Vícekriteriální optimalizace

- Optimalizace více funkcí najednou
- Je zapotřebí další matematický aparát
- Obecně:

$$\text{minimize } \mathbf{y} = \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x}))$$

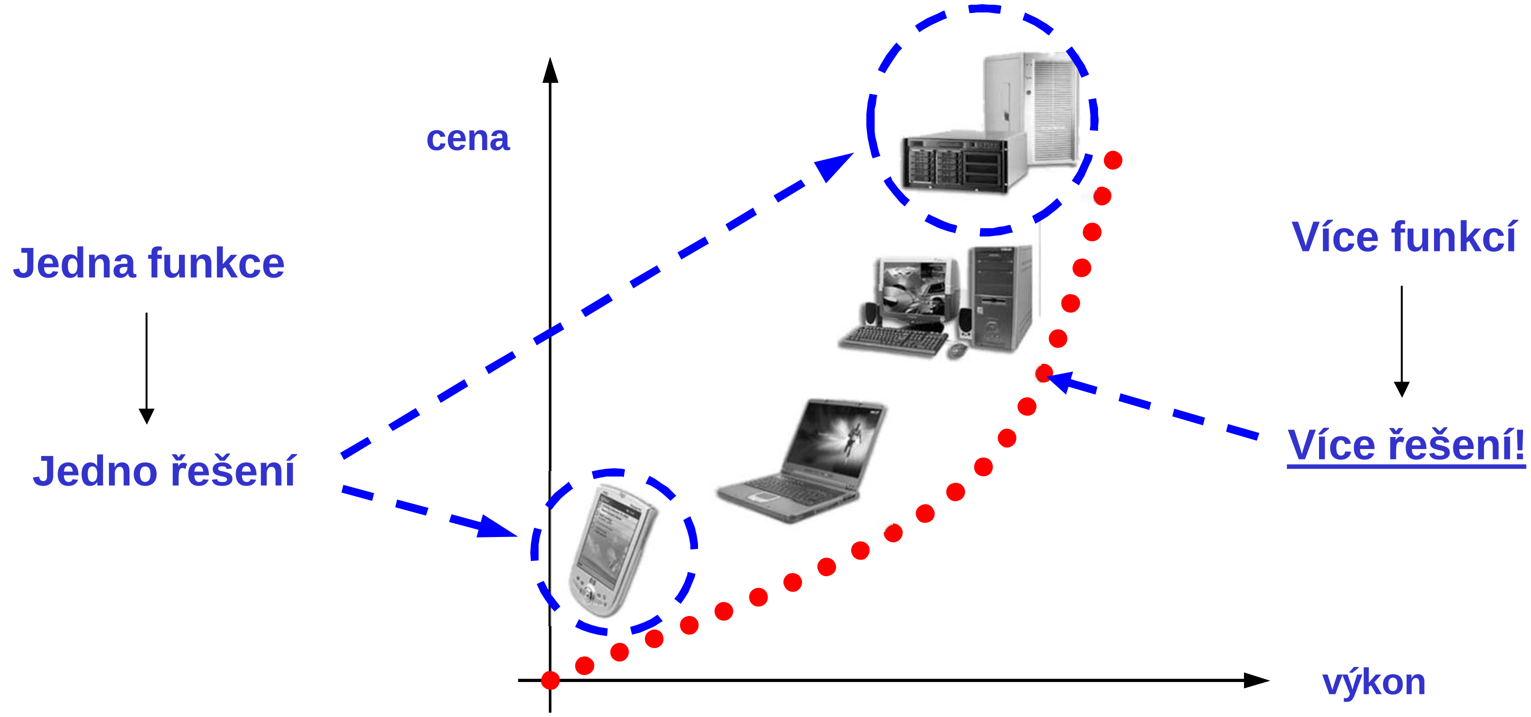
$$\text{subjected to } g_j(\mathbf{x}) = 0\,, \quad j = 1, \ldots, ne\,,$$

$$g_j(\mathbf{x}) \leq 0\,, \quad j = ne + 1, \ldots, m = ne + ni\,,$$

$$\text{where } \mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathbf{X}\,, \quad \mathbf{X} \subset \{N, R\}^n\,,$$

$$\mathbf{y} = (y_1, y_2, \ldots, y_k) \in \mathbf{Y}\,, \quad \mathbf{Y} \subseteq R^k\,,$$

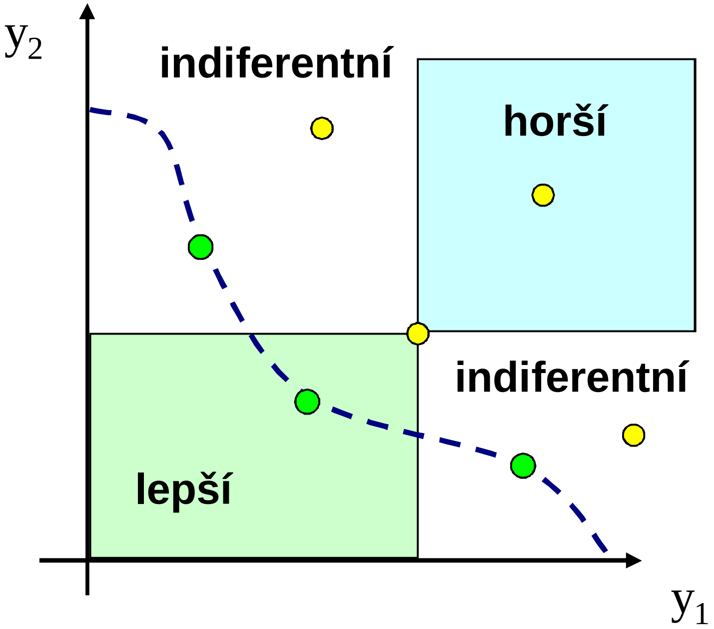
Motivace

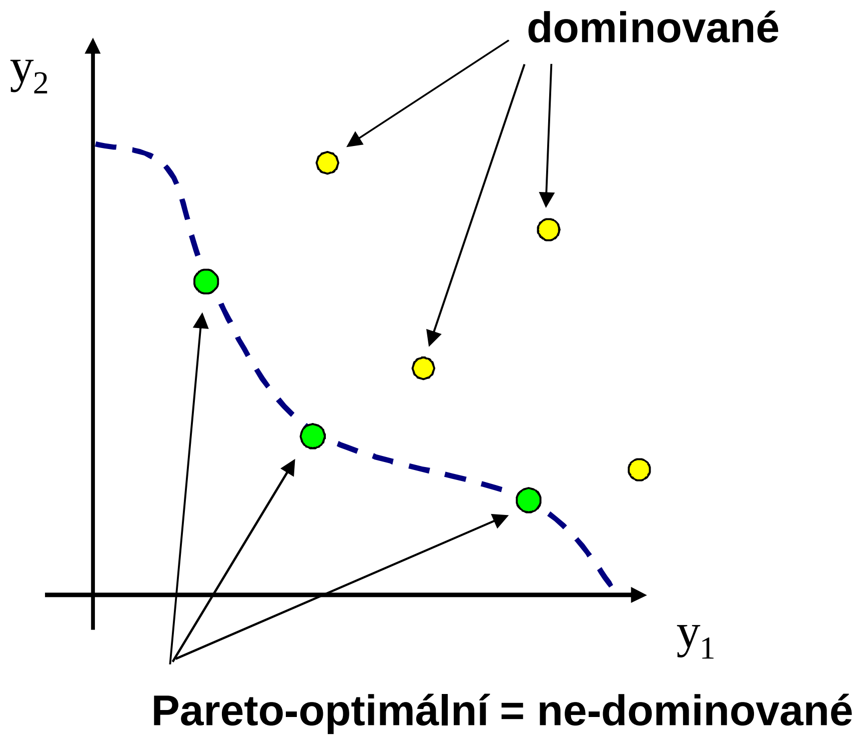


Jedno-kriteriální optimalizace je speciální případ multi-kriteriální optimalizace (ale nikoliv naopak!).

Základní pojmy

Min $(y_1, y_2, \ldots, y_k) = \mathbf{f}\,(x_1, x_2, \ldots, x_n)$





$\mathbf{a} \succ \mathbf{b}$ (**a** dominates **b**) *iff* $\quad \forall i : f_i(\mathbf{a}) \leq f_i(\mathbf{b}) \wedge \exists i : f_i(\mathbf{a}) < f_i(\mathbf{b}),$

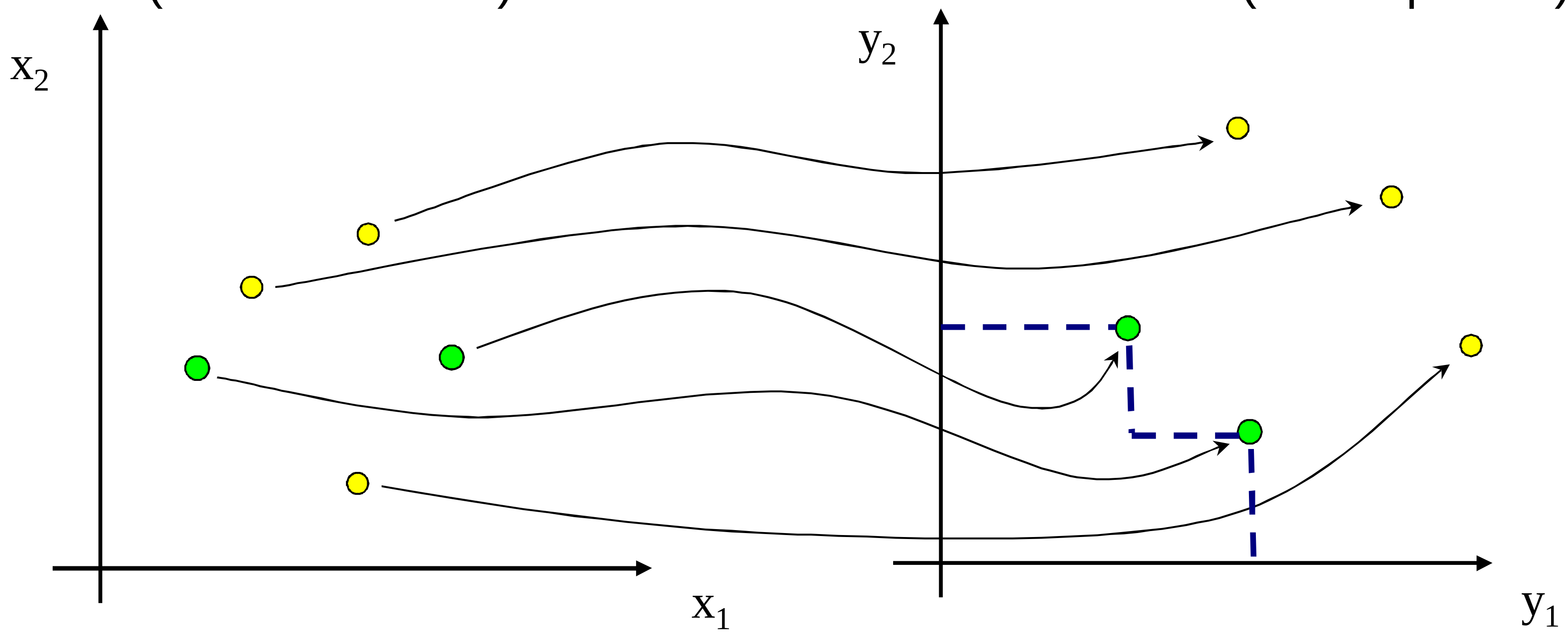
Základní pojmy

Decision/design space X (Prostor proměnných)

Objective space Y (Prostor funkcí)

Pareto set (Pareto množina)

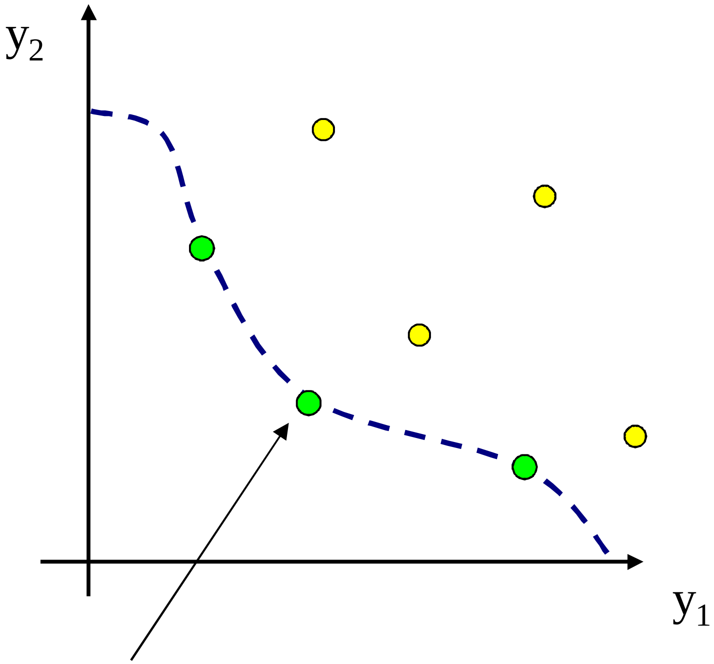
Pareto front/surface (Pareto povrch)



decision vector $(x_1,x_2,\ldots,x_n) \longrightarrow$ **f** $\longrightarrow (y_1,y_{,2},\ldots,y_k)$ **objective vector**

(rozhodovací vektor) (vektor funkce)

Optimalizace & Rozhodování
(Decision making)



Pareto-optimalita: optimální/kompromisní množina řešení (všechny funkce mají stejnou důležitost)

- Rozhodování **před optimalizací** (definice jediné cílové funkce) => EA
- Rozhodování **po optimalizaci** (nalezení Pareto-optimální množiny) => MOEA
- Rozhodování **v průběhu optimalizace** (interaktivně) => interaktivní algoritmy, např. NIMBUS

Single-objectivization

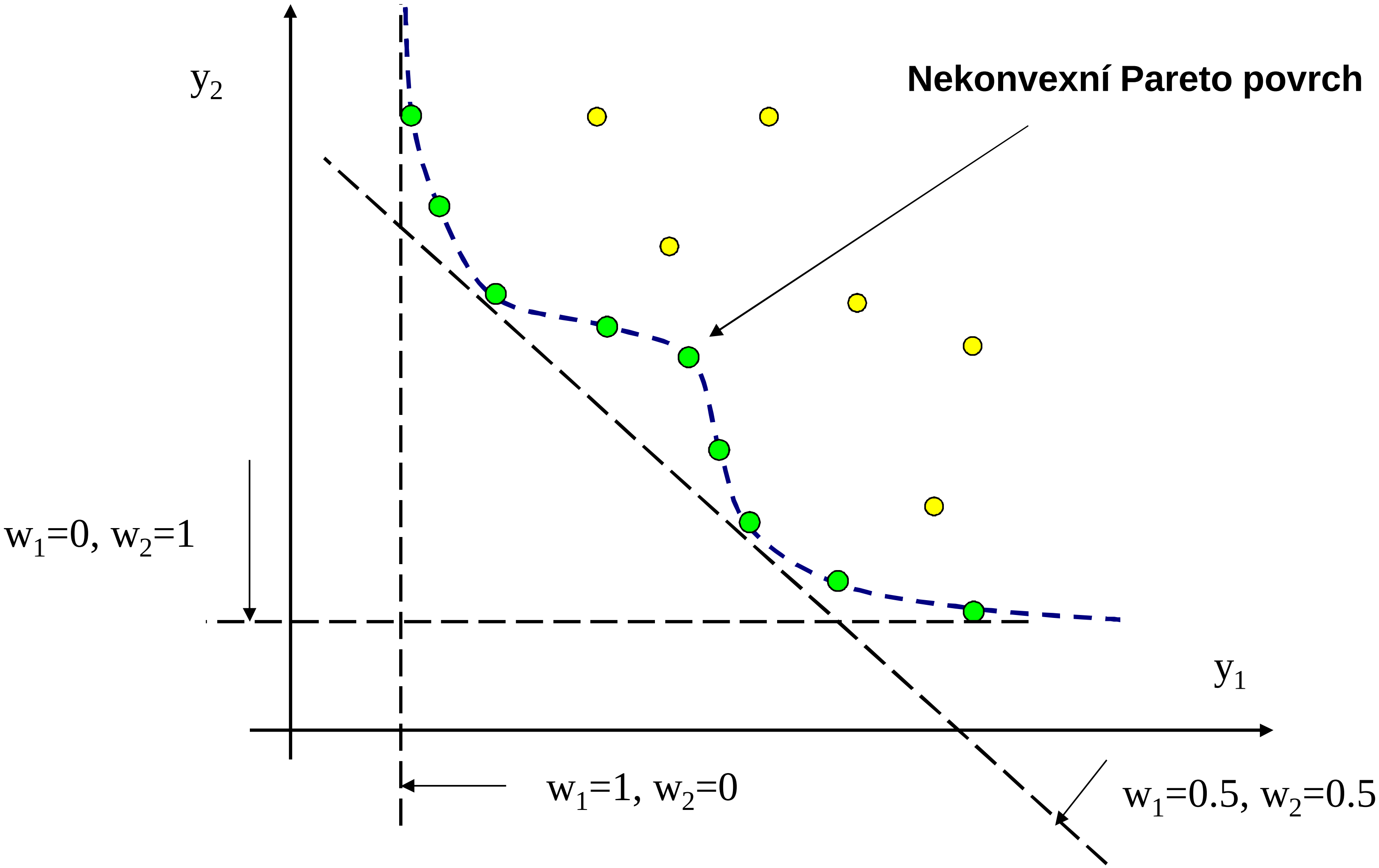
(Převedení MO problému na jednu funkci)

- Nejčastěji pomocí vah:

$$F'(\mathbf{x}) = \sum_{i}^{k} w_i f_i(\mathbf{x}), \qquad \text{kde} \sum_{i}^{k} w_i = 1$$

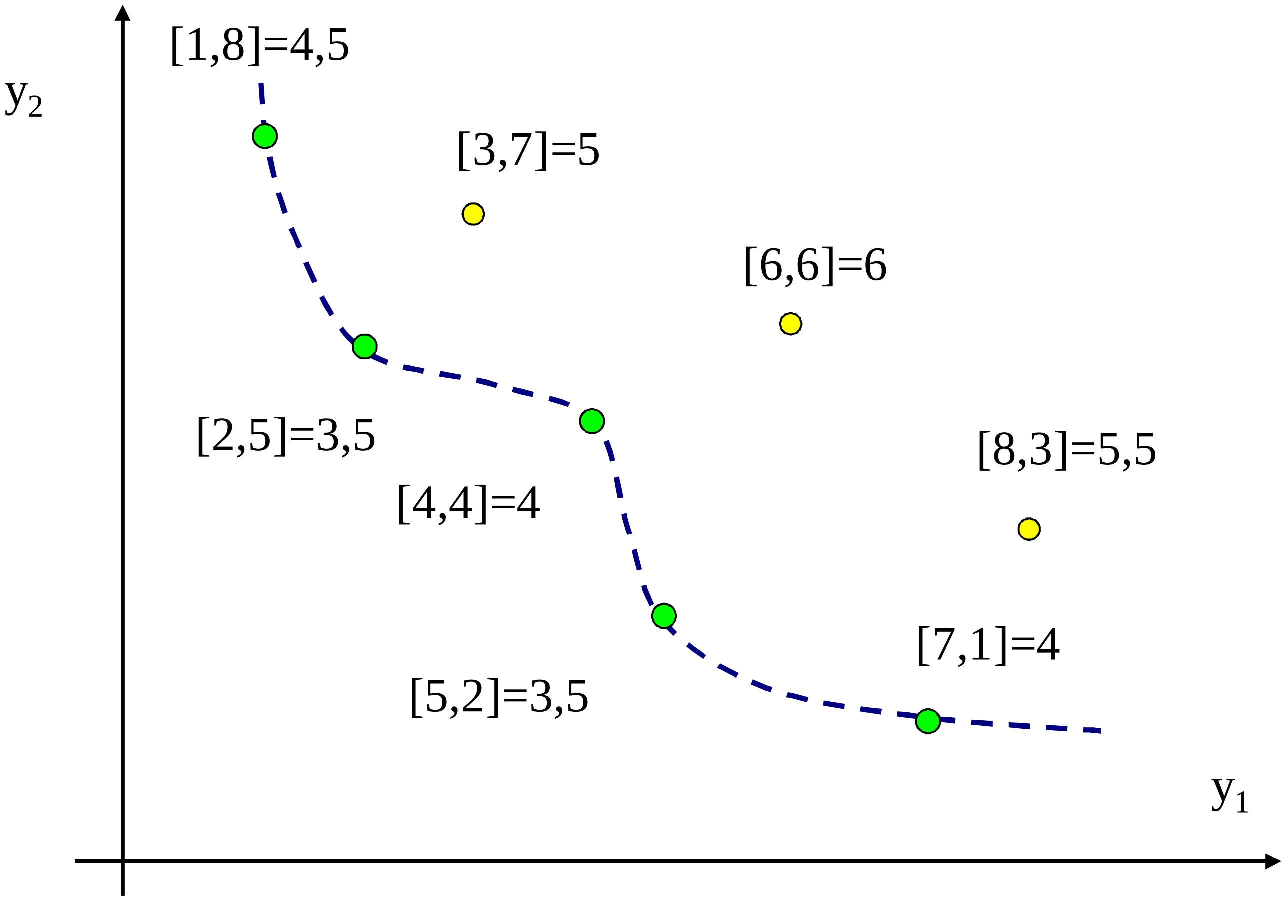
- Problém:
  - Najít kombinaci vah jež vede na kompletní Pareto množinu (nebo-li optimalizace další optimalizace)
  - Neschopnost postihnout nekonvexní Pareto povrch

Single-objectivization



Average ranking

(Průměrné pořadí)



Multikriteriální EA (MOEA)

- V multikriteriální optimalizaci je potřeba udržovat množinu Pareto-optimálních řešení => analogie s populací u EA

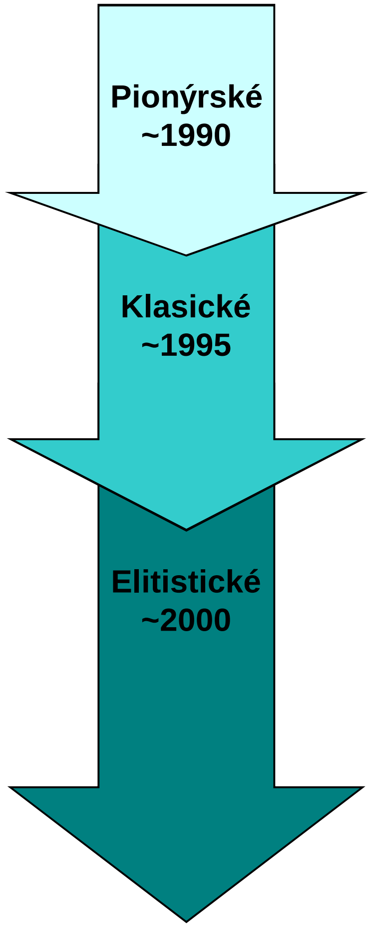
- Nejjednodušší implementace – Pareto-dominance začleněná do procesu selekce (např. Generalized Differential Evolution)

Historie MOEA

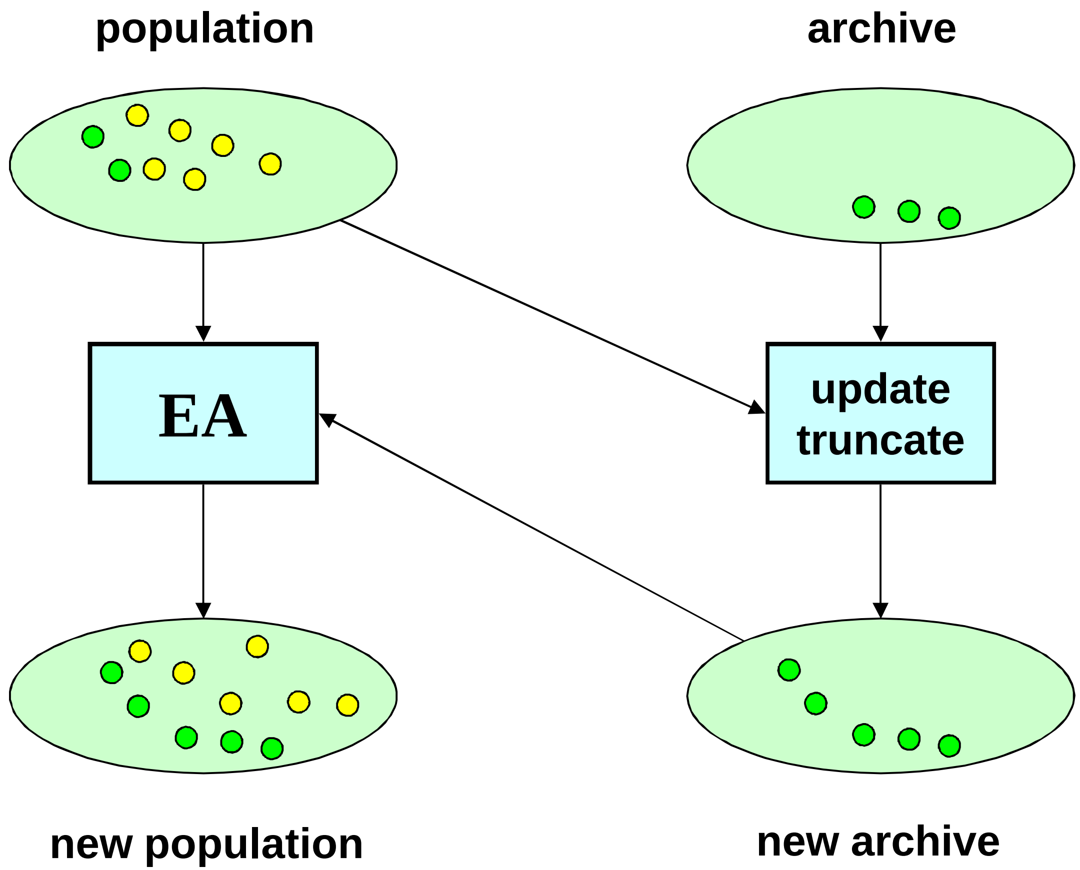
VEGA [Schaffer, 1985]

MOGA [Fonesca & Fleming, 1993]
NPGA [Horn & Nafpliotis, 1993]
NSGA [Srinivas & Deb, 1994]

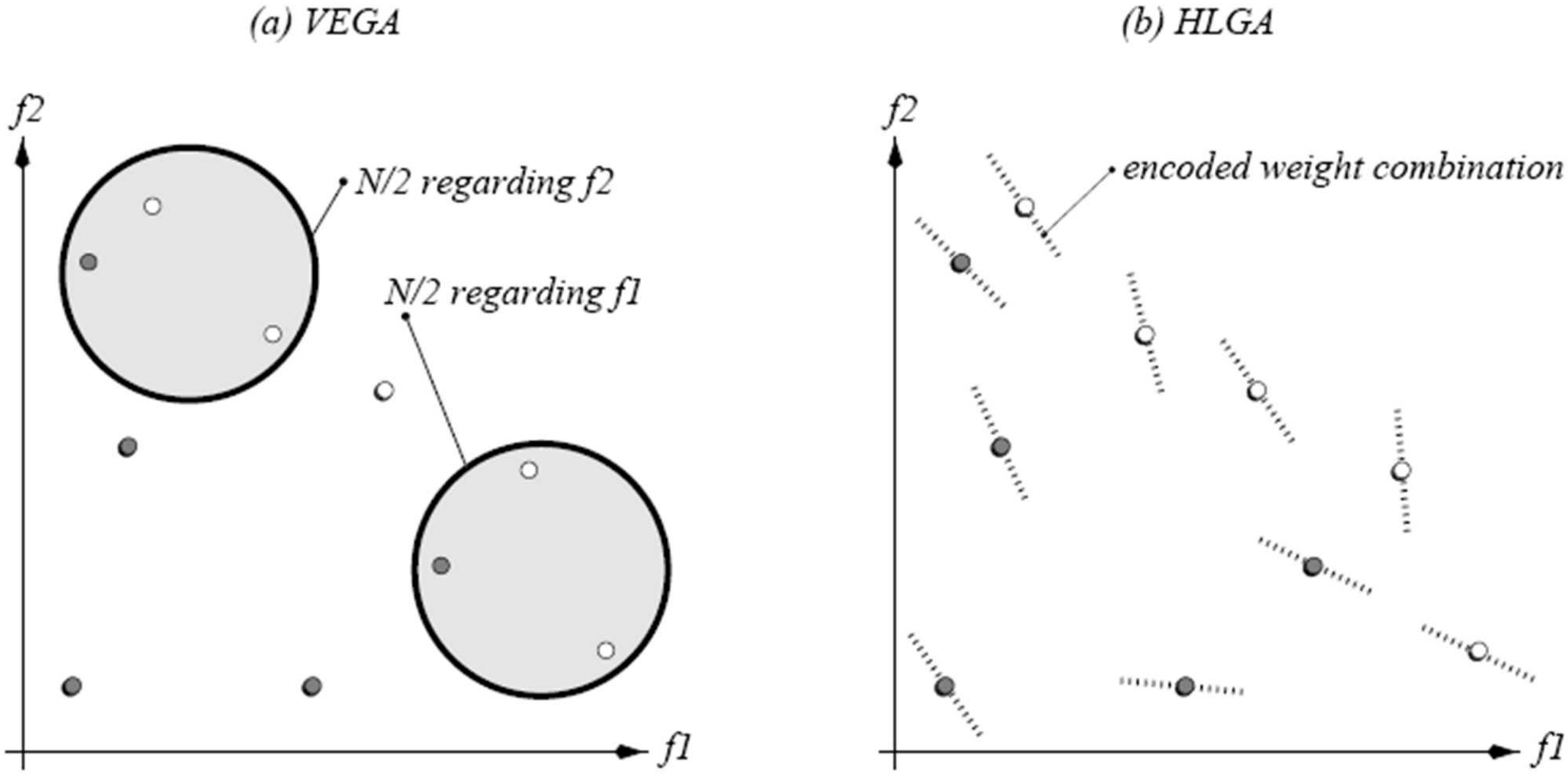
SPEA [Zitzler & Thiele, 1999]
PAES, PESA [Knowles & Corne, 1999]
NSGA-II [Deb et al., 2000]
SPEA 2 [Zitzler & Thiele, 2001]



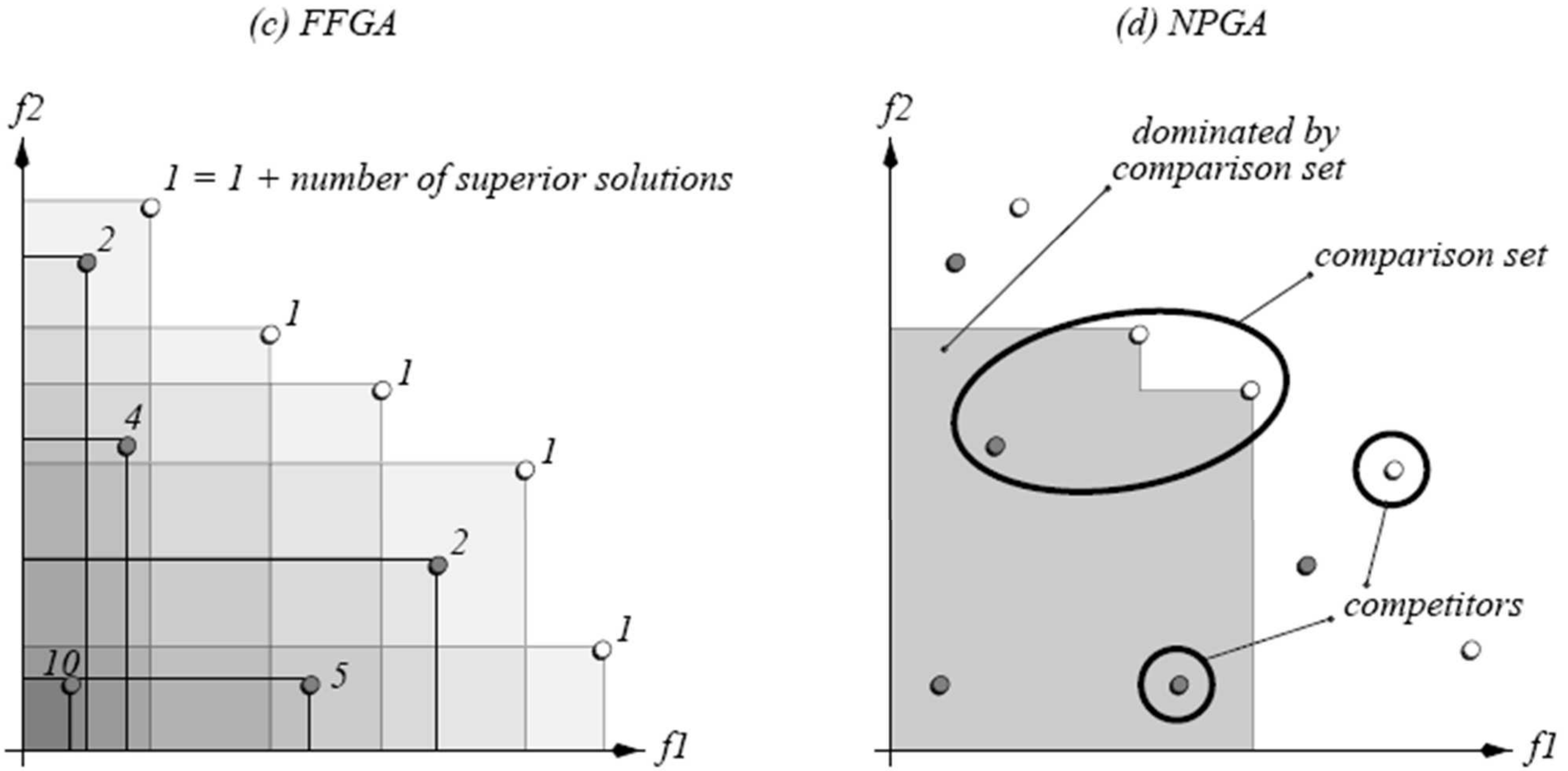
Obecný „elitistický“ MOEA



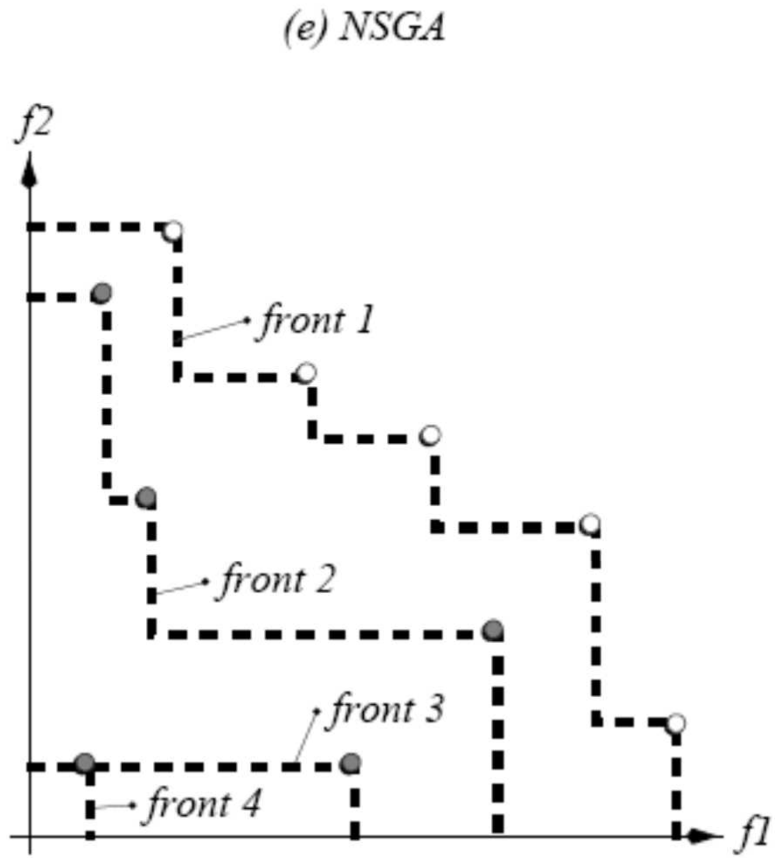
Jak ohodnotit řešení?

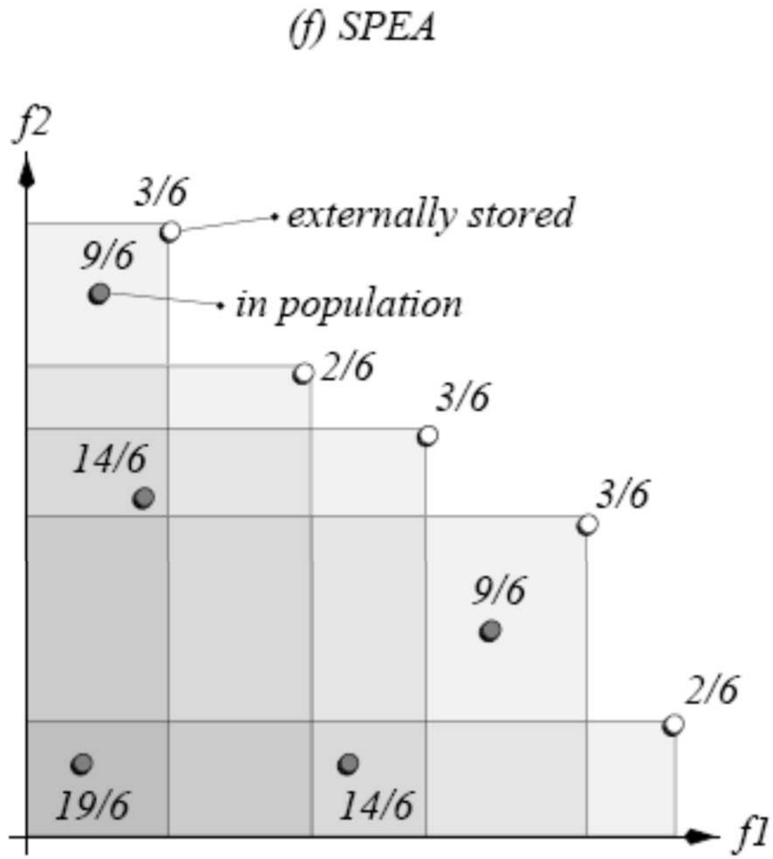


Jak ohodnotit řešení?



Jak ohodnotit řešení?





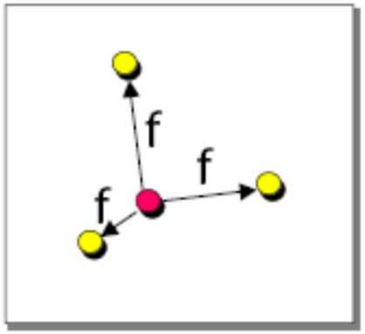
Jak zajistit diverzitu?

Density estimation techniques: [Silverman 86]

Kernel

MOGA, NPGA

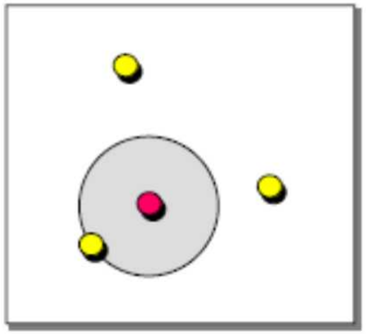
density estimate = sum of f values where f is a function of the distance



Nearest neighbor

NSGA-II, SPEA2

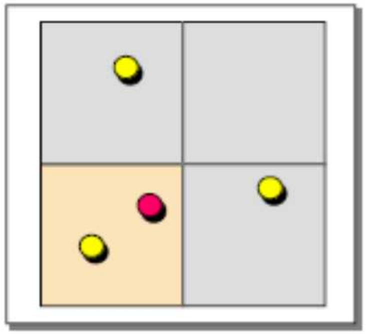
density estimate = volume of the sphere defined by the nearest neighbor



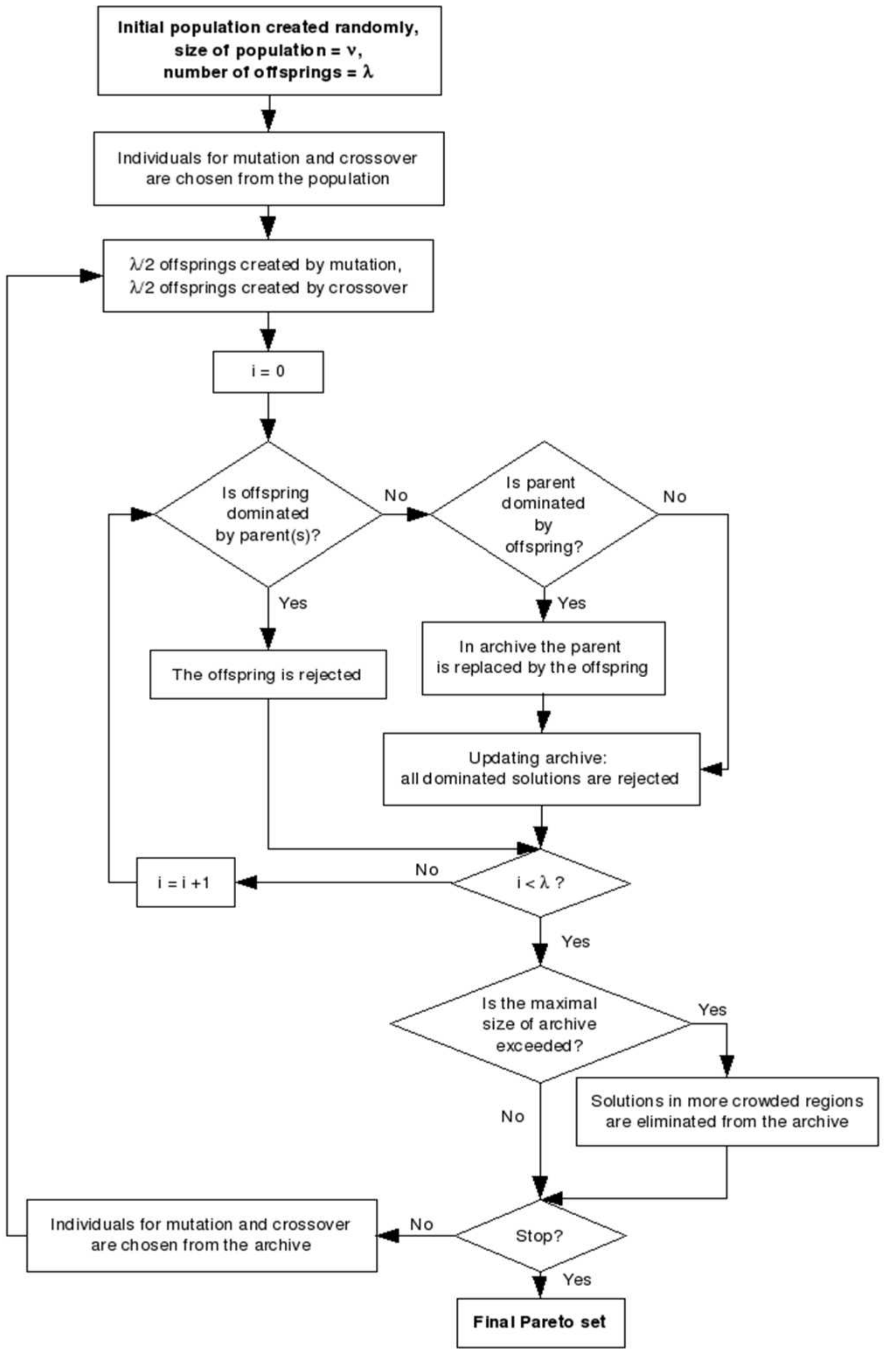
Histogram

PAES, PESA

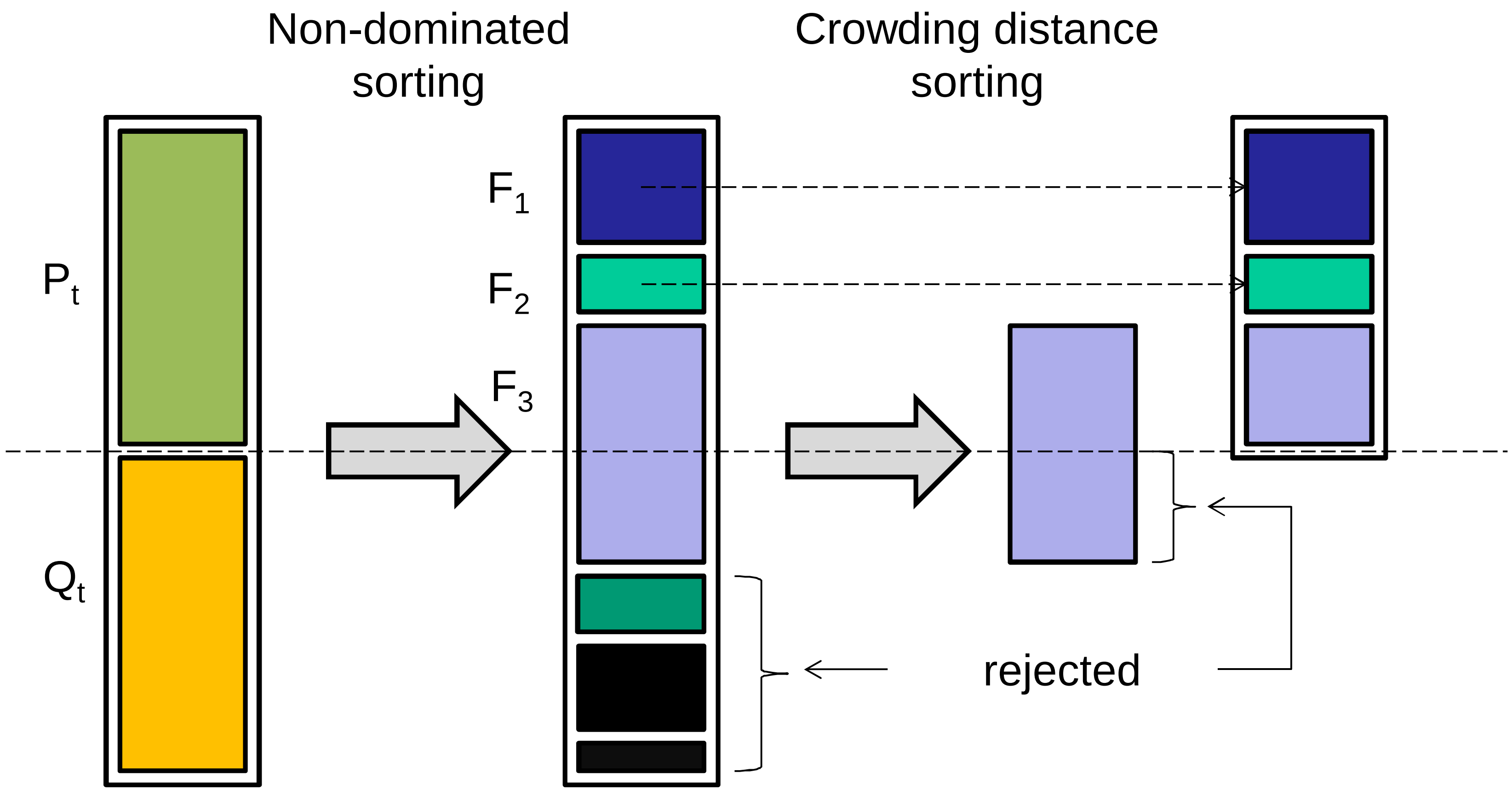
density estimate = number of solutions in the same box



PAES


Initial population created randomly, size of population = ν, number of offsprings = λ
Individuals for mutation and crossover are chosen from the population
λ/2 offsprings created by mutation, λ/2 offsprings created by crossover
i = 0
Is offspring dominated by parent(s)?
No
Is parent dominated by offspring?
No
Yes
The offspring is rejected
Yes
In archive the parent is replaced by the offspring
Updating archive: all dominated solutions are rejected
No
i = i +1
i < λ ?
Yes
Is the maximal size of archive exceeded?
Yes
Solutions in more crowded regions are eliminated from the archive
No
Individuals for mutation and crossover are chosen from the archive
No
Stop?
Yes
Final Pareto set

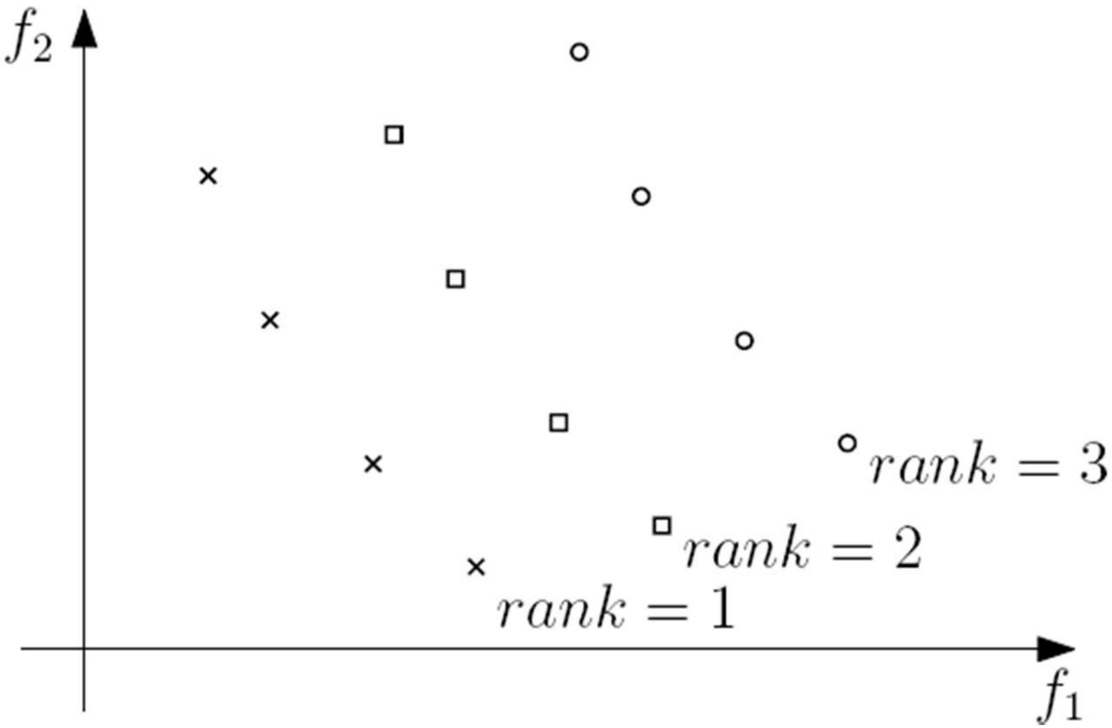
NSGA II



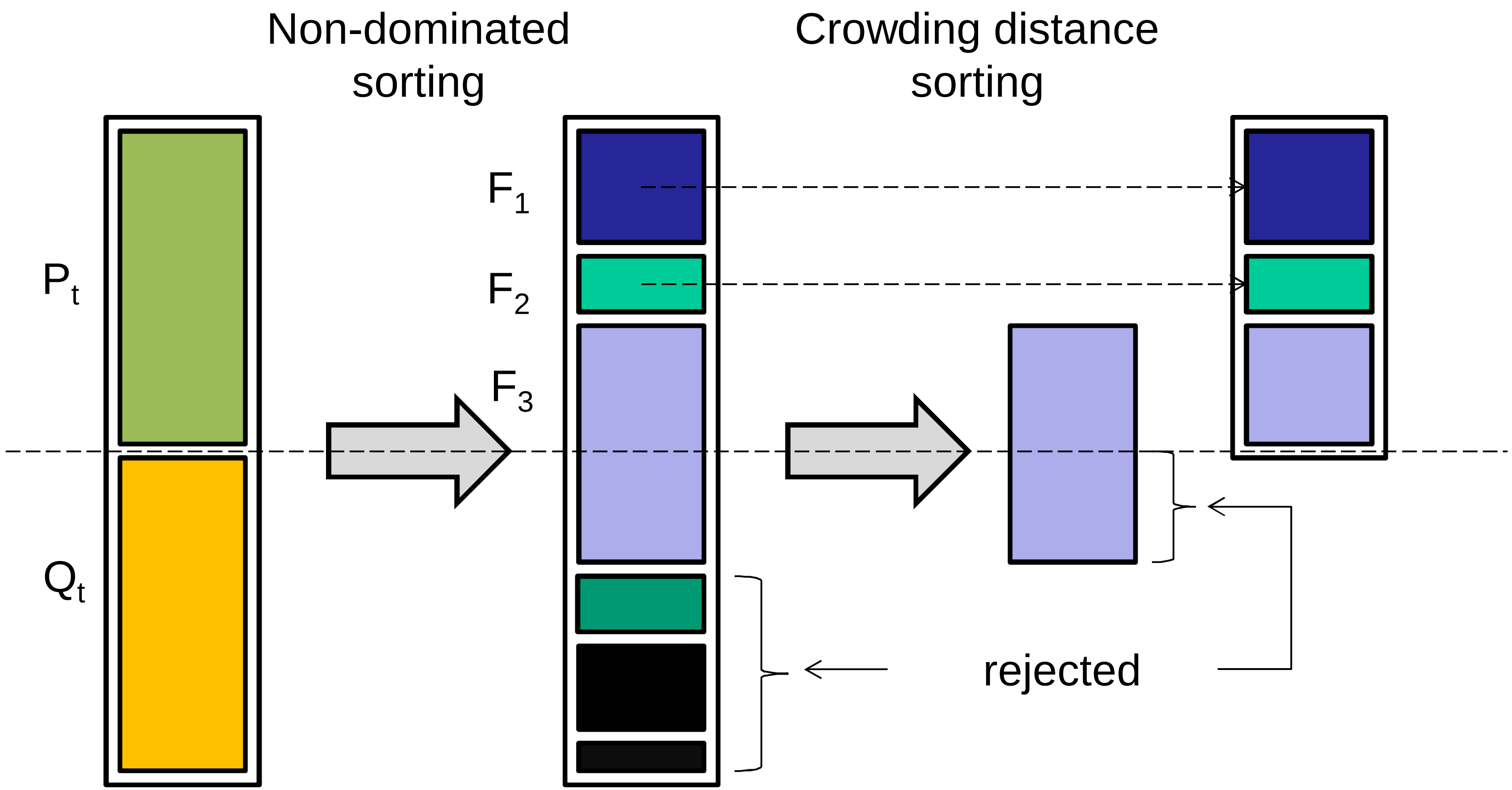
Non-dominated sorting
Crowding distance sorting
P_t
Q_t
F_1
F_2
F_3
rejected

NSGA II

Non-dominated ranking



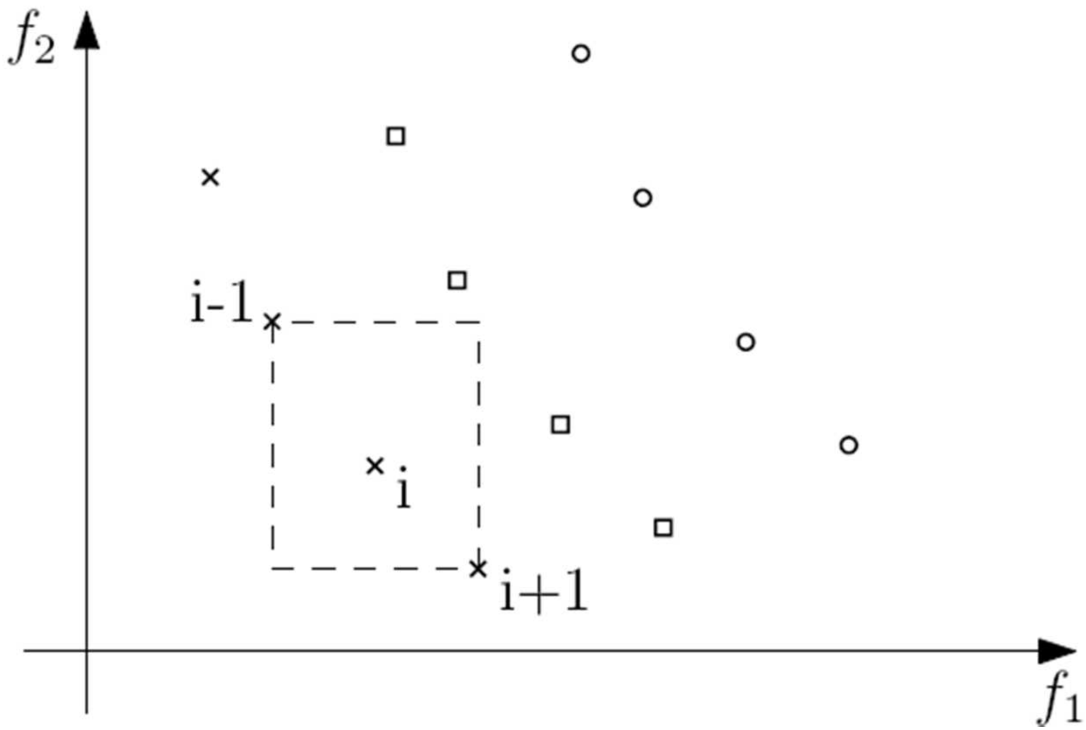
NSGA II



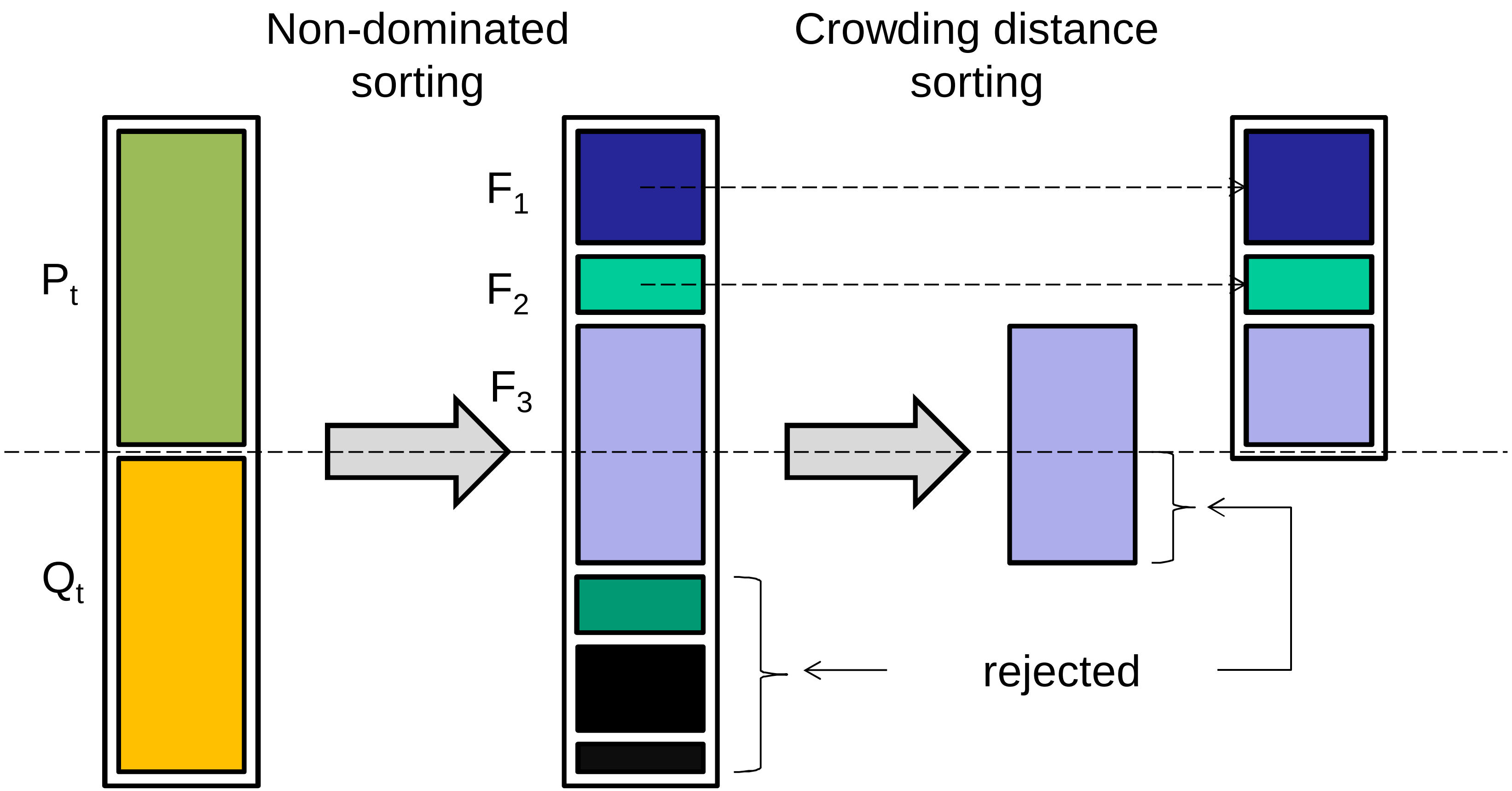
Non-dominated sorting
Crowding distance sorting
P_t
Q_t
F_1
F_2
F_3
rejected

NSGA II

Crowding distance

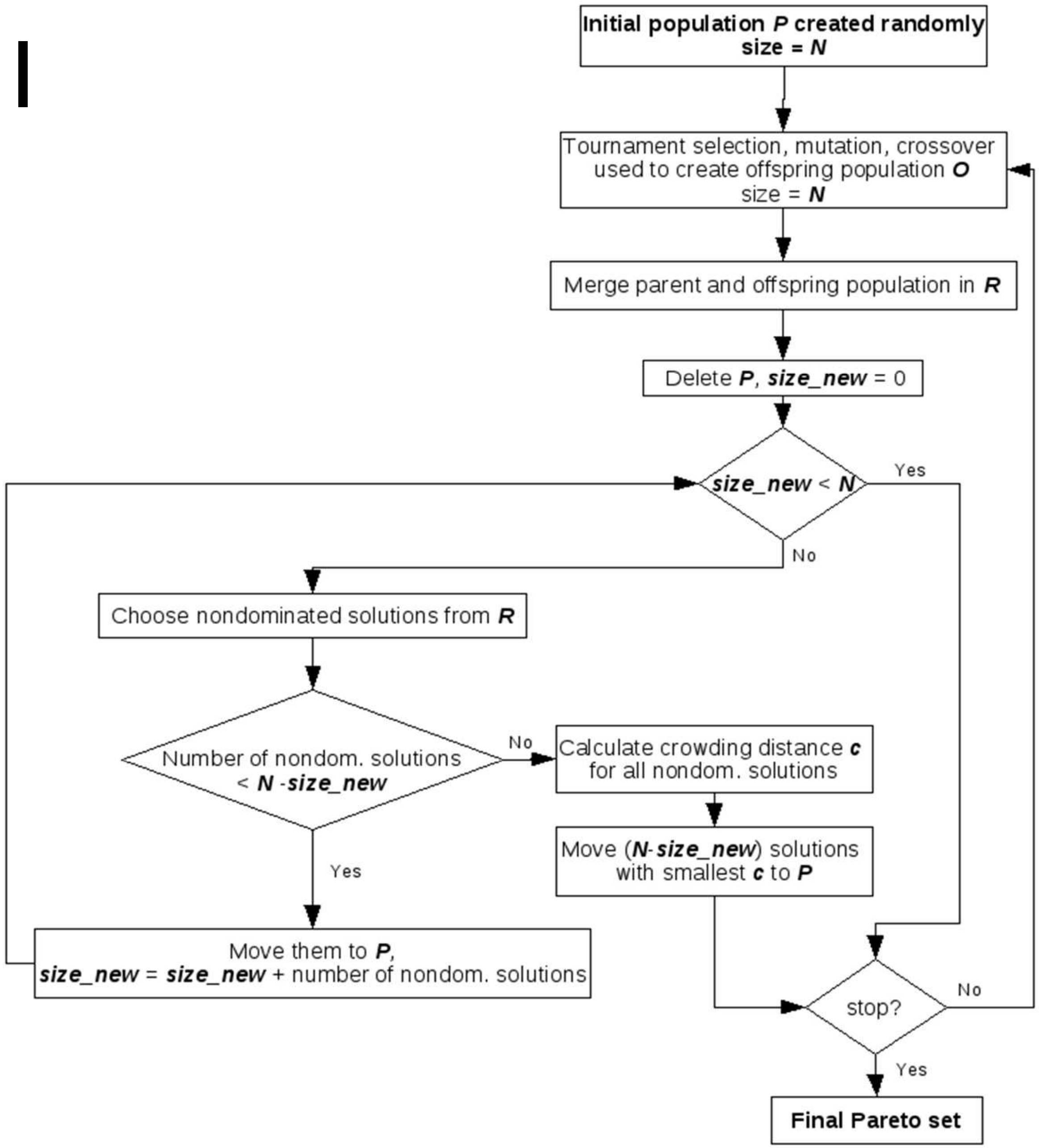


NSGA II



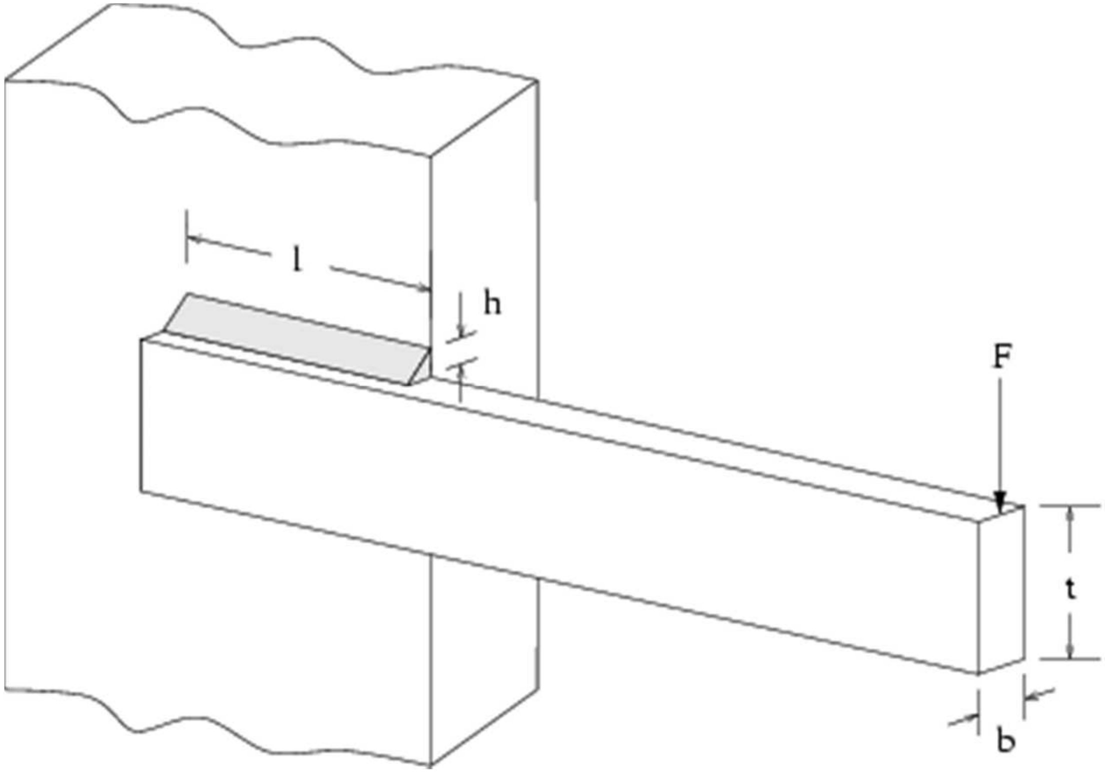
Non-dominated sorting
Crowding distance sorting
P_t
Q_t
F_1
F_2
F_3
rejected

NSGA II



Initial population P created randomly
size = N
Tournament selection, mutation, crossover
used to create offspring population O
size = N
Merge parent and offspring population in R
Delete P, size_new = 0
size_new < N
Yes
No
Choose nondominated solutions from R
Number of nondom. solutions
< N -size_new
No
Calculate crowding distance c
for all nondom. solutions
Move (N-size_new) solutions
with smallest c to P
Yes
Move them to P,
size_new = size_new + number of nondom. solutions
stop?
No
Yes
Final Pareto set

Příklad



F = 6 000 lb

h,b ∈ (0.125, 5.0) [in]

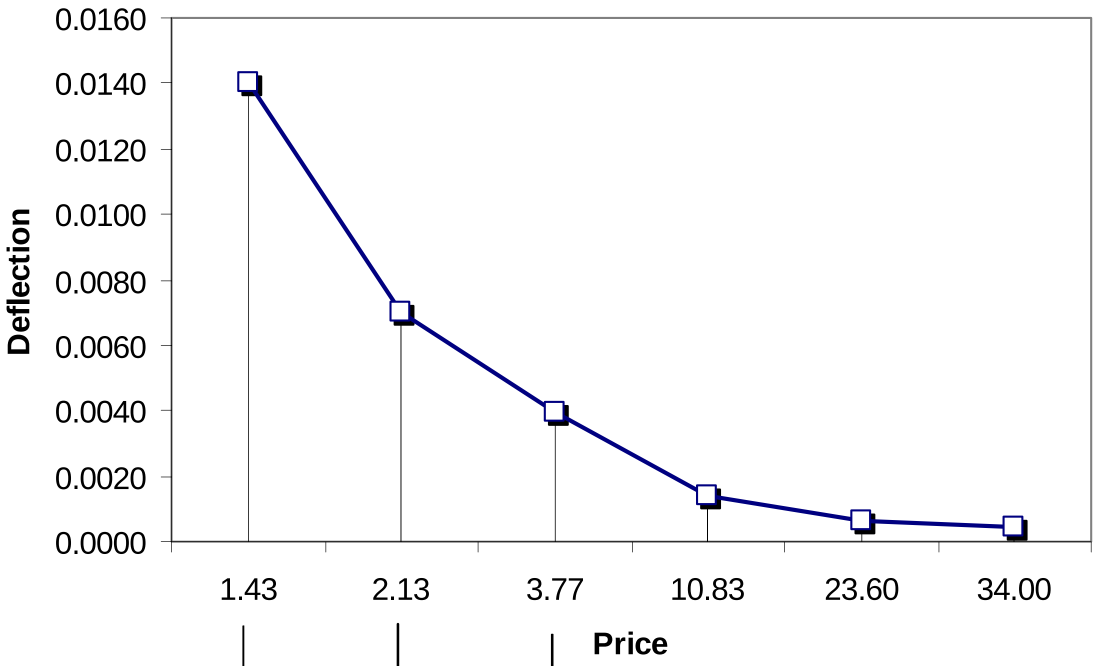
l,t ∈ (0.1, 10.0) [in]

Minimalizuj:

f_1 = cenu

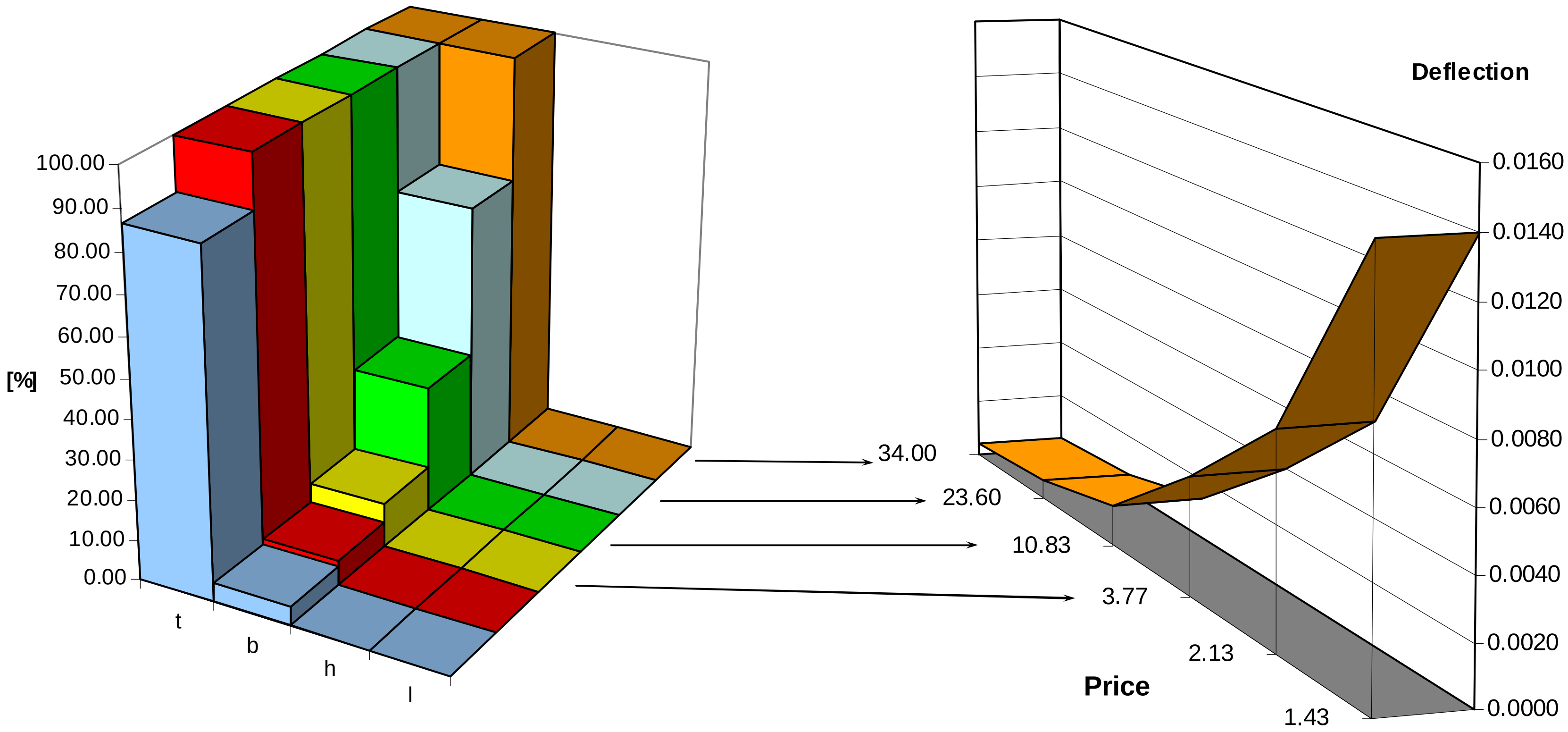
f_2 = průhyb

Pareto povrch:



Pareto množina:

Výsledky



100.00
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60.00
50.00
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30.00
20.00
10.00
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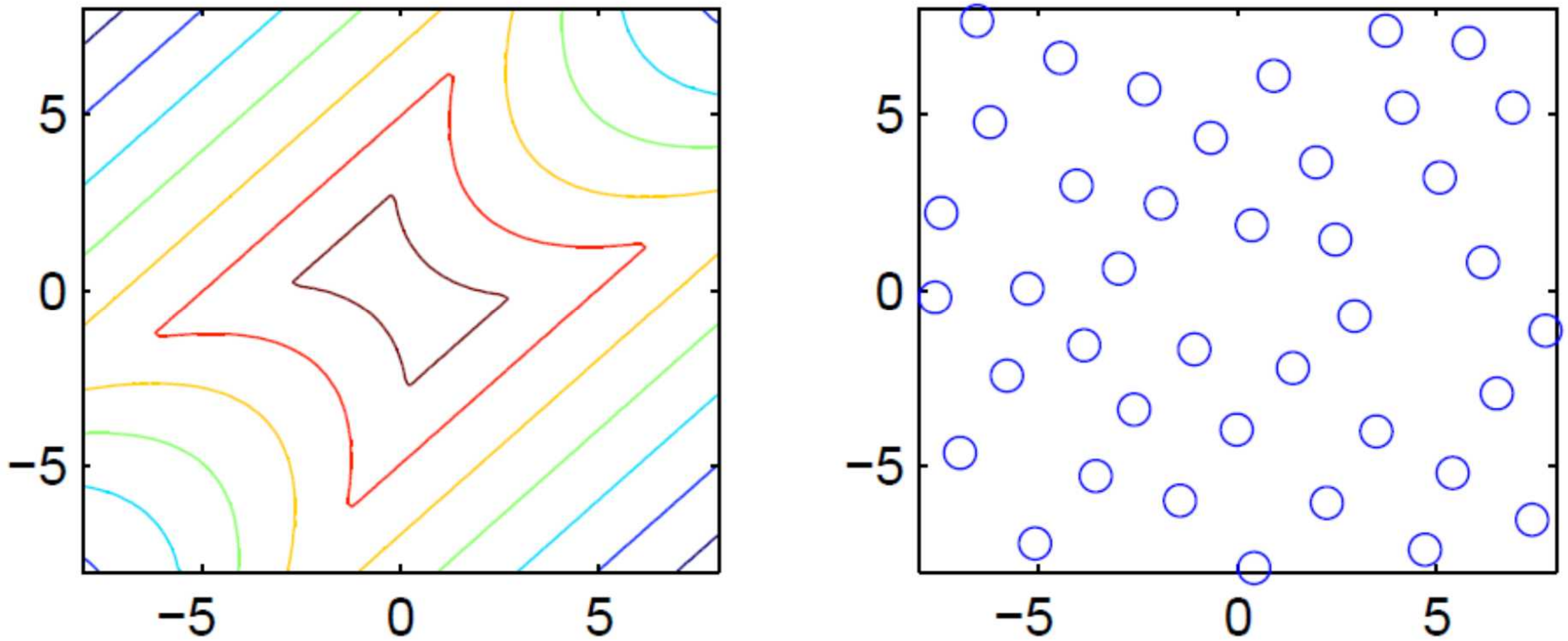
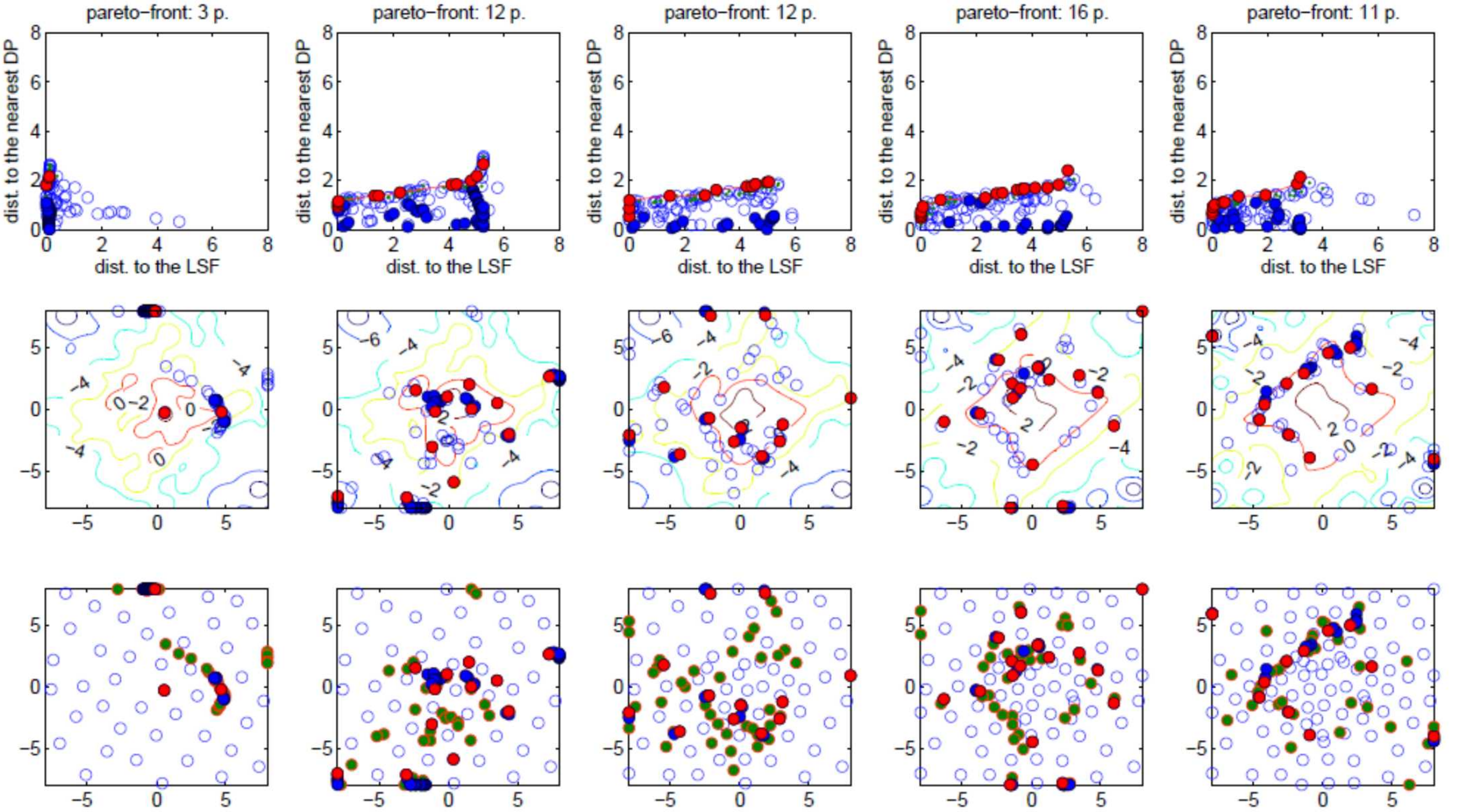
34.00
23.60
10.83
3.77
2.13
1.43
Price
Deflection
0.0160
0.0140
0.0120
0.0100
0.0080
0.0060
0.0040
0.0020
0.0000

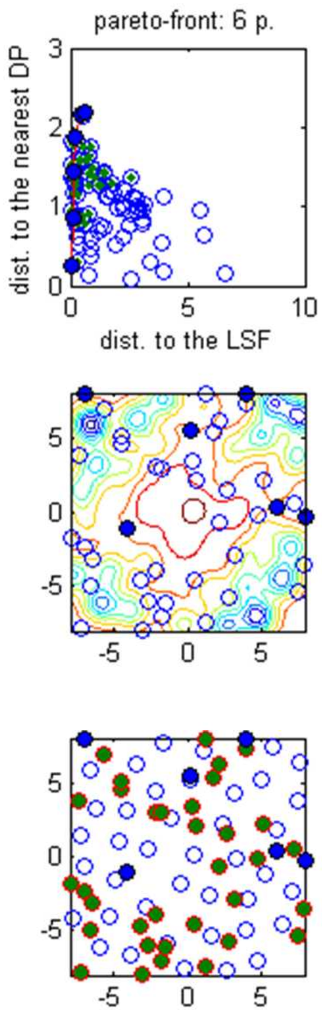
Example 2:
Adaptive update of meta-model



Contours of the example (left) and starting DoE (right). Note that the red contour is for F(x) = 0.



Pareto front (top), contours of the problem with DoEs (middle) and DoEs' points (bottom).
Key: Red – added and computed solutions, Blue – points that were too close to other Pareto front points, Green – the remaining points of population and Blue empty points – the original DoE.



Pareto front (top), contours of the problem with DoEs (middle) and DoEs' points (bottom).
Key: Red – added and computed solutions, Blue – points that were too close to other Pareto front points, Green – the remaining points of population and Blue empty points – the original DoE.

Reference

[1] Zitzler, E. (1999). Evolutionary Algorithms for Multiobjective Optimization: Methods and Applications. PhD thesis, Swiss Federal Institute of Technology (ETH), Zurich, Switzerland.

[2] Zitzler, E., Laumanns, M., and Thiele, L. (2001). SPEA2: Improving the Strength Pareto Evolutionary Algorithm. In Giannakoglou, K., Tsahalis, D., Periaux, J., Papailou, P., and Fogarty, T., editors, EUROGEN 2001. Evolutionary Methods for Design, Optimization and Control with Applications to Industrial Problems, Athens, Greece.

[3] Miettinen, K. (1999). Nonlinear Multiobjective Optimization. Kluwer Academic Publishers, Dordrecht.

Reference

[4] Knowles, J. D. and Corne, D.W. (2000). Approximating the Nondominated Front Using the Pareto Archived Evolution Strategy. Evolutionary Computation, 8(2):149–172.

[5] Kukkonen, S. and Lampinen, J. (2004). Comparison of generalized differential evolution to other multi-objective evolutionary algorithms. In European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2004).

[6] Coello, C. A. C. (2004). List of references on evolutionary multiobjective optimization. http://www.lania.mx/~ccoello/EMOO/EMOObib.html.

[7] Lepš, M. (2005). Single and Multi-Objective Optimization in Civil Engineering with Applications, PhD thesis, CTU in Prague.

[8] NIMBUS: https://wwwnimbus.it.jyu.fi/

Při přípravě této přednášky byla použita řada materiálů laskavě poskytnutých Ing. Adélou Pospíšilovou ze Stavební fakulty ČVUT.

Prosba. V případě, že v textu objevíte nějakou chybu nebo budete mít námět na jeho vylepšení, ozvěte se prosím na **`matej.leps@fsv.cvut.cz`**.

Oprava 18.11.2009: Přidán slide na Average Ranking, NSGA II a PAES

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