2) \ , \tag{175}$$

$$\frac{\partial N_{15}^{(2)}}{\partial \eta} = -\frac{1}{4}(1-\xi)(1-\zeta^2) \ , \tag{176}$$

$$\frac{\partial N_{16}^{(2)}}{\partial \eta} = -\frac{1}{4}(1+\xi)(1-\zeta^2) \ , \tag{177}$$

$$\frac{\partial N_{17}^{(2)}}{\partial \eta} = \frac{1}{4}(1-\xi^2)(1-\zeta) \ , \tag{178}$$

$$\frac{\partial N_{18}^{(2)}}{\partial \eta} = -\frac{1}{2}(1-\xi)\eta(1-\zeta) \ , \tag{179}$$

$$\frac{\partial N_{19}^{(2)}}{\partial \eta} = -\frac{1}{4}(1-\xi^2)(1-\zeta) \ , \tag{180}$$

$$\frac{\partial N_{20}^{(2)}}{\partial \eta} = -\frac{1}{2}(1+\xi)\eta(1-\zeta) \ . \tag{181}$$

Šestistěn s kvadratickými aproximačními funkcemi (20 uzlů)

Parciální derivace podle ζ

$$\frac{\partial N_1^{(2)}}{\partial \zeta} = \frac{1}{8}(1+\xi)(1+\eta)(\xi+\eta+\zeta-2)+\frac{1}{8}(1+\xi)(1+\eta)(1+\zeta)\ , \tag{182}$$

$$\frac{\partial N_2^{(2)}}{\partial \zeta} = \frac{1}{8}(1-\xi)(1+\eta)(-\xi+\eta+\zeta-2)+\frac{1}{8}(1-\xi)(1+\eta)(1+\zeta)\ , \tag{183}$$

$$\frac{\partial N_3^{(2)}}{\partial \zeta} = \frac{1}{8}(1-\xi)(1-\eta)(-\xi-\eta+\zeta-2)+\frac{1}{8}(1-\xi)(1-\eta)(1+\zeta)\ , \tag{184}$$

$$\frac{\partial N_4^{(2)}}{\partial \zeta} = \frac{1}{8}(1+\xi)(1-\eta)(\xi-\eta+\zeta-2)+\frac{1}{8}(1+\xi)(1-\eta)(1+\zeta)\ , \tag{185}$$

$$\frac{\partial N_5^{(2)}}{\partial \zeta} = -\frac{1}{8}(1+\xi)(1+\eta)(\xi+\eta-\zeta-2)-\frac{1}{8}(1+\xi)(1+\eta)(1-\zeta)\ , \tag{186}$$

$$\frac{\partial N_6^{(2)}}{\partial \zeta} = -\frac{1}{8}(1-\xi)(1+\eta)(-\xi+\eta-\zeta-2)-\frac{1}{8}(1-\xi)(1+\eta)(1-\zeta)\ , \tag{187}$$

$$\frac{\partial N_7^{(2)}}{\partial \zeta} = -\frac{1}{8}(1-\xi)(1-\eta)(-\xi-\eta-\zeta-2)-\frac{1}{8}(1-\xi)(1-\eta)(1-\zeta)\ , \tag{188}$$

$$\frac{\partial N_8^{(2)}}{\partial \zeta} = -\frac{1}{8}(1+\xi)(1-\eta)(\xi-\eta-\zeta-2)-\frac{1}{8}(1+\xi)(1-\eta)(1-\zeta)\ , \tag{189}$$

$$\frac{\partial N_9^{(2)}}{\partial \zeta} = \frac{1}{4}(1-\xi^2)(1+\eta) \ , \tag{190}$$

$$\frac{\partial N_{10}^{(2)}}{\partial \zeta} = \frac{1}{4}(1-\xi)(1-\eta^2) \ , \tag{191}$$

$$\frac{\partial N_{11}^{(2)}}{\partial \zeta} = \frac{1}{4}(1-\xi^2)(1-\eta) \ , \tag{192}$$

$$\frac{\partial N_{12}^{(2)}}{\partial \zeta} = \frac{1}{4}(1+\xi)(1-\eta^2) \ , \tag{193}$$

$$\frac{\partial N_{13}^{(2)}}{\partial \zeta} = -\frac{1}{2}(1+\xi)(1+\eta)\zeta \ , \tag{194}$$

$$\frac{\partial N_{14}^{(2)}}{\partial \zeta} = -\frac{1}{2}(1-\xi)(1+\eta)\zeta \ , \tag{195}$$

$$\frac{\partial N_{15}^{(2)}}{\partial \zeta} = -\frac{1}{2}(1-\xi)(1-\eta)\zeta \ , \tag{196}$$

$$\frac{\partial N_{16}^{(2)}}{\partial \zeta} = -\frac{1}{2}(1+\xi)(1-\eta)\zeta \ , \tag{197}$$

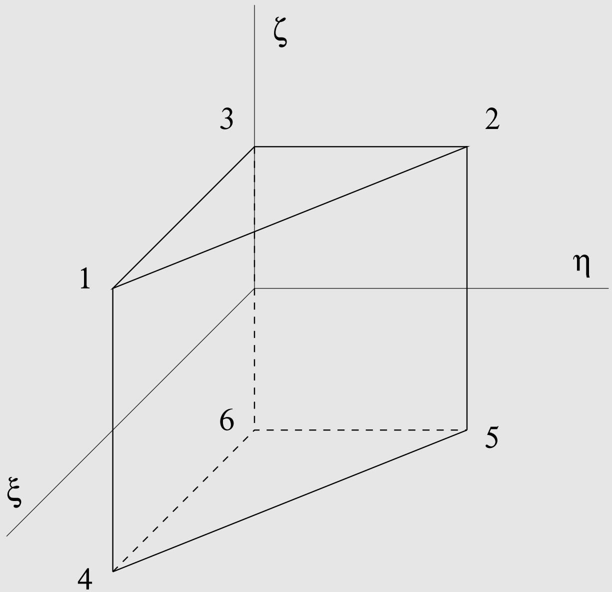
$$\frac{\partial N_{17}^{(2)}}{\partial \zeta} = -\frac{1}{4}(1-\xi^2)(1+\eta) \ , \tag{198}$$

$$\frac{\partial N_{18}^{(2)}}{\partial \zeta} = -\frac{1}{4}(1-\xi)(1-\eta^2) \ , \tag{199}$$

$$\frac{\partial N_{19}^{(2)}}{\partial \zeta} = -\frac{1}{4}(1-\xi^2)(1-\eta)\ , \tag{200}$$

$$\frac{\partial N_{20}^{(2)}}{\partial \zeta} = -\frac{1}{4}(1+\xi)(1-\eta^2)\ . \tag{201}$$

Klín s lineárními aproximačními funkcemi (6 uzlů)



Klín s lineárními aproximačními funkcemi (6 uzlů)

Aproximační funkce:

$$N_1^{(1)} = \frac{1}{2}\xi(1+\zeta)\ , \tag{202}$$

$$N_2^{(1)} = \frac{1}{2}\eta(1+\zeta)\ , \tag{203}$$

$$N_3^{(1)} = \frac{1}{2}(1-\xi-\eta)(1+\zeta)\ , \tag{204}$$

$$N_4^{(1)} = \frac{1}{2}\xi(1-\zeta)\ , \tag{205}$$

$$N_5^{(1)} = \frac{1}{2}\eta(1-\zeta)\ , \tag{206}$$

$$N_6^{(1)} = \frac{1}{2}(1-\xi-\eta)(1-\zeta)\ , \tag{207}$$

Klín s lineárními aproximačními funkcemi (6 uzlů)

Parciální derivace podle ξ

$$\frac{\partial N_1^{(1)}}{\partial \xi} = \frac{1}{2}(1+\zeta) \; , \tag{208}$$

$$\frac{\partial N_2^{(1)}}{\partial \xi} = 0 \; , \tag{209}$$

$$\frac{\partial N_3^{(1)}}{\partial \xi} = -\frac{1}{2}(1+\zeta) \; , \tag{210}$$

$$\frac{\partial N_4^{(1)}}{\partial \xi} = \frac{1}{2}(1-\zeta) \; , \tag{211}$$

$$\frac{\partial N_5^{(1)}}{\partial \xi} = 0 \; , \tag{212}$$

$$\frac{\partial N_6^{(1)}}{\partial \xi} = -\frac{1}{2}(1-\zeta) \; , \tag{213}$$

Klín s lineárními aproximačními funkcemi (6 uzlů)

Parciální derivace podle η

$$\frac{\partial N_1^{(1)}}{\partial \eta} = 0 \;, \tag{214}$$

$$\frac{\partial N_2^{(1)}}{\partial \eta} = \frac{1}{2}(1+\zeta) \;, \tag{215}$$

$$\frac{\partial N_3^{(1)}}{\partial \eta} = -\frac{1}{2}(1+\zeta) \;, \tag{216}$$

$$\frac{\partial N_4^{(1)}}{\partial \eta} = 0 \;, \tag{217}$$

$$\frac{\partial N_5^{(1)}}{\partial \eta} = \frac{1}{2}(1-\zeta) \;, \tag{218}$$

$$\frac{\partial N_6^{(1)}}{\partial \eta} = -\frac{1}{2}(1-\zeta) \;, \tag{219}$$

Klín s lineárními aproximačními funkcemi (6 uzlů)

Parciální derivace podle ζ

$$\frac{\partial N_1^{(1)}}{\partial \zeta} = \frac{\xi}{2} \, , \tag{220}$$

$$\frac{\partial N_2^{(1)}}{\partial \zeta} = \frac{\eta}{2} \, , \tag{221}$$

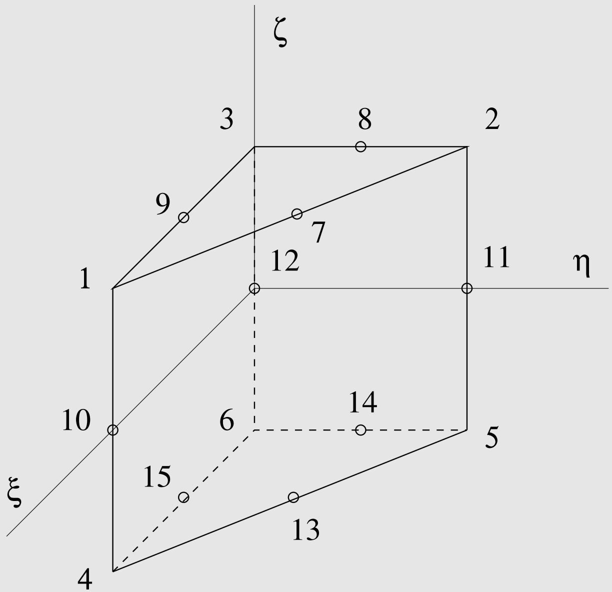
$$\frac{\partial N_3^{(1)}}{\partial \zeta} = \frac{1}{2}(1 - \xi - \eta) \, , \tag{222}$$

$$\frac{\partial N_4^{(1)}}{\partial \zeta} = -\frac{\xi}{2} \, , \tag{223}$$

$$\frac{\partial N_5^{(1)}}{\partial \zeta} = -\frac{\eta}{2} \, , \tag{224}$$

$$\frac{\partial N_6^{(1)}}{\partial \zeta} = -\frac{1}{2}(1 - \xi - \eta) \, , \tag{225}$$

Klín s kvadratickými aproximačními funkcemi
(15 uzlů)



Klín s kvadratickými aproximačními funkcemi (15 uzlů)

Aproximační funkce:

$$
\begin{aligned}
N_1^{(2)} &= \xi(\xi - 0.5)(1+\zeta)\zeta \ , && (226)\\
N_2^{(2)} &= \eta(\eta - 0.5)(1+\zeta)\zeta \ , && (227)\\
N_3^{(2)} &= (1-\xi-\eta)(0.5-\xi-\eta)(1+\zeta)\zeta \ . && (228)\\
N_4^{(2)} &= \xi(\xi - 0.5)(\zeta-1)\zeta \ , && (229)\\
N_5^{(2)} &= \eta(\eta - 0.5)(\zeta-1)\zeta \ , && (230)\\
N_6^{(2)} &= (1-\xi-\eta)(0.5-\xi-\eta)(\zeta-1)\zeta \ . && (231)\\
N_7^{(2)} &= 2\xi\eta(1+\zeta) \ , && (232)\\
N_8^{(2)} &= 2\eta(1-\xi-\eta)(1+\zeta) \ , && (233)\\
N_9^{(2)} &= 2\xi(1-\xi-\eta)(1+\zeta) \ , && (234)\\
N_{10}^{(2)} &= \xi(1-\zeta^2) \ , && (235)\\
N_{11}^{(2)} &= \eta(1-\zeta^2) \ , && (236)\\
N_{12}^{(2)} &= (1-\xi-\eta)(1-\zeta^2) \ , && (237)\\
N_{13}^{(2)} &= 2\xi\eta(1-\zeta) \ , && (238)\\
N_{14}^{(2)} &= 2\eta(1-\xi-\eta)(1-\zeta) \ , && (239)\\
N_{15}^{(2)} &= 2\xi(1-\xi-\eta)(1-\zeta) \ , && (240)
\end{aligned}
$$

Klín s kvadratickými aproximačními funkcemi (15 uzlů)

Parciální derivace podle ξ

$$\frac{\partial N_1^{(2)}}{\partial \xi} = (2\xi - 0.5)(1+\zeta)\zeta \ , \tag{241}$$

$$\frac{\partial N_2^{(2)}}{\partial \xi} = 0 \ , \tag{242}$$

$$\frac{\partial N_3^{(2)}}{\partial \xi} = (2\xi + 2\eta - \frac{3}{2})(1+\zeta)\zeta \ . \tag{243}$$

$$\frac{\partial N_4^{(2)}}{\partial \xi} = (2\xi - 0.5)(\zeta - 1)\zeta \ , \tag{244}$$

$$\frac{\partial N_5^{(2)}}{\partial \xi} = 0 \ , \tag{245}$$

$$\frac{\partial N_6^{(2)}}{\partial \xi} = (2\xi + 2\eta - \frac{3}{2})(\zeta - 1)\zeta \ . \tag{246}$$

$$\frac{\partial N_7^{(2)}}{\partial \xi} = 2\eta(1+\zeta) \ , \tag{247}$$

$$\frac{\partial N_8^{(2)}}{\partial \xi} = -2\eta(1+\zeta) \ , \tag{248}$$

$$\frac{\partial N_9^{(2)}}{\partial \xi} = (2 - 4\xi - 2\eta)(1+\zeta) \ , \tag{249}$$

$$\frac{\partial N_{10}^{(2)}}{\partial \xi} = (1-\zeta^2)\ , \tag{250}$$

$$\frac{\partial N_{11}^{(2)}}{\partial \xi} = 0\ , \tag{251}$$

$$\frac{\partial N_{12}^{(2)}}{\partial \xi} = \zeta^2 - 1\ , \tag{252}$$

$$\frac{\partial N_{13}^{(2)}}{\partial \xi} = 2\eta(1-\zeta)\ , \tag{253}$$

$$\frac{\partial N_{14}^{(2)}}{\partial \xi} = 2\eta(\zeta-1)\ , \tag{254}$$

$$\frac{\partial N_{15}^{(2)}}{\partial \xi} = (2-4\xi-2\eta)(1-\zeta)\ . \tag{255}$$

Klín s kvadratickými aproximačními funkcemi (15 uzlů)

Parciální derivace podle η

$$\frac{\partial N_1^{(2)}}{\partial \eta} = 0 \; , \tag{256}$$

$$\frac{\partial N_2^{(2)}}{\partial \eta} = (2\eta - 0.5)(1+\zeta)\zeta \; , \tag{257}$$

$$\frac{\partial N_3^{(2)}}{\partial \eta} = (2\xi + 2\eta - \frac{3}{2})(1+\zeta)\zeta \; , \tag{258}$$

$$\frac{\partial N_4^{(2)}}{\partial \eta} = 0 \; , \tag{259}$$

$$\frac{\partial N_5^{(2)}}{\partial \eta} = (2\eta - 0.5)(\zeta - 1)\zeta \; , \tag{260}$$

$$\frac{\partial N_6^{(2)}}{\partial \eta} = (2\xi + 2\eta - \frac{3}{2})(\zeta - 1)\zeta \; . \tag{261}$$

$$\frac{\partial N_7^{(2)}}{\partial \eta} = 2\xi(1+\zeta) \; , \tag{262}$$

$$\frac{\partial N_8^{(2)}}{\partial \eta} = (2 - 2\xi - 4\eta)(1+\zeta) \; , \tag{263}$$

$$\frac{\partial N_9^{(2)}}{\partial \eta} = -2\xi(1+\zeta) \; , \tag{264}$$

$$\frac{\partial N_{10}^{(2)}}{\partial \eta} = 0 \ , \tag{265}$$

$$\frac{\partial N_{11}^{(2)}}{\partial \eta} = 1 - \zeta^2 \ , \tag{266}$$

$$\frac{\partial N_{12}^{(2)}}{\partial \eta} = \zeta^2 - 1 \ , \tag{267}$$

$$\frac{\partial N_{13}^{(2)}}{\partial \eta} = 2\xi(1 - \zeta) \ , \tag{268}$$

$$\frac{\partial N_{14}^{(2)}}{\partial \eta} = (2 - 2\xi - 4\eta)(1 - \zeta) \ , \tag{269}$$

$$\frac{\partial N_{15}^{(2)}}{\partial \eta} = 2\xi(\zeta - 1) \ , \tag{270}$$

Klín s kvadratickými aproximačními funkcemi (15 uzlů)

Parciální derivace podle ζ

$$\frac{\partial N_1^{(2)}}{\partial \zeta} = \xi(\xi - 0.5)(1 + 2\zeta) \ , \tag{271}$$

$$\frac{\partial N_2^{(2)}}{\partial \zeta} = \eta(\eta - 0.5)(1 + 2\zeta) \ , \tag{272}$$

$$\frac{\partial N_3^{(2)}}{\partial \zeta} = (1 - \xi - \eta)(0.5 - \xi - \eta)(1 + 2\zeta) \ . \tag{273}$$

$$\frac{\partial N_4^{(2)}}{\partial \zeta} = \xi(\xi - 0.5)(2\zeta - 1) \ , \tag{274}$$

$$\frac{\partial N_5^{(2)}}{\partial \zeta} = \eta(\eta - 0.5)(2\zeta - 1) \ , \tag{275}$$

$$\frac{\partial N_6^{(2)}}{\partial \zeta} = (1 - \xi - \eta)(0.5 - \xi - \eta)(2\zeta - 1) \ . \tag{276}$$

$$\frac{\partial N_7^{(2)}}{\partial \zeta} = 2\xi\eta \ , \tag{277}$$

$$\frac{\partial N_8^{(2)}}{\partial \zeta} = 2\eta(1 - \xi - \eta) \ , \tag{278}$$

$$\frac{\partial N_9^{(2)}}{\partial \zeta} = 2\xi(1 - \xi - \eta) \ , \tag{279}$$

$$\frac{\partial N_{10}^{(2)}}{\partial \zeta} = -2\xi\zeta \; , \tag{280}$$

$$\frac{\partial N_{11}^{(2)}}{\partial \zeta} = -2\eta\zeta \; , \tag{281}$$

$$\frac{\partial N_{12}^{(2)}}{\partial \zeta} = -2\zeta(1-\xi-\eta) \; , \tag{282}$$

$$\frac{\partial N_{13}^{(2)}}{\partial \zeta} = -2\xi\eta \; , \tag{283}$$

$$\frac{\partial N_{14}^{(2)}}{\partial \zeta} = -2\eta(1-\xi-\eta) \; , \tag{284}$$

$$\frac{\partial N_{15}^{(2)}}{\partial \zeta} = -2\xi(1-\xi-\eta) \; , \tag{285}$$

Transformace derivací

Transformace derivací ve 2D

Nechť je funkce f funkcí dvou proměnnćh x a y. Proměnné můžeme aproximovat:

$$x(\xi,\eta) = \sum_{i=1}^{n} N_i(\xi,\eta)x_i \ , \qquad (286)$$

$$y(\xi,\eta) = \sum_{i=1}^{n} N_i(\xi,\eta)y_i \ . \qquad (287)$$

Závislost funkce f lze vyjádřit $f(x,y) = f(x(\xi,\eta), y(\xi,\eta)) = f(\xi,\eta)$. První derivace funkce f podle proměnných ξ a η mají následující tvar

$$\frac{\partial f}{\partial \xi} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \xi} \ , \qquad (288)$$

$$\frac{\partial f}{\partial \eta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \eta} \ . \qquad (289)$$

Přepíšeme předchozí rovnice:

$$\begin{pmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{pmatrix} \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial \xi} \\ \frac{\partial f}{\partial \eta} \end{pmatrix} \ , \qquad (290)$$

kde řešení soustavy má tvar:

$$\frac{\partial f}{\partial x} = \frac{1}{J}\left(\frac{\partial f}{\partial \xi}\frac{\partial y}{\partial \eta} - \frac{\partial f}{\partial \eta}\frac{\partial y}{\partial \xi}\right) , \quad (291)$$

$$\frac{\partial f}{\partial y} = \frac{1}{J}\left(\frac{\partial f}{\partial \eta}\frac{\partial x}{\partial \xi} - \frac{\partial f}{\partial \xi}\frac{\partial x}{\partial \eta}\right) . \quad (292)$$

J značí jakobián derivace:

$$J = \frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta} - \frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \xi} . \quad (293)$$

Pokud lze funkci f is aproximovat m počtem funkcí}

$$f(\xi,\eta) = \sum_{j=1}^{m} M_j(\xi,\eta) f_j , \quad (294)$$

první derivace podle proměnných x a y jsou

$$\frac{\partial f}{\partial x} = \frac{1}{J}\left(\sum_{j=1}^{m}\frac{\partial M_j}{\partial \xi} f_j \sum_{i=1}^{n}\frac{\partial N_i}{\partial \eta} y_i - \sum_{j=1}^{m}\frac{\partial M_j}{\partial \eta} f_j \sum_{i=1}^{n}\frac{\partial N_i}{\partial \xi} y_i\right) , \quad (295)$$

$$\frac{\partial f}{\partial y} = \frac{1}{J}\left(\sum_{j=1}^{m}\frac{\partial M_j}{\partial \eta} f_j \sum_{i=1}^{n}\frac{\partial N_i}{\partial \xi} x_i - \sum_{j=1}^{m}\frac{\partial M_j}{\partial \xi} f_j \sum_{i=1}^{n}\frac{\partial N_i}{\partial \eta} x_i\right) , \quad (296)$$

Transformace derivací ve 3D

Tentokrát je funkce f funkcí tří proměnnćh x,y and z. Proměnné můžeme aproximovat:

$$x(\xi,\eta,\zeta) = \sum_{i=1}^{n} N_i(\xi,\eta,\zeta) x_i \ , \tag{297}$$

$$y(\xi,\eta,\zeta) = \sum_{i=1}^{n} N_i(\xi,\eta,\zeta) y_i \ , \tag{298}$$

$$z(\xi,\eta,\zeta) = \sum_{i=1}^{n} N_i(\xi,\eta,\zeta) z_i \ . \tag{299}$$

Závislost funkce f lze vyjádřit $f(x,y,z) = f(x(\xi,\eta,\zeta), y(\xi,\eta,\zeta), z(\xi,\eta,\zeta)) = f(\xi,\eta,\zeta)$. První derivace funkce f podle proměnných ξ, η a ζ mají následující tvar

$$\frac{\partial f}{\partial \xi} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \xi} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \xi} \ , \tag{300}$$

$$\frac{\partial f}{\partial \eta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \eta} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \eta} \ , \tag{301}$$

$$\frac{\partial f}{\partial \zeta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \zeta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \zeta} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \zeta} \ . \tag{302}$$

Přepíšeme předchozí rovnice:

$$\begin{pmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{pmatrix} \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial \xi} \\ \frac{\partial f}{\partial \eta} \\ \frac{\partial f}{\partial \zeta} \end{pmatrix} \ , \tag{303}$$

kde řešení soustavy má tvar:

$$\frac{\partial f}{\partial x} = \frac{1}{J}\Big(\frac{\partial f}{\partial \xi}\frac{\partial y}{\partial \eta}\frac{\partial z}{\partial \zeta} + \frac{\partial f}{\partial \zeta}\frac{\partial y}{\partial \xi}\frac{\partial z}{\partial \eta} + \frac{\partial f}{\partial \eta}\frac{\partial y}{\partial \zeta}\frac{\partial z}{\partial \xi} - \tag{304}$$

$$- \frac{\partial f}{\partial \zeta}\frac{\partial y}{\partial \eta}\frac{\partial z}{\partial \xi} - \frac{\partial f}{\partial \xi}\frac{\partial y}{\partial \zeta}\frac{\partial z}{\partial \eta} - \frac{\partial f}{\partial \eta}\frac{\partial y}{\partial \xi}\frac{\partial z}{\partial \zeta}\Big) \ , \tag{305}$$

$$\frac{\partial f}{\partial y} = \frac{1}{J}\Big(\frac{\partial f}{\partial \eta}\frac{\partial x}{\partial \xi}\frac{\partial z}{\partial \zeta} + \frac{\partial f}{\partial \xi}\frac{\partial x}{\partial \zeta}\frac{\partial z}{\partial \eta} + \frac{\partial f}{\partial \zeta}\frac{\partial x}{\partial \eta}\frac{\partial z}{\partial \xi} - \tag{306}$$

$$- \frac{\partial f}{\partial \eta}\frac{\partial x}{\partial \zeta}\frac{\partial z}{\partial \xi} - \frac{\partial f}{\partial \zeta}\frac{\partial x}{\partial \xi}\frac{\partial z}{\partial \eta} - \frac{\partial f}{\partial \xi}\frac{\partial x}{\partial \eta}\frac{\partial z}{\partial \zeta}\Big) \ , \tag{307}$$

$$\frac{\partial f}{\partial z} = \frac{1}{J}\Big(\frac{\partial f}{\partial \zeta}\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta} + \frac{\partial f}{\partial \eta}\frac{\partial x}{\partial \zeta}\frac{\partial y}{\partial \xi} + \frac{\partial f}{\partial \xi}\frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \zeta} - \tag{308}$$

$$- \frac{\partial f}{\partial \xi}\frac{\partial x}{\partial \zeta}\frac{\partial y}{\partial \eta} - \frac{\partial f}{\partial \eta}\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \zeta} - \frac{\partial f}{\partial \zeta}\frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \xi}\Big) \ . \tag{309}$$

J značí jakobián derivace:

$$J = \frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta}\frac{\partial z}{\partial \zeta} + \frac{\partial x}{\partial \zeta}\frac{\partial y}{\partial \xi}\frac{\partial z}{\partial \eta} + \frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \zeta}\frac{\partial z}{\partial \xi} - \frac{\partial x}{\partial \zeta}\frac{\partial y}{\partial \eta}\frac{\partial z}{\partial \xi} - \frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \zeta}\frac{\partial z}{\partial \eta} - \frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \xi}\frac{\partial z}{\partial \zeta} \ . \tag{310}$$

Pokud lze funkci f aproximovat m počtem funkcí}

$$f(\xi,\eta,\zeta) = \sum_{j=1}^{m} M_j(\xi,\eta,\zeta) f_j \ , \tag{311}$$

první derivace podle proměnných x, y a z jsou

$$\frac{\partial f}{\partial x} = \frac{1}{J}\Big(\sum_{j=1}^{m}\frac{\partial M_j}{\partial \xi}f_j\sum_{i=1}^{n}\frac{\partial N_i}{\partial \eta}y_i\sum_{i=1}^{n}\frac{\partial N_i}{\partial \zeta}z_i + \sum_{j=1}^{m}\frac{\partial M_j}{\partial \zeta}f_j\sum_{i=1}^{n}\frac{\partial N_i}{\partial \xi}y_i\sum_{i=1}^{n}\frac{\partial N_i}{\partial \eta}z_i +$$

$$
\begin{aligned}
&+ \sum_{j=1}^{m} \frac{\partial M_j}{\partial \eta} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} y_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} z_i - \sum_{j=1}^{m} \frac{\partial M_j}{\partial \zeta} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} y_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} z_i - \\
&- \sum_{j=1}^{m} \frac{\partial M_j}{\partial \xi} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} y_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} z_i - \sum_{j=1}^{m} \frac{\partial M_j}{\partial \eta} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} y_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} z_i \Bigg), \quad (312)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f}{\partial y} = \frac{1}{J} \Bigg(&\sum_{j=1}^{m} \frac{\partial M_j}{\partial \eta} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} z_i + \sum_{j=1}^{m} \frac{\partial M_j}{\partial \xi} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} z_i + \\
&+ \sum_{j=1}^{m} \frac{\partial M_j}{\partial \zeta} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} z_i - \sum_{j=1}^{m} \frac{\partial M_j}{\partial \eta} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} z_i - \\
&- \sum_{j=1}^{m} \frac{\partial M_j}{\partial \zeta} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} z_i - \sum_{j=1}^{m} \frac{\partial M_j}{\partial \xi} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} z_i \Bigg), \quad (313)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f}{\partial z} = \frac{1}{J} \Bigg(&\sum_{j=1}^{m} \frac{\partial f}{\partial \zeta} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} y_i + \sum_{j=1}^{m} \frac{\partial f}{\partial \eta} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} y_i + \\
&+ \sum_{j=1}^{m} \frac{\partial f}{\partial \xi} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} y_i - \sum_{j=1}^{m} \frac{\partial f}{\partial \xi} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} y_i - \\
&- \sum_{j=1}^{m} \frac{\partial f}{\partial \eta} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} y_i - \sum_{j=1}^{m} \frac{\partial f}{\partial \zeta} f_j \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} x_i \sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} y_i \Bigg), \quad (314)
\end{aligned}
$$