Modern Methods of Optimization

Lecture 2: Portfolio management example

Faculty of Civil Engineering / CTU in Prague

Portfolio and risk management

Objective invest your money with the *minimal risk*

Constraint achieve at least the given *profit*

Stock	Price [CZK]				
	2001	2002	2003	2004	...
Commercial Bank (KB ČR)	1000	2000	2250	3100	...
ČEZ group	850	990	140	330	...
Unipetrol group	45	40	60	80	...
Philip Morris ČR	8,000	11,000	15,000	17,500	...

- Data source files:
  `https://gitlab.com/jan.zeman4/132mmo` > codes
- Run MATLAB
- Start new script

Input data

- Values $V_{i,j}$ – $n \times m$ matrix (number of years $\times$ number of products)
- Minimum required profit p

```
V=load('shares_history.dat');
[n,m]=size(V);
p=3;

plot(V,'o');
legend('KOMB','CEZ','UNIPE','TABAK');
xlabel('Year');
ylabel('Value');
```

Modeling: Profit

- Annual return – $(n-1) \times m$ matrix

$$R_{ij} = \frac{V_{i+1,j} - V_{i,j}}{V_{i,j}} \times 100$$

- Expected Annual return

$$ER_j = \frac{1}{n-1} \sum_{i=1}^{n-1} R_{ij}$$

```
R=zeros(n-1,m);
for␣i=1:n-1
␣␣␣␣R(i,:)=(V(i+1,:)-V(i,:))./V(i,:)*100;
end
ER=mean(R);
```

Modeling profit (constraint)

```
plot(R,'*');
legend('KOMB','CEZ','UNIPE','TABAK');
xlabel('Annual␣return');
ylabel('Value');
```

- Expected profit must exceed the minimum profit p

$$\sum_{j=1}^{m} ER_j x_j \geq p,$$

- x_j: relative amount of investment

$$x_j \geq 0 \qquad \sum_{j=1}^{m} x_j = 1$$

Modeling risk

- Risk of a single investment into j-th stock

$$\approx x_j \text{ Variance}_j = \frac{1}{n-2}\sum_{i=1}^{n-1}(R_{ij} - ER_j)^2$$

- Risk of simultanous investment into j-th and k-th stocks

$$\approx x_j x_k \text{ CoVariance}_{jk} = \frac{1}{n-2}\sum_{i=1}^{n-1}(R_{ij} - ER_j)(R_{ik} - ER_k)$$

- Total risk (of investment)

$$\sim \sum_{j=1}^{m}\sum_{k=1}^{m} C_{jk} x_j x_k$$

```
C=cov(R);
```

Problem formulation

Objective invest your money with the *minimal risk*

$$\min \sum_{j=1}^{m} \sum_{k=1}^{m} C_{jk} x_j x_k$$

Constraint achieve at least the given *profit* p

$$\sum_{j=1}^{m} x_j ER_j \geq p$$

$$x_j \geq 0$$

$$\sum_{j=1}^{m} x_j = 1$$

Problem formulation with matrices

- Quadratic programming problem (`doc quadprog`)

Quadprog function

```
x = quadprog(H,f,A,b,Aeq,beq,lb,ub)
```

Finds a minimum for a problem specified by

$$\min \frac{1}{2} x^T H x + f^T x$$

$$\text{such that } Ax \le b$$

$$Aeq\, x = beq$$

$$lb \le x \le ub$$

$H =$

$f =$

$A =$

$b =$

$Aeq =$

$beq =$

$lb =$

$ub =$

Solving the problem

```
x=quadprog(C,zeros(m,1),-ER,-p,ones(1,m),1,zeros,[]);

x
ER*x
```

- How good is the model?

```
Vreal=load('shares_real.dat');

Rreal=(Vreal-V(n,:))./V(n,:)*100;
Rreal*x
```